5 Algebraic Expressions and Properties

5.1 Algebraic Expressions

5.2 Writing Expressions

5.3 Properties of Addition and Multiplication

5.4 The Distributive Property

5.5 Factoring Expressions

Chapter Learning Target:
Understand algebraic expressions.

Chapter Success Criteria:

- I can identify parts of an algebraic expression.
- I can write algebraic expressions.
- I can solve a problem using algebraic expressions.
- I can interpret algebraic expressions in real-life problems.

Laurie's Notes

Chapter 5 Overview

This chapter is a continuation of the algebra strand that students explored in prior courses. Students will now extend this understanding to include: writing and evaluating algebraic expressions, using properties with algebraic expressions, and factoring expressions.

Students used variables in prior courses, often in the context of finding the area or the perimeter of a geometric figure. Formulas were written as verbal models and then variables were introduced. For example:

Area of rectangle = length times width
Area of rectangle = length $\times$ width
$A = \ell \times w$

Keep in mind that the different notations used to represent operations, particularly multiplication and division, are not understood by all students. In the first lesson, where algebraic expressions are introduced, take time to review the different representations. For example, in prior courses 3__ or 3 ▢ meant thirty-something. In algebra, $3x$ means 3 times a quantity called x.

A major goal for this chapter is that students become confident in writing and evaluating algebraic expressions. The order of operations, including exponents and grouping symbols, is extended to algebraic expressions.

In prior courses, students were introduced to the Commutative and Associative Properties. They may forget, or mix up, the names of these two properties, but students generally have a good sense of how the properties apply to numbers. Students should also be able to provide examples of why the Commutative and Associative Properties do not apply to subtraction and division. The Addition Property of Zero and the Multiplication Properties of Zero and One are also presented in this chapter. All the properties are shown with words, numbers, and variables.

Students have some experience using the Distributive Property with numerical expressions. A common misconception students often have is that the Distributive Property is *only* about multiplying. They see the equal sign as an arrow. For example, $3(x + 7) \rightarrow 3x + 21$. Students may not recognize that factoring is represented in the Distributive Property as well. For example, $3x + 21 = 3(x + 7)$. The last two lessons of the chapter clarify and connect these ideas. In Section 5.4, when students are working on the exploration, probe their understanding of what the equal sign implies. For example, $3(x + 4) = 3x + 12$ and $3x + 12 = 3(x + 4)$.

Suggested Pacing

Chapter Opener	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	2 Days
Section 4	2 Days
Section 5	3 Days
Connecting Concepts	1 Day
Chapter Review	1 Day
Chapter Test	1 Day
Total Chapter 5	15 Days
Year-to-Date	78 Days

Chapter Learning Target

Understand algebraic expressions.

Chapter Success Criteria

- Identify parts of an algebraic expression.
- Write algebraic expressions.
- Solve a problem using algebraic expressions.
- Interpret algebraic expressions in real-life problems.

Chapter 5 Learning Targets and Success Criteria

Section	Learning Target	Success Criteria
5.1 Algebraic Expressions	Evaluate algebraic expressions given values of their variables.	• Identify parts of an algebraic expression. • Evaluate algebraic expressions with one or more variables. • Evaluate algebraic expressions with one or more operations.
5.2 Writing Expressions	Write algebraic expressions and solve problems involving algebraic expressions.	• Write numerical expressions. • Write algebraic expressions. • Write and evaluate algebraic expressions that represent real-life problems.
5.3 Properties of Addition and Multiplication	Identify equivalent expressions and apply properties to generate equivalent expressions.	• Explain the meaning of equivalent expressions. • Use properties of addition to generate equivalent expressions. • Use properties of multiplication to generate equivalent expressions.
5.4 The Distributive Property	Apply the Distributive Property to generate equivalent expressions.	• Explain how to apply the Distributive Property. • Use the Distributive Property to simplify algebraic expressions. • Use the Distributive Property to combine like terms.
5.5 Factoring Expressions	Factor numerical and algebraic expressions.	• Use the Distributive Property to factor numerical expressions. • Identify the greatest common factor of terms including variables. • Use the Distributive Property to factor algebraic expressions. • Interpret factored expressions in real-life problems.

Progressions

Through the Grades		
Grade 5	**Grade 6**	**Grade 7**
• Use parentheses, brackets, or braces in numerical expressions. • Write and interpret numerical expressions.	• Use the distributive property to factor algebraic expressions. • Write and evaluate algebraic expressions. • Apply the properties of operations to show expressions are equivalent.	• Add, subtract, factor, and expand linear expressions with rational coefficients. • Understand that rewriting expressions in different forms can show how the quantities are related.

Through the Chapter					
Standard	**5.1**	**5.2**	**5.3**	**5.4**	**5.5**
6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.					★
6.EE.A.2a Write expressions that record operations with numbers and with letters standing for numbers.		★			
6.EE.A.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.	●			●	★
6.EE.A.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).	●				
6.EE.A.3 Apply the properties of operations to generate equivalent expressions.			●	●	★
6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).			●	●	★

Key

▲ = preparing ★ = complete

● = learning ■ = extending

STEAM Video

1. *Sample answer:* so that the Sun is at the same angle and the time between measurements is the same
2. *Sample answer:* 10.5 in.; 10.5 is halfway between 7 and 14.
3. *Sample answer:* 30 in.; 38 in.; 47 in.

Performance Task

Sample answer: no; The pattern may not continue long-term.

Laurie's Notes

STEAM Video

Before the Video

- To introduce the STEAM Video, read aloud the first paragraph of Shadow Drawings and discuss the question with your students.

? "Can you think of any other real-life situations in which you would want to use an expression to represent a changing quantity?"

During the Video

- The video shows shadow drawings of a plant after 1, 2, and 3 weeks of growth.

? Pause the video at 1:25 and ask, "What has happened in the video so far?"
Tory is making a shadow drawing of a plant she got three weeks before. Each week she made a shadow drawing on the same day at the same time. Robert suggests measuring the drawings to predict the growth rate of the plant.

? "How can measuring the drawings help predict the growth rate of the plant?"
Sample answer: Measuring the height each week may show a pattern in the amount of growth in one week.

- Watch the remainder of the video.

After the Video

- Have students work with a partner to answer Questions 1–3.
- As students discuss and answer the questions, listen for understanding and knowledge of writing and evaluating algebraic expressions.

Performance Task

- Use this information to spark students' interest and promote thinking about real-life problems.

? Ask, "Do the expressions provide accurate predictions far into the future?"

- After completing the chapter, students will have gained the knowledge needed to complete "Describing Change."

STEAM Video

Shadow Drawings

Expressions can be used to represent the growth of living things over time. Can you think of any other real-life situations in which you would want to use an expression to represent a changing quantity?

Watch the STEAM Video "Shadow Drawings." Then answer the following questions.

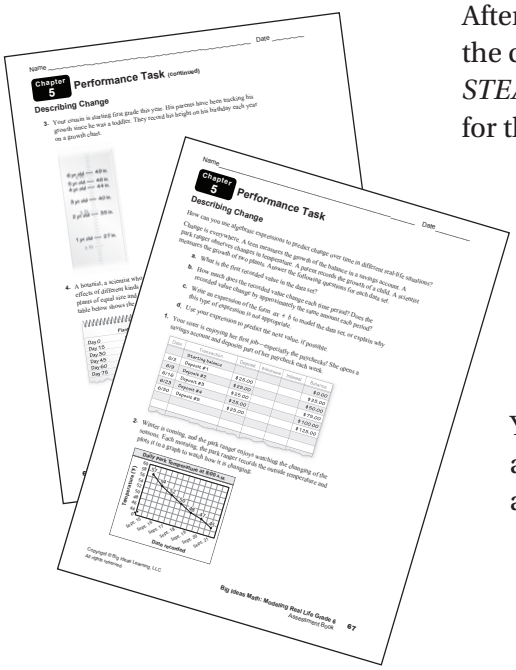
1. Tory traces the shadow of a plant each week on the same day of the week and at the same time of day. Why does she need to be so careful about the timing of the drawing?

2. The table shows the height of the plant each week for the first three weeks. About how tall was the plant after 1.5 weeks? Explain your reasoning.

Week	1	2	3
Height (inches)	7	14	22

3. Predict the height of the plant when Tory makes her next three weekly drawings.

Performance Task


Name _____ Date _____

Chapter 5 Performance Task (continued)

Describing Change

Name _____ Date _____

Chapter 5 Performance Task

Describing Change

How can you use algebraic expressions to predict change over time in different real-life situations?

Big Ideas Math: Modeling Real Life, Grade 6 Assessment Book 67

Describing Change

After completing this chapter, you will be able to use the concepts you learned to answer the questions in the *STEAM Video Performance Task*. You will be given data sets for the following real-life situations.

Savings account

Temperature

Human growth

Plant growth

You will be asked to use given data to write expressions and make predictions. Do the expressions provide accurate predictions far into the future?

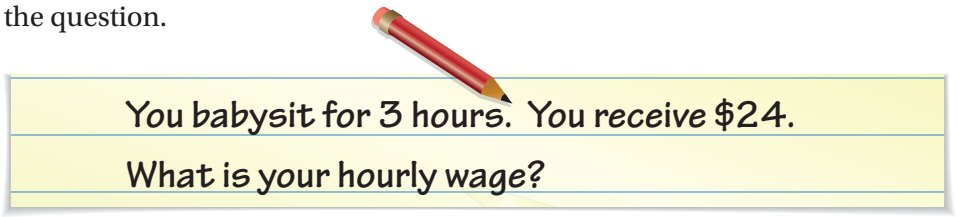
Getting Ready for Chapter 5

Chapter Exploration

1. Work with a partner.

a. You babysit for 3 hours. You receive $24. What is your hourly wage?

- Write the problem. Underline the important numbers and units you need to solve the problem.
- Read the problem carefully a second time. Circle the key phrase for the question.



- Write each important number or phrase, with its units, on a piece of paper. Write $+$, $-$, $\times$, $\div$, and $=$ on five other pieces of paper.

- Arrange the pieces of paper to answer the question, "What is your hourly wage?"
- Evaluate the expression that represents the hourly wage.

hourly wage = ☐ ÷ ☐ Write.

= ☐ Evaluate.

So, your hourly wage is $ ☐ per hour.

b. How can you use your hourly wage to find how much you will receive for any number of hours worked?

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

algebraic expression
variable
constant
equivalent expressions
factoring an expression

Laurie's Notes

Check out the digital flash cards.
BigIdeasMath.com

Chapter Exploration

- Ask students to name jobs they have had where they earned money. Students may say babysitting, delivering newspapers, odd jobs for neighbors, grocery shopping for an elderly person, or dog walking.
- Discuss the difference between being paid by the hour and being paid for completing a job. "What are the advantages and disadvantages of each method of payment?" Listen for responses that include *hours worked* compared to *hours paid*.
- Manipulating the pieces of paper allows students to make sense of writing a mathematical model.
- **Common Question:** Students may ask, "Why go through all this work when I already knew the answer?" Tell them that you are modeling a process: read and re-read, underline important numbers and units, circle what you are trying to solve for, and then write an expression.
- **Another Method:** You can also set up the problem as: \$24 = hourly wage (\$ per hour) × 3 hours. Although this is a true statement, it is helpful to have all of the computations on one side of the equation. Let students know that they will learn how to solve equations similar to this at a later time, when the answer may not be so obvious.

ELL Support

Explain that it is not uncommon for American teenagers to earn money by taking care of younger children. An adult may ask a teenager to care for their children so they can go out for an evening. This is known as *babysitting*. ELLs may come from cultures in which childcare is a communal activity that does not involve financial compensation. Explain that the phrase *hourly wage* describes the amount of money a person is paid for every hour worked. In this situation, the hourly wage describes the amount of money you will be paid for every hour you care for children.

Vocabulary

- These terms represent some of the vocabulary that students will encounter in Chapter 5. Discuss the terms as a class.
- Where have students heard the word *constant* outside of a math classroom? In what contexts? Students may not be able to write the actual definition, but they may write phrases associated with constant.
- Allowing students to discuss these terms now will prepare them for understanding the terms as they are presented in the chapter.
- When students encounter a new definition, encourage them to write in their *Student Journals*. They will revisit these definitions during the Chapter Review.

Topics for Review

- Factors of Whole Numbers
- Greatest Common Factor
- Interpreting Numerical Expressions
- Multiples of Whole Numbers
- Using Order of Operations

Chapter Exploration

1. a. Check that students have underlined 3 hours and \$12, circled hourly wage, written those values and the operations on pieces of paper, formed the correct equation and arrived at an answer of \$8.

b. Multiply \$8 by the number of hours worked.

Learning Target

Evaluate algebraic expressions given values of their variables.

Success Criteria

- Identify parts of an algebraic expression.
- Evaluate algebraic expressions with one or more variables.
- Evaluate algebraic expressions with one or more operations.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Clarify the meaning of the word *variable*. Explain that *to vary* is *to change*, so a variable is something that can change. For example, weather is variable. There may be variable cloudiness from day to day. In the context of math, a variable is a symbol that represents one or more numbers, such as x or y. When the value of a variable in an expression changes, the value of the expression may change as well.

Exploration 1

a. number of hours worked;
Sample answer: 4; $\$24 \div 4 = \$6$

b. price of each baking mold;
Sample answer: $\$4$; $5 \times \$4 = \$20$

c. total race distance; *Sample answer:* 5000 ft;
$5000 \text{ ft} - 2000 \text{ ft} = 3000 \text{ ft}$

d. number of months;
Sample answer: 5;
$25 \text{ cm} + 1.6 \times 5 = 33 \text{ cm}$

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
6.EE.A.2b, 6.EE.A.2c

Preparing to Teach

- In prior chapters, students performed operations with different representations of numbers and compared their magnitudes. As students continue to problem-solve, they will need to consider both numerical and algebraic expressions.
- Many students struggle with solving word problems, but not because the mathematics is too difficult. They struggle because they do not know how to approach word problems. Help students hone their reading skills so that they can become better problem solvers.
- Several new vocabulary terms are introduced in this lesson. Consistently using precise mathematical language will help students master the first success criterion.

Motivate

- Ask six students to stand at the front of the room. Give each student an index card with a number written on the card (possible numbers: 1, 3, 4, 5, 8, 9). Write simple expressions on the board, drawing boxes where the **variables** would be. Tell the six students holding cards to place their cards in the boxes and announce the values of each expression for their numbers. Examples:

$\square + 14$ $\quad 27 - \square$ $\quad \square^2$ $\quad 6 \cdot \square$

- You can vary the expressions and numbers to fit the level of your class. The goal is for students to recognize that the value of the expression changes (or varies) for each number substituted.
- ? After several examples, erase the boxes and replace them with variables. Then ask, "What do the letters mean?" The letters (or variables) have the same meanings as the boxes. They are unknown values that represent numbers. "Does anyone know what these expressions are called?" **algebraic expressions**

Exploration 1

- This exploration requires a variety of different skills. First, students must ask, "What do I need to know?" This is a literacy skill. Do they understand what they need to know to answer the question in each part? Secondly, students must choose a reasonable value for the missing amount. Using that value, can they find a numerical solution?
- Work through part (a) as a class. Begin by asking what they need to know and then discuss a reasonable number of hours. Listen for suggestions of "easy" numbers (factors of 24).
- ◉ After students complete parts (b)–(d), ask pairs to share their solutions on the board. Point out the different values chosen to complete each expression. This is a preview of evaluating algebraic expressions.
- **MP6 Attend to Precision:** Students need opportunities to communicate using precise mathematical language. In the exploration, their language is something you want to pay attention to. Listen to how students refer to the quantities, particularly their references to units for each of the parts.

5.1 Algebraic Expressions

Learning Target: Evaluate algebraic expressions given values of their variables.

Success Criteria:
- I can identify parts of an algebraic expression.
- I can evaluate algebraic expressions with one or more variables.
- I can evaluate algebraic expressions with one or more operations.

EXPLORATION 1 Evaluating Expressions

Math Practice

Make Sense of Quantities

What are the units in the problem? How does this help you write an expression?

Work with a partner. Identify any missing information that is needed to answer each question. Then choose a reasonable quantity and write an expression for each problem. After you have written the expression, evaluate it using mental math or some other method.

a. You receive $24 for washing cars. How much do you earn per hour?

b. You buy 5 silicone baking molds at a craft store. How much do you spend?

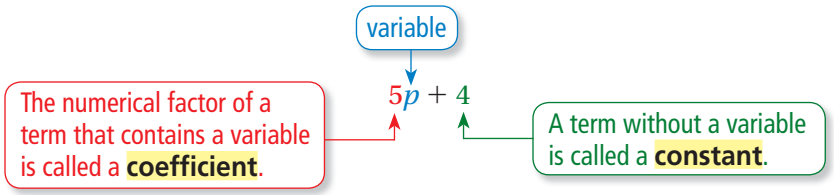
c. You are running in a mud race. How much farther do you have to go after running 2000 feet?

d. A rattlesnake is 25 centimeters long when it hatches. The snake grows at a rate of about 1.6 centimeters per month for several months. What is the length of the rattlesnake?

5.1 Lesson

Key Vocabulary
algebraic expression, *p. 202*
variable, *p. 202*
term, *p. 202*
coefficient, *p. 202*
constant, *p. 202*

An **algebraic expression** is an expression that may contain numbers, operations, and one or more *variables*. A **variable** is a symbol that represents one or more numbers. Each number or variable by itself, or product of numbers and variables in an algebraic expression, is called a **term**.



EXAMPLE 1 Identifying Parts of an Algebraic Expression

Identify the terms, coefficients, and constants in each expression.

a. $5x + 13$

$5x + 13$

Terms: $5x$, 13
Coefficient: 5
Constant: 13

b. $2z^2 + y + 3$

$2z^2 + y + 3$

Terms: $2z^2$, $1y$, 3
Coefficients: 2, 1
Constant: 3

A variable by itself has a coefficient of 1. So, the term y in Example 1(b) has a coefficient of 1.

Try It **Identify the terms, coefficients, and constants in the expression.**

1. $12 + 10c$ **2.** $15 + 3w + \frac{1}{2}$ **3.** $z^2 + 9z$

EXAMPLE 2 Writing Algebraic Expressions Using Exponents

Write each expression using exponents.

a. $d \cdot d \cdot d \cdot d$

Because d is used as a factor 4 times, its exponent is 4.

So, $d \cdot d \cdot d \cdot d = d^4$.

b. $1.5 \cdot h \cdot h \cdot h$

Because h is used as a factor 3 times, its exponent is 3.

So, $1.5 \cdot h \cdot h \cdot h = 1.5h^3$.

Try It **Write the expression using exponents.**

4. $j \cdot j \cdot j \cdot j \cdot j \cdot j$ **5.** $9 \cdot k \cdot k \cdot k \cdot k \cdot k$

Laurie's Notes

Scaffolding Instruction

- Going from words and sentences to numbers, variables, and operations emphasizes the efficiency of **algebraic expressions**.
- **Emerging:** Students may be able to identify the information in a real-life problem but struggle to interpret its meaning in relation to evaluating algebraic expressions. Examples 1–5 provide additional practice evaluating algebraic expressions before using them in real-life problems.
- **Proficient:** Students use precise mathematical language and evaluate algebraic expressions with ease. Have them self-assess using Try It Exercises 1–3 and 9–16.

EXAMPLE 1

- Write $5x + 13$. Discuss the words: **term**, **variable**, **coefficient**, and **constant**. Reassure students that these words will become familiar with use.

❓ "How many terms are there in $5x + 13$?" 2 "Name the terms." $5x$ and 13 Be sure students understand that in this expression 5 is not a term, however, $5x$ is a term.

- In working through part (b), ask students to identify the variables first.

❓ "What is the coefficient of the z^2 term?" 2 "What is the coefficient of y term?" 1 It's common for students to say that there isn't a coefficient of y, or mistakenly identify the coefficient as 0.

- Explain that $1y = y$. Tell students that the coefficient 1 is not usually written in front of the term.

Try It

◉ Have students work in pairs to complete the exercises. Then have students share their answers verbally or on the board. Use *Fist of Five* to check their understanding of the first success criterion.

EXAMPLE 2

❓ Write 3^2 and ask, "How do you read this?" 3 squared or 3 raised to the second power "What does the exponent mean?" how many times 3 is a factor

- This example connects to a prior lesson on powers and exponents, that is now extending to variables.
- In part (b), tell students that only the variable h is being raised to the power.

Try It

- **Neighbor Check:** Have students work independently, and then have their neighbors check their work. Have students discuss any discrepancies.

Extra Example 1

Identify the terms, coefficients, and constants in each expression.

a. $3a + 17$
Terms: $3a$, 17
Coefficient: 3
Constant: 17

b. $4x^2 + 5x + 7$
Terms: $4x^2$, $5x$, 7
Coefficients: 4, 5
Constant: 7

Try It

1. Terms: 12, $10c$
Coefficient: 10
Constant: 12

2. Terms: 15, $3w$, $\frac{1}{2}$
Coefficient: 3
Constants: 15, $\frac{1}{2}$

3. Terms: z^2, $9z$
Coefficients: 1, 9
Constant: none

Extra Example 2

Write each expression using exponents.

a. $x \cdot x \cdot x \cdot x \cdot x$ x^5

b. $7 \cdot d \cdot d \cdot d$ $7d^3$

Try It

4. j^6

5. $9k^5$

Extra Example 3

a. Evaluate $15 - y$ when $y = 3$. 12

b. Evaluate $24 \div w$ when $w = 4$. 6

Try It

6. 33

7. 13

8. 36

Extra Example 4

a. Evaluate $f + g$ when $f = 11$ and $g = 7$. 18

b. Evaluate $j \cdot k$ when $j = 12$ and $k = \frac{3}{4}$. 9

ELL Support

Allow students to work in groups to complete Try It Exercises 9–12. Circulate and listen to discussions. Expect students to perform according to their different language levels.

Beginner: Write the expression using the values of the variables and evaluate.

Intermediate: State the answer using a complete sentence. For example, "When p is twenty-four and q is eight, p divided by q is three."

Advanced: Explain the process.

Try It

9. 3

10. 32

11. 16

12. 192

Laurie's Notes

EXAMPLE 3

- Work through the example as a class. Use color-coding to help students recognize that substitution has occurred.
- There is only one operation in each part, so the order of operations is not necessary.
- Discuss the push-pin note. Students should be comfortable seeing multiplication represented in each of the three forms.

? "Can you use all three forms to represent multiplication of two numbers? Explain." No, if the numbers are 3 and 4, $3 \cdot 4$ and $3(4)$ are okay, but writing 34 is not.

- **Note:** Now that variables have been introduced, multiplication will not be represented with $\times$ when variables are present. This is to avoid confusion with the variable x.

Try It

- Have students work independently on the exercises and then compare their answers with a neighbor.
- Listen to their answers for Exercise 8. When dividing by $\frac{1}{2}$, did they multiply by the reciprocal or use a different strategy?

EXAMPLE 4

- Encourage students to use color-coding to differentiate between the two values being substituted.
- Check that students replace each variable with the correct value. Tell them to be careful when the values of the variables are not written in the same order as the variables in the expression.

? In part (b), "How do you divide a whole number by a fraction?" Multiply by the reciprocal of the fraction.

? "What strategies can you use to multiply 16 by $\frac{3}{2}$?" Listen for a variety of strategies. For example, students may say, "Multiply 16 by 3 to get 48 and then divide 48 by 2," or "$\frac{3}{2}$ is $1\frac{1}{2}$ and half of 16 is 8, so add 8 to 16."

- **MP2 Reason Abstractly and Quantitatively:** Allowing students to share different strategies of performing a computation helps them to develop computational fluency and the ability to reason quantitatively.

Try It

- Have students work in pairs to complete the exercises. Then have students share their answers verbally or on the board. Check students' work.

? "In Exercise 12, what other ways can you write $p \cdot q$?" pq or $(p)(q)$

◉ Students are continuing to work on the second success criterion.

To evaluate an algebraic expression, substitute a number for each variable. Then use the order of operations to find the value of the numerical expression.

EXAMPLE 3 Evaluating Algebraic Expressions

a. Evaluate $k + 10$ when $k = 25$.

$k + 10 = 25 + 10$ Substitute 25 for k.

$= 35$ Add 25 and 10.

b. Evaluate $4 \cdot n$ when $n = 12$.

$4 \cdot n = 4 \cdot 12$ Substitute 12 for n.

$= 48$ Multiply 4 and 12.

You can write the product of 4 and n in several ways.

$4 \cdot n$

$4n$

$4(n)$

Try It

6. Evaluate $24 + c$ when $c = 9$.

7. Evaluate $d - 17$ when $d = 30$.

8. Evaluate $18 \div q$ when $q = \frac{1}{2}$.

EXAMPLE 4 Evaluating an Expression with Two Variables

a. Evaluate $n - m$ when $m = 12$ and $n = 30$.

$n - m = 30 - 12$ Substitute 30 for n and 12 for m.

$= 18$ Subtract 12 from 30.

b. Evaluate $a \div b$ when $a = 16$ and $b = \frac{2}{3}$.

$a \div b = 16 \div \frac{2}{3}$ Substitute 16 for a and $\frac{2}{3}$ for b.

$= 16 \cdot \frac{3}{2}$ Multiply by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$.

$= 24$ Multiply.

Try It

Evaluate the expression when $p = 24$ and $q = 8$.

9. $p \div q$

10. $q + p$

11. $p - q$

12. $p \cdot q$

EXAMPLE 5 Evaluating Expressions with Two Operations

a. Evaluate $3x - 14$ when $x = 5$.

$3x - 14 = 3(5) - 14$ Substitute 5 for x.

$= 15 - 14$ Using order of operations, multiply 3 and 5.

$= 1$ Subtract 14 from 15.

b. Evaluate $n^2 + 8.5$ when $n = 2$.

$n^2 + 8.5 = 2^2 + 8.5$ Substitute 2 for n.

$= 4 + 8.5$ Using order of operations, evaluate 2^2.

$= 12.5$ Add 4 and 8.5.

Try It **Evaluate the expression when $y = 6$.**

13. $5y + 1$

14. $30 - 24 \div y$

15. $y^2 - 7$

16. $1.5 + y^2$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$2x + 1$ $5w \cdot c$ $3(4) + 5$ $2y \cdot z$

18. ALGEBRAIC EXPRESSIONS Identify the terms, coefficients, and constants in the expression $9h + 1$.

EVALUATING EXPRESSIONS **Evaluate the expression when $m = 8$.**

19. $m - 7$

20. $5m + 4$

21. MP NUMBER SENSE Does the value of the expression $20 - x$ *increase*, *decrease*, or *stay the same* as x increases? Explain.

22. OPEN-ENDED Write an algebraic expression using more than one operation. When you evaluate the expression, how do you know which operation to perform first?

23. MP STRUCTURE Is the expression $8.2 \div m \cdot m \cdot m \cdot m$ the same as the expression $8.2 \div m^4$? Explain your reasoning.

Laurie's Notes

EXAMPLE 5

- Students are asked to evaluate expressions involving more than one operation. They need to remember the order of operations.
- Write the expressions and ask, "How many operations will be performed in part (a)? in part (b)?" 2; 2
- "In part (a), which operation should you perform first?" multiplication "In part (b), which operation should you perform first?" evaluate the exponent
- These questions usually elicit a comment about the order of operations, at which time you can probe for understanding.
- Make students aware that they are working on the third success criterion, so that they can assess their understanding in the Self-Assessment for Concepts & Skills.

Try It

- Have students complete the exercises independently and then display their answers on whiteboards. Check students' work and look for errors involving the order of operations.

Self-Assessment for Concepts & Skills

- **MP6 Attend to Precision:** Have students work in groups to complete Exercise 17. This exercise should lead to a rich discussion that includes vocabulary, such as *terms*, *constants*, *coefficients*, *variables*, *expressions*, and *operations*. Students should share their reasoning using precise mathematical language.
- Have students complete Exercises 18–23 independently.
- In Exercises 22 and 23, check their level of understanding of the second and third success criteria.

ELL Support

Allow students to practice language by working in pairs. Have two pairs compare their answers. If there is a disagreement, pairs should work together to reach a consensus. Check comprehension of Exercises 17–19 by having each group write their final answers on a whiteboard. Discuss Exercises 21–23 with students to check for understanding.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Extra Example 5

a. Evaluate $\frac{t}{5} + 6$ when $t = 45$. 15

b. Evaluate $v^2 - 4$ when $v = 9$. 77

Try It

13. 31

14. 26

15. 29

16. 37.5

Self-Assessment for Concepts & Skills

17. $3(4) + 5$; the other three are algebraic expressions.

18. Terms: $9h$, 1
Coefficient: 9
Constant: 1

19. 1 **20.** 44

21. decrease; When you subtract greater and greater values from 20, you will have less and less left.

22. *Sample answer:* $5x + 4$; Use the order of operations.

23. no;
$8.2 \div m \cdot m \cdot m \cdot m$
$= \left(\frac{8.2}{m}\right) \cdot m^3$
and $8.2 \div m^4 = \frac{8.2}{m^4}$

Extra Example 6

You are saving money to buy a video game that costs $50. Your uncle gives you $20 and you save $4 each week. The expression $20 + 4w$ gives the amount of money you save after w weeks. Can you buy the video game after 8 weeks? yes

Self-Assessment *for Problem Solving*

24. yes

25. yes; You need $63 to buy the jacket and have $66.

Formative Assessment Tip

Misconception Check

This technique gives students the opportunity to think about their own understanding of a concept or process. Write a worked-out problem on the board that demonstrates a common misconception, a mistake that students often make about a concept or process. Ask students if they agree or disagree with the solution and to explain why. Allow time for students to think about the problem independently and write an explanation. Then ask volunteers to share their explanations with the class. Listening to the thinking of others may solidify or modify their own beliefs.

Learning Target

Evaluate algebraic expressions given values of their variables.

Success Criteria

- Identify parts of an algebraic expression.
- Evaluate algebraic expressions with one or more variables.
- Evaluate algebraic expressions with one or more operations.

Laurie's Notes

EXAMPLE 6

- You may want to begin by displaying the first two sentences only. Then ask students to write an expression that represents the situation.
- **Think-Pair-Share:** Allow time for students to read the problem independently and consider the next step. Then have students work in pairs to discuss the problem, make a plan, use the plan to solve, and check their solutions. Have each pair compare their answer with another pair, and then explain their method. Ask volunteers to share their methods with the class.
- Discuss the Another Method note. Encourage students to think about how this strategy is similar to their own and why the strategy makes sense. You want students to see and appreciate different methods of solving a problem. Do not be surprised if some students substitute values for the variable until they get to, or exceed, $125. The guess-and-check method is valid but not always efficient.

Self-Assessment *for Problem Solving*

- The goal for all students is to feel comfortable with the problem-solving plan. It is important for students to problem-solve in class, where they may receive support from you and their peers. Keep in mind that some students may only be ready to complete the first step.
- Have students work on the exercises independently. Each exercise requires students to read the problem, comprehend the situation, and determine what is being asked.
- Ask volunteers to share their strategies and allow others to ask questions for clarification.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Closure

Misconception Check: "Do you agree or disagree with this solution? Explain."
Evaluate $5n - 3m$ when $m = 4$ and $n = 6$.

$$\begin{aligned} 5n - 3m &= 5(4) - 3(6) \\ &= 20 - 18 \\ &= 2 \end{aligned}$$

Sample answer: disagree; The wrong values were substituted for the variables.

EXAMPLE 6 Modeling Real Life

You are saving to buy a meteorite fragment for $125. You begin with $45 and you save $3 each week. The expression $45 + 3w$ gives the amount of money you save after w weeks. Can you buy the meteorite after 20 weeks?

Understand the problem.

You are given an expression that represents your savings after w weeks. You are asked whether you have enough money to buy a $125 meteorite after 20 weeks.

Make a plan.

To find the amount of money you save after 20 weeks, evaluate the expression when $w = 20$. Then compare the value of the expression to the price of the meteorite.

Solve and check.

$$45 + 3w = 45 + 3(20) \quad \text{Substitute 20 for } w.$$
$$= 45 + 60 \quad \text{Multiply 3 and 20.}$$
$$= 105 \quad \text{Add 45 and 60.}$$

You cannot buy the $125 meteorite after 20 weeks because you only have $105.

Another Method You start with $45, so you need to save another $125 - 45 = \$80$. At $3 per week, it will take you $\frac{80}{3} \approx 27$ weeks of saving.

$$45 + 3(27) = 45 + 81 = \$126 \checkmark$$

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

24. The expression $12.25m + 29.99$ gives the cost (in dollars) of a gym membership for m months. You have $180 to spend on a membership. Can you buy a one-year membership?

25. DIG DEEPER! The expression $p - 15$ gives the amount (in dollars) you pay after using the coupon when the original amount of a purchase is p dollars. The expression $30 + 6n$ gives the amount of money (in dollars) you save after n weeks. A jacket costs $78. Can you buy the jacket after 6 weeks? Explain.

Coupon

Good for $15 off any purchase of $75 or more

5.1 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

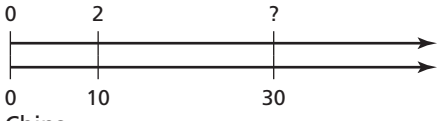
You ask 40 students which of three items from the cafeteria they like the best. You record the results on the piece of paper shown.

What is your favorite cafeteria food?
Pizza: 30%
Pasta: 18 students
Salad: 10 students

1. What percent of students answered salad?
2. How many students answered pizza?
3. What percent of students answered pasta?

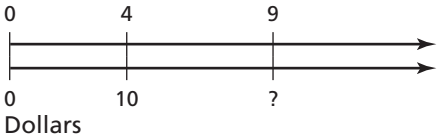
Find the missing quantity in the double number line.

4. Fiber (grams)



Chips

5. Pastries



Dollars

Divide. Write the answer in simplest form.

6. $1\frac{3}{8} \div \frac{3}{4}$
7. $2\frac{7}{9} \div 2$
8. $4 \div 4\frac{2}{5}$
9. $3\frac{2}{3} \div 1\frac{2}{7}$

Concepts, Skills, & Problem Solving

EVALUATING EXPRESSIONS **Write and evaluate an expression for the problem.** (See Exploration 1, p. 201.)

10. The scores on your first two history tests are 82 and 95. By how many points did you improve on your second test?
11. You buy a hat for $12 and give the cashier a $20 bill. How much change do you receive?
12. You receive $8 for raking leaves for 2 hours. What is your hourly wage?
13. Music lessons cost $20 per week. How much do 6 weeks of lessons cost?

ALGEBRAIC EXPRESSIONS **Identify the terms, coefficients, and constants in the expression.**

14. $7h + 3$
15. $g + 12 + 9g$
16. $5c^2 + 7d$
17. $2m^2 + 15 + 2p^2$
18. $6 + n^2 + \frac{1}{2}d$
19. $8x + \frac{x^2}{3}$

Terms: $2, x^2, y$
Coefficient: 2
Constant: none

20. **YOU BE THE TEACHER** Your friend finds the terms, coefficients, and constants in the algebraic expression $2x^2y$. Is your friend correct? Explain your reasoning.

Assignment Guide and Concept Check

Check out the Dynamic Assessment System.
BigIdeasMath.com

Level	Assignment 1	Assignment 2
Emerging	2, 3, 5, 9, 12, 14, 15, 22, 25, 26, 31, 33, 37, 38, 51	16, 18, 20, 27, 28, 47, 48, 55, 57, 60, 61
Proficient	2, 3, 5, 9, 12, 15, 17, 23, 24, 26, 33, 37, 39, 44, 51	19, 20, 27, 28, 29, 49, 50, 53, 56, 58, 60, 61, 62
Advanced	2, 3, 5, 9, 12, 18, 19, 27, 35, 39, 44, 45, 55	20, 28, 30, 49, 50, 53, 58, 60, 61, 62, 64, 65

- Assignment 1 is for use after students complete the Self-Assessment for Concepts & Skills.
- Assignment 2 is for use after students complete the Self-Assessment for Problem Solving.
- The red exercises can be used as a concept check.

Review & Refresh Prior Skills

Exercises 1 and 3 Finding Percents
Exercise 2 Finding the Percent of a Number
Exercises 4 and 5 Using a Double Number Line
Exercises 6–9 Dividing with Mixed Numbers

Common Errors

- **Exercises 14–19** Students may not list all of the coefficients of the variable terms. Remind them that a variable term such as g has a coefficient of 1, and that coefficients may also be fractions as in Exercises 18 and 19.

Review & Refresh

1. 25%
2. 12
3. 45%
4. 6
5. 22.5
6. $1\frac{5}{6}$
7. $1\frac{7}{18}$
8. $\frac{10}{11}$
9. $2\frac{23}{27}$

Concepts, Skills, & Problem Solving

10. $95 - 82$; 13
11. $20 - 12$; \$8
12. $8 \div 2$; \$4
13. 20×6; \$120
14. Terms: $7h$, 3
    Coefficient: 7
    Constant: 3
15. Terms: g, 12, $9g$
    Coefficients: 1, 9
    Constant: 12
16. Terms: $5c^2$, $7d$
    Coefficients: 5, 7
    Constant: none
17. Terms: $2m^2$, 15, $2p^2$
    Coefficients: 2, 2
    Constant: 15
18. Terms: 6, n^2, $\frac{1}{2}d$
    Coefficients: 1, $\frac{1}{2}$
    Constant: 6
19. Terms: $8x$, $\frac{x^2}{3}$
    Coefficients: 8, $\frac{1}{3}$
    Constant: none
20. no; The only term is $2x^2y$.

Concepts, Skills, & Problem Solving

21. a. Terms: 2ℓ, $2w$
Coefficients: 2, 2
Constant: none

b. The coefficient 2 of ℓ represents that there are 2 lengths on the rectangle. The coefficient 2 of w represents that there are 2 widths on the rectangle.

22. b^3 **23.** g^5

24. $8w^4$ **25.** $5.2y^3$

26. a^2c^2 **27.** $2.1xz^4$

28. yes; There are 4 factors of n.

29. $(5d)^2$

30. $x^4 + x^3 + x^2 + x$

31. 9 **32.** 10

33. 11 **34.** 9

35. 10 **36.** 17

37. 6 **38.** 2

39. 5 **40.** 15

41. 9 **42.** 1

43. 4 **44.** 6

45. 24 **46.** 36

47. \$15; \$105

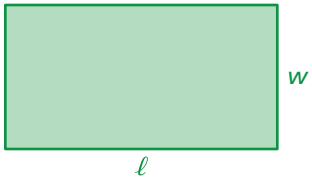
48. 24; 48; 72

49. 32; 16; 8

Common Errors

- **Exercises 24–27** Students may raise all the factors to the same power. Discuss the difference between $8w^4 = 8 \cdot w \cdot w \cdot w \cdot w$ and $(8w)^4 = 8w \cdot 8w \cdot 8w \cdot 8w$.
- **Exercises 31–46** Students may substitute the wrong value(s) for the variable(s). Tell students to write out the expression and then write the value(s) of the variable(s) underneath the variable(s) before substituting the value(s).

21. **PERIMETER** You can use the expression $2\ell + 2w$ to find the perimeter of a rectangle, where ℓ is the length and w is the width.



a. Identify the terms, coefficients, and constants in the expression.

b. Interpret the coefficients of the terms.

USING EXPONENTS **Write the expression using exponents.**

22. $b \cdot b \cdot b$

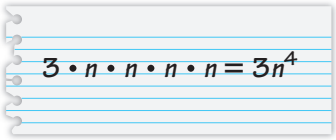
23. $g \cdot g \cdot g \cdot g \cdot g$

24. $8 \cdot w \cdot w \cdot w \cdot w$

25. $5.2 \cdot y \cdot y \cdot y$

26. $a \cdot a \cdot c \cdot c$

27. $2.1 \cdot x \cdot z \cdot z \cdot z \cdot z$



28. **YOU BE THE TEACHER** Your friend writes the product using exponents. Is your friend correct? Explain your reasoning.

29. **AREA** Write an expression using exponents that represents the area of the square.

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat had seven kits
Kits, cats, sacks, wives
How many were going to St. Ives?

30. MP **REASONING** Suppose the man in the St. Ives poem has x wives, each wife has x sacks, each sack has x cats, and each cat has x kits. Write an expression using exponents that represents the total number of kits, cats, sacks, and wives.

EVALUATING EXPRESSIONS **Evaluate the expression when $a = 3$, $b = 2$, and $c = 12$.**

31. $6 + a$

32. $b \cdot 5$

33. $c - 1$

34. $27 \div a$

35. $12 - b$

36. $c + 5$

37. $2a$

38. $c \div 6$

39. $a + b$

40. $c + a$

41. $c - a$

42. $a - b$

43. $\frac{c}{a}$

44. $\frac{c}{b}$

45. $b \cdot c$

46. $c(a)$

47. MP **PROBLEM SOLVING** You earn $15n$ dollars for mowing n lawns. How much do you earn for mowing 1 lawn? 7 lawns?

EVALUATING EXPRESSIONS **Copy and complete the table.**

48.

x	3	6	9
$x \cdot 8$			

49.

x	2	4	8
$64 \div x$			

50. **MODELING REAL LIFE** Due to gravity, an object falls $16t^2$ feet in t seconds. You drop a rock from a bridge that is 75 feet above the water. Will the rock hit the water in 2 seconds? Explain.

EVALUATING EXPRESSIONS **Evaluate the expression when $a = 10$, $b = 9$, and $c = 4$.**

51. $2a + 3$

52. $4c - 7.8$

53. $\frac{a}{4} + \frac{1}{3}$

54. $\frac{24}{b} + 8$

55. $c^2 + 6$

56. $a^2 - 18$

57. $a + 9c$

58. $bc + 12.3$

59. $3a + 2b - 6c$

$5m + 3 = 5 \cdot 8 + 3$

$= 5 \cdot 11$

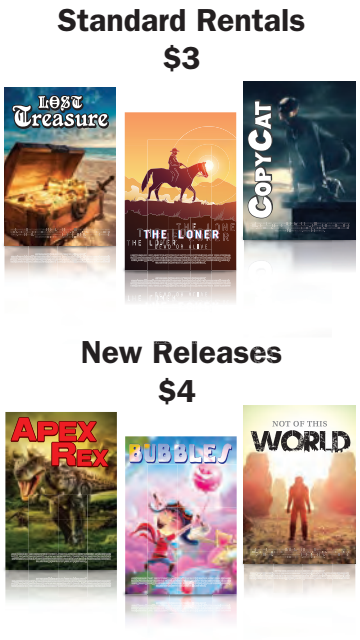
$= 55$

60. **YOU BE THE TEACHER** Your friend evaluates the expression when $m = 8$. Is your friend correct? Explain your reasoning.

61. **MP PROBLEM SOLVING** After m months, the height of a plant is $(10 + 3m)$ millimeters. How tall is the plant after 8 months? 3 years?

62. **MP STRUCTURE** You use a video streaming service to rent x new releases and y standard rentals. Which expression tells you how much money you will need?

$3x + 4y$ $\quad$ $4x + 3y$ $\quad$ $7(x + y)$



63. **OPEN-ENDED** You float 2000 feet along a lazy river water ride. The ride takes less than 10 minutes. Give two examples of possible times and speeds.

64. **DIG DEEPER!** The expression $20a + 13c$ is the cost (in dollars) for a adults and c students to enter a science center.

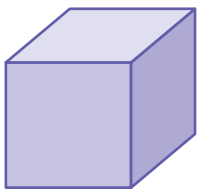
a. How much does it cost for an adult? a student? Explain your reasoning.

b. Find the total cost for 4 adults and 24 students.

c. You find the cost for a group. Then the numbers of adults and students in the group both double. Does the cost double? Explain your answer using an example.

d. In part (b), the number of adults is cut in half, but the number of students doubles. Is the cost the same? Explain your answer.

65. **MP REASONING** The volume of the cube (in cubic inches) is equal to four times the area of one of its faces (in square inches). What is the volume of the cube?



Common Errors

- **Exercises 51–59** Students may forget about the order of operations, or they may substitute the wrong value(s) for the variable(s). Review the order of operations. Before substituting, have students write or identify which operations are involved in the problem and which should be evaluated first.
- **Exercise 62** Students may rush through the problem and choose the first expression. Remind them to read the problem carefully.

Mini-Assessment

Evaluate the expression when $x = 2$, $y = 5$, and $z = 10$.

1. $5 + x$ 7
2. $3y$ 15
3. $z - y$ 5
4. $x \cdot y$ 10
5. Your friend earns $7x$ dollars for working x hours. How much does your friend earn for working 16 hours? 40 hours? $112; $280

Section Resources

Surface Level	Deep Level
Resources by Chapter • Extra Practice • Reteach • Puzzle Time Student Journal • Self-Assessment • Practice Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension Graphic Organizers Dynamic Assessment System • Section Practice

Concepts, Skills, & Problem Solving

50. no; In 2 seconds, the rock has only fallen 64 feet.

51. 23

52. 8.2

53. $2\frac{5}{6}$

54. $10\frac{2}{3}$

55. 22

56. 82

57. 46

58. 48.3

59. 24

60. no; $5m + 3 = 40 + 3 = 43$

61. 34 mm; 118 mm

62. $4x + 3y$

63. *Sample answer:* 8 min at 250 ft/min; 9 min at $222\frac{2}{9}$ ft/min

64. **a.** $20; $13; The coefficients represent the ticket prices.

b. $392

c. yes; *Sample answer:* $20(3) + 13(10) = 190$, $20(6) + 13(20) = 380$

d. no; *Sample answer:* $20(2) + 13(48) = 664$

65. 64 in.^{3}

Learning Target

Write algebraic expressions and solve problems involving algebraic expressions.

Success Criteria

- Write numerical expressions.
- Write algebraic expressions.
- Write and evaluate algebraic expressions that represent real-life problems.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Discuss the meaning of the word *expression*. In everyday language, an expression is used to describe something using descriptive language rather than the literal meaning. For example, when describing heavy rain, someone may use the expression, "It's raining cats and dogs!" In mathematics, an expression is used to represent a mathematical phrase containing numbers, operations, and/or variables.

Exploration 1

a. See Additional Answers.

b. *Sample answer:* $20 - p$

c. 20: customer has \$20; $4.65s$: total cost; 4.65: price per sandwich; s: number of sandwiches; no; Egg salad and grilled cheese both cost \$4.65, and the number of sandwiches is unknown.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
6.EE.A.2a

Preparing to Teach

- Students evaluated algebraic expressions in the last section and now they will build on that knowledge to write algebraic expressions that represent real-life problems.
- It is important for students to know that a variable can represent any value in an algebraic expression, but some values may not be reasonable. Writing an algebraic expression to represent a context provides an efficient method for testing multiple values.
- **MP8 Look for and Express Regularity in Repeated Reasoning:** When students write and then evaluate algebraic expressions for different values, they develop a general method, or shortcut. This occurs as a result of the calculations being repeated. Look for this understanding as students work through the exploration.

Motivate

- A synonym is one of two or more words or phrases that have the same meaning. For example, *loud* and *noisy* both can mean producing much noise.
- On the board, write the expression $3 \cdot n$. Ask multiple volunteers to read the expression aloud. Listen for various phrases, such as "3 times n," "3 multiplied by n," and "the product of 3 and n."
- **Discuss:** In math class, you often hear words or phrases that suggest a particular operation. In fact, there are several words that may imply the same operation. Today's lesson provides practice reading a situation and translating it into an algebraic expression.

Exploration 1

- **MP8 Look for and Express Regularity in Repeated Reasoning:** This exploration encourages students to see the differences between numerical and algebraic expressions. When writing an algebraic expression, students should recognize that the operations can be performed repeatedly by substituting different numbers for the variable(s).
- Introduce the exploration by asking about students' favorite sandwiches and which sandwich they would order from those recommended by the chef.
- In part (a), check to see that students are writing a numerical expression versus writing the result of the numerical expression. You want students to see the repetition in the last column, so that they can visualize the algebraic expression.
- In part (b), students can choose any variable for their algebraic expressions. Have them discuss what the variable represents in the situation. Ask students to share and explain their algebraic expressions.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** In part (c), ask a volunteer to give his or her explanation of the expression $20 - 4.65s$ and then ask if there are any other interpretations. Students should feel comfortable justifying their statements and discussing other explanations.

5.2 Writing Expressions

Learning Target: Write algebraic expressions and solve problems involving algebraic expressions.

Success Criteria:
- I can write numerical expressions.
- I can write algebraic expressions.
- I can write and evaluate algebraic expressions that represent real-life problems.

EXPLORATION 1 Writing Expressions

Work with a partner. You use a $20 bill to buy lunch at a café. You order a sandwich from the menu board shown.

Math Practice

Use Expressions

How do the numerical expressions help you generalize the situation and write an algebraic expression?

a. Complete the table. In the last column, write a numerical expression for the amount of change you receive.

Sandwich	Price (dollars)	Change Received (dollars)
Reuben		
BLT		
Egg salad		
Roast beef		

b. MP **REPEATED REASONING** Write an algebraic expression that represents the amount of change you receive when you order any sandwich from the menu board.

c. The expression $20 - 4.65s$ represents the amount of change one customer receives after ordering from the menu board. Explain what each part of the expression represents. Do you know what the customer ordered? Explain your reasoning.

5.2 Lesson

Some words can imply math operations.

Operation	Addition	Subtraction	Multiplication	Division
Key Words and Phrases	added to plus sum of more than increased by total of and	subtracted from minus difference of less than decreased by fewer than take away	multiplied by times product of twice	divided by quotient of

EXAMPLE 1 Writing Numerical Expressions

Write each phrase as an expression.

a. 8 fewer than 21

$21 - 8$ The phrase *fewer than* means *subtraction*.

b. the product of 30 and 9

30×9, or $30 \cdot 9$ The phrase *product of* means *multiplication*.

Try It **Write the phrase as an expression.**

1. the sum of 18 and 35

2. 6 times 50

EXAMPLE 2 Writing Algebraic Expressions

Write each phrase as an expression.

a. 14 more than a number x

$x + 14$ The phrase *more than* means *addition*.

b. a number y minus 75

$y - 75$ The word *minus* means *subtraction*.

c. the quotient of 3 and a number z

$3 \div z$, or $\frac{3}{z}$ The phrase *quotient of* means *division*.

Try It **Write the phrase as an expression.**

3. 25 less than a number b

4. a number x divided by 4

5. the total of a number t and 11

6. 100 decreased by a number k

Common Error

When writing expressions involving subtraction or division, order is important. For example, the quotient of a number x and 2 means

$x \div 2$, not $2 \div x$.

Laurie's Notes

Scaffolding Instruction

- In the exploration, students extended their knowledge of writing numerical expressions to writing algebraic expressions. They used numerical expressions to explore writing an algebraic expression to represent a situation.
- **Emerging:** Students may struggle to choose a variable to represent an unknown value, or need practice writing the variable with an operation to indicate the meaning of a phrase. Students will benefit from guided instruction for the examples.
- **Proficient:** Students can translate phrases into algebraic expressions. They have strong literacy skills and can justify their expressions. Have students self-assess using the Try It exercises.

Discuss

- Review the table of words that can imply math operations. Ask students to add to the list.
- Remind students that these words and phrases must *always* be read in context. They cannot just pick out a word or phrase and assume that it will represent the same operation every time.
  - Clarify this misconception using the two scenarios. First, Ann buys a pen for $2.50 and a notebook for $3.25. What is the total of her purchases? Second, Ann buys 5 pens for $2.50 each. What is the total of her purchases? Although "total of" is used in both situations, the expressions require two different operations (addition and multiplication, respectively).

EXAMPLE 1

- In part (a), remind students that for subtraction phrases they should always look for the key words *from* and *than* to know that the order of the numbers must be switched from the way the phrase is written.

EXAMPLE 2

- Refer students to the Common Error note and ask, "Is order important for addition and multiplication? Explain." No, addition and multiplication are commutative.
- Work through the example. Remind students that subtraction and division are not commutative. In part (b), the expression is *not* $75 - y$. In part (c), the expression is *not* $z \div 3$.

Try It

- **Neighbor Check:** Have students work independently, and then have their neighbors check their work. Have students discuss any discrepancies.
- **MP6 Attend to Precision:** Students communicate precisely to their peers by translating phrases into algebraic expressions and referring to the parts of the expressions using the correct vocabulary.

Extra Example 1

Write each phrase as an expression.

a. 34 divided by 2 $34 \div 2$

b. 35 increased by 43 $35 + 43$

Try It

1. $18 + 35$
2. $6 \cdot 50$

Extra Example 2

Write each phrase as an expression.

a. the difference of a number t and 95 $t - 95$

b. 2 times a number w $2w$

c. the total of a number x and 8 $x + 8$

Try It

3. $b - 25$
4. $x \div 4$
5. $t + 11$
6. $100 - k$

ELL Support

Have students work in pairs to discuss and complete Try It Exercises 3–6. Write each phrase on the board and underline *less than*, *divided by*, *total*, and *decreased by*. Explain that these phrases imply mathematical operations.
Beginner: Write the expression.
Intermediate: State the expression. For example, "*b* minus twenty-five."
Advanced: Explain why their expression is correct.

Extra Example 3

The number of students in the science club is 4 more than twice the number of students in the math club. Let m be the number of students in the math club. Which expression can you use to represent the number of students in the science club?

A. $4m + 2$ **B.** $2m + 4$

C. $2m - 4$ **D.** $4 - 2m$

B

Try It

7. $2t + 5$

Self-Assessment for Concepts & Skills

8. $7 + 11$

9. $9 - 5$

10. x take away 12; $x - 12$; $x + 12$

11. no; The first means $x - 12$, and the second means $12 - x$.

Laurie's Notes

EXAMPLE 3

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Ask a volunteer to read the problem. Given the amount of words, some students may have difficulty getting started. Encourage students to re-read the problem and underline key words.
- Some students may be unfamiliar with interstates. Ask if anyone has been on these interstates. Are they close to home or in your state? If not, which interstates are close? What purpose do interstates serve?
- **Common Error:** Choice C is a very common *wrong* answer. Ask students to think about the value of the expression when $m = 100$. "Something negative" is a common response.
- Refer students to the push-pin note. Remind them that they have seen capital letters as variables before. A variable is just a symbol that represents a value and it can be lowercase or uppercase, as long as you are consistent. Just avoid any letter that may be confused with a number (e.g., O, I).

Try It

- In Exercise 7, some students may write $5 + 2t$ and others may write $2t + 5$. Addition is commutative, so this is fine. Caution students that subtraction is *not* commutative, so there is only one way to correctly write "five less than twice as many tokens."

Self-Assessment for Concepts & Skills

- These exercises encompass many of the words and phrases that students have been translating into operations.
- Students should work on the exercises independently. Then have them compare their answers with another student and discuss any differences.
- In Exercises 9 and 11, listen for students confusing the order of the terms. Ask volunteers to explain why only one expression is correct.

ELL Support

Allowing students to work in pairs provides extra support from peers and the opportunity to practice language. Check comprehension of Exercises 8–10 by having pairs write their answers on whiteboards. Discuss explanations for Exercise 11 to check for understanding.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

EXAMPLE 3 Writing an Algebraic Expression

The length of Interstate 90 from the West Coast to the East Coast is 153.5 miles more than 2 times the length of Interstate 15 from southern California to northern Montana. Let m be the length of Interstate 15. Which expression can you use to represent the length of Interstate 90?

A. $2m + 153.5$ **B.** $2m - 153.5$

C. $153.5 - 2m$ **D.** $153.5m + 2$

Variables can be lowercase or uppercase. Make sure you consistently use the same case for a variable when solving a problem.

The word *times* means *multiplication*. So, multiply 2 and m.

The phrase *more than* means *addition*. So, add $2m$ and 153.5.

$2m + 153.5$

The correct answer is **A**.

Try It

7. Your friend has 5 more than twice as many game tokens as you. Let t be the number of game tokens you have. Write an expression for the number of game tokens your friend has.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

WRITING EXPRESSIONS **Write the phrase as an expression.**

8. the sum of 7 and 11

9. 5 subtracted from 9

10. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" expressions.

12 more than x

x increased by 12

x take away 12

the sum of x and 12

11. MP **PRECISION** Your friend says that the phrases below have the same meaning. Is your friend correct? Explain your reasoning.

the difference of a number x and 12

the difference of 12 and a number x

EXAMPLE 4 Modeling Real Life

You plant a cypress tree that is 10 inches tall. Each year, its height increases by 15 inches. Write an expression that represents the height (in inches) after t years. What is the height after 9 years?

Make a table showing the height of the tree each year for the first several years. Use the results to write an expression and evaluate the expression when $t = 9$.

The height is *increasing*, so *add* 15 each year, as shown in the table.

Year, t	Height (inches)
0	10
1	$10 + 15(1) = 25$
2	$10 + 15(2) = 40$
3	$10 + 15(3) = 55$
4	$10 + 15(4) = 70$

When t is 0, the height is 10 inches.

You can see that an expression is $10 + 15t$.

Sometimes, as in Example 3, a variable represents a single value. Other times, as in Example 4, a variable can represent more than one value.

Evaluate $10 + 15t$ when $t = 9$.

$$10 + 15t = 10 + 15(9) = 145$$

So, the height (in inches) after t years is $10 + 15t$. After 9 years, the height of the tree is 145 inches.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

12. A company rents paddleboards by charging a rental fee plus an hourly rate. Write an expression that represents the cost (in dollars) of renting a paddleboard for h hours. How much does an eight-hour rental cost?

13. DIG DEEPER! A county fair charges an entry fee of \$7 and \$0.75 for each ride token. You have \$15. Write an expression that represents the amount (in dollars) you have left after entering the fair and purchasing n tokens. How many tokens can you purchase? How much money do you have left after purchasing 6 tokens?

Laurie's Notes

EXAMPLE 4

- Say, "You want to organize some information about this problem, so set up a table to keep track of the height each year."
- Make a table with two columns (year, height). It is not natural to start with the year 0, so be sure to discuss this. Each year students should add 15 to the last number.

❓ "How can you find the height after 9 years without computing all the years between 4 and 9?" Write an expression and evaluate it for 9 years.

- Work with students to develop the general expression $10 + 15t$ and verify that it works for years 0 through 4.

❓ "Check your answer. Does it make sense?" Students may not be able to visualize 145 inches, so tell them to convert to feet to see if it is reasonable.

◉ This example addresses the third success criterion.

Self-Assessment for Problem Solving

- Allow time in class for students to practice using the problem-solving plan. Remember, some students may only be able to complete the first step.
- Have students work independently, and then discuss the exercises as a class. Select two volunteers to display their work on the board.
- Exercise 12 is very similar to Example 4. As you circulate, remind students who are struggling to refer back to the example to jumpstart their thinking.

◉ Are students showing progress in relating words to numbers, variables, and symbols?

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Closure

- **Four Corners:** Designate each of four corners in your classroom as one of the four operations: addition, subtraction, multiplication, and division. Give each student an index card with a word or a phrase that represents one of the four operations (some phrases may be repeated). Tell students to move to the corners of the room that represent their cards.

◉ This is an informal, whole-class assessment of the first two success criteria.

Extra Example 4

A bamboo shoot is 4 inches tall. During its growing season, it grows 3 inches a day. Write an expression that represents the height (in inches) after d days. What is the height after 14 days?

$4 + 3d$; 46 inches

Self-Assessment for Problem Solving

12. $15 + 12h$; \$111

13. $8 - 0.75n$; 10; \$3.50

Formative Assessment Tip

Four Corners

This technique allows students to place a problem into one of four categories or answer choices. Designate each of four corners in your classroom as a different category or answer. Give each student an index card with a problem written on it and ask them to move to the corners of the room that correspond to their cards. Have students discuss with others in their corner and determine if they are all in the appropriate corner. Allow students to change corners if necessary. Have each group read their cards to the class, so that everyone hears which problems are related to each category or answer. Discuss any discrepancies as a class.

Learning Target

Write algebraic expressions and solve problems involving algebraic expressions.

Success Criteria

- Write numerical expressions.
- Write algebraic expressions.
- Write and evaluate algebraic expressions that represent real-life problems.

Review & Refresh

1. Terms: $4f$, 8
Coefficient: 4
Constant: 8

2. Terms: $\frac{4}{5}$, $3s$, 2
Coefficient: 3
Constants: $\frac{4}{5}$, 2

3. Terms: $9h^2$, $\frac{8}{9}p$, 1
Coefficients: 9, $\frac{8}{9}$
Constant: 1

4. 7.5

5. 2361.6 or 2400

6. 4860 or 4909.09

7. $\frac{4}{5}$

8. $\frac{4}{9}$

9. $\frac{2}{15}$

10. $3\frac{1}{2}$

Concepts, Skills, & Problem Solving

11. 10: you have \$10; $5.25n$: total cost; 5.25: price per sandwich; n: number of sandwiches

12. 20: you have \$20; $4.95n$: total cost; 4.95: price per sandwich; n: number of sandwiches

13. 100: you have \$100; $6.75n$: total cost; 6.75: price per sandwich; n: number of sandwiches

14. $8 - 5$

15. $3 \cdot 12$

16. $28 \div 7$

17. $6 + 10$

18. $18 - 3$

19. $15 + 17$

20. $x - 13$

21. $5 \cdot d$

22. $18 \div a$

23. $s - 6$

24. $7 + w$

25. t^3

26. no; The expression is $\frac{8}{y}$.

27. yes; The expression is correct.

28. **a.** $x \div 5$

b. *Sample answer:* If the total cost is \$30, then the cost per person is $x \div 5 = 30 \div 5 = \$6$.

29. **a.**

Days	1	2	3	4	5
Samples	15	30	45	60	75

b. $15n$

Assignment Guide and Concept Check

Check out the Dynamic Assessment System.
BigIdeasMath.com

Level	Assignment 1	Assignment 2
Emerging	3, 6, 9, 13, 14, 16, 17, 21, 22, 26	24, 25, 27, 28, 29, 30, 31, 37, 39
Proficient	3, 6, 9, 13, 14, 17, 21, 22, 23, 26	24, 25, 27, 29, 30, 33, 37, 39, 40
Advanced	3, 6, 9, 13, 14, 21, 22, 24, 25, 26	27, 33, 34, 38, 40, 41, 42, 43

- Assignment 1 is for use after students complete the Self-Assessment for Concepts & Skills.
- Assignment 2 is for use after students complete the Self-Assessment for Problem Solving.
- The red exercises can be used as a concept check.

Review & Refresh Prior Skills

Exercises 1–3 Identifying Parts of Algebraic Expressions
Exercises 4–6 Using Conversion Factors
Exercises 7–10 Dividing Fractions

Common Errors

- **Exercises 14–25** Students may write subtraction problems in the wrong order. Have students look for the key words *from* and *than* to know that the order of the numbers must be switched from the way the phrase is written. For example, "5 less *than* 8" means $8 - 5$, not $5 - 8$.
- **Exercises 14–25** Students may write division problems in the wrong order. For example, they may write "28 divided by 7" as $7 \div 28$ instead of $28 \div 7$.

5.2 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Identify the terms, coefficients, and constants in the expression.

1. $4f + 8$

2. $\frac{4}{5} + 3s + 2$

3. $9h^2 + \frac{8}{9}p + 1$

Copy and complete the statement.

4. $\frac{2\text{ c}}{\text{min}} = \frac{\square\text{ gal}}{\text{h}}$

5. $\frac{12\text{ m}}{\text{sec}} \approx \frac{\square\text{ ft}}{\text{min}}$

6. $\frac{3\text{ lb}}{\text{sec}} \approx \frac{\square\text{ kg}}{\text{h}}$

Divide. Write the answer in simplest form.

7. $\frac{1}{2} \div \frac{5}{8}$

8. $\frac{1}{3} \div \frac{3}{4}$

9. $\frac{2}{5} \div 3$

10. $3 \div \frac{6}{7}$

Concepts, Skills, & Problem Solving

MP STRUCTURE **The expression represents the amount of change you receive after buying n sandwiches. Explain what each part of the expression represents.** (See Exploration 1, p. 209.)

11. $10 - 5.25n$

12. $20 - 4.95n$

13. $100 - 6.75n$

WRITING EXPRESSIONS **Write the phrase as an expression.**

14. 5 less than 8

15. the product of 3 and 12

16. 28 divided by 7

17. the total of 6 and 10

18. 3 fewer than 18

19. 17 added to 15

20. 13 subtracted from a number x

21. 5 times a number d

22. the quotient of 18 and a number a

23. the difference of a number s and 6

24. 7 increased by a number w

25. a number t cubed

YOU BE THE TEACHER **Your friend writes the phrase as an expression. Is your friend correct? Explain your reasoning.**

26. The quotient of 8 and a number y is $\frac{y}{8}$.

27. 16 decreased by a number x is $16 - x$.

28. **MP NUMBER SENSE** Five friends share the cost of a dinner equally.

a. Write an expression that represents the cost (in dollars) per person.

b. Make up a reasonable total cost and test your expression.

29. **MODELING REAL LIFE** A biologist analyzes 15 bacteria samples each day.

a. Copy and complete the table.

b. Write an expression that represents the total number of samples analyzed after n days.

Days	1	2	3	4	5
Total Samples					

30. **PROBLEM SOLVING** To rent a moving truck for the day, it costs \$33 plus \$1 for each mile driven.

a. Write an expression that represents the cost (in dollars) to rent the truck.

b. You drive the truck 300 miles. How much do you pay?

WRITING PHRASES **Give two ways to write the expression as a phrase.**

31. $n + 6$ **32.** $4w$ **33.** $15 - b$ **34.** $14 - 3z$

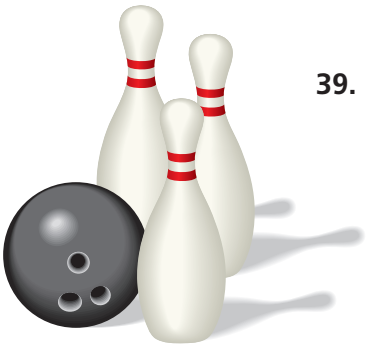
EVALUATING EXPRESSIONS **Write the phrase as an expression. Then evaluate the expression when $x = 5$ and $y = 20$.**

35. 3 less than the quotient of a number y and 4

36. the sum of a number x and 4, all divided by 3

37. 6 more than the product of 8 and a number x

38. the quotient of 40 and the difference of a number y and 16



39. **MODELING REAL LIFE** It costs \$3 to bowl a game and \$2 for shoe rental.

a. Write an expression that represents the total cost (in dollars) of g games.

b. Use your expression to find the total cost of 8 games.

40. **MODELING REAL LIFE** Florida has 8 less than 5 times the number of counties in Arizona. Georgia has 25 more than twice the number of counties in Florida.

a. Write an expression that represents the number of counties in Florida.

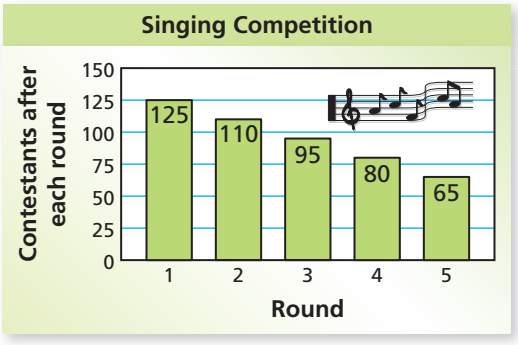
b. Write an expression that represents the number of counties in Georgia.

c. Arizona has 15 counties. How many do Florida and Georgia have?

41. **PATTERNS** There are 140 people in a singing competition. The graph shows the results for the first five rounds.

a. Write an expression that represents the number of people after each round.

b. Assuming this pattern continues, how many people compete in the ninth round? Explain your reasoning.



42. **NUMBER SENSE** The difference between two numbers is 8. The lesser number is a. Write an expression that represents the greater number.

43. **NUMBER SENSE** One number is four times another. The greater number is x. Write an expression that represents the lesser number.

Common Errors

- **Exercises 35–38** Students may write the problem in the wrong order, such as $\frac{y-3}{4}$ instead of $\frac{y}{4} - 3$. Tell students to write out the phrase and put parentheses around the different parts of the expression before writing an algebraic expression. Then evaluate the expression using the order of operations. For example, "3 less than (the quotient of a number y and 4)."

Mini-Assessment

Write the phrase as an expression.

1. the product of 5 and 7 $5 \cdot 7$
2. the sum of a number b and 3 $b + 3$
3. the quotient of 9 and a number y $\frac{9}{y}$
4. 8 less than a number x $x - 8$
5. It costs \$15 for one surfing lesson and \$25 per day to rent a surfboard. Write an expression for the total cost of one lesson and renting a surfboard for y days. $15 + 25y$

Section Resources

Surface Level	Deep Level
Resources by Chapter • Extra Practice • Reteach • Puzzle Time Student Journal • Self-Assessment • Practice Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension Graphic Organizers Dynamic Assessment System • Section Practice
Transfer Level	
Dynamic Assessment System • Mid-Chapter Quiz	Assessment Book • Mid-Chapter Quiz

Concepts, Skills, & Problem Solving

30. **a.** $33 + m$

b. \$333

31–34. Sample answers given.

31. the sum of n and 6; 6 more than a number n

32. 4 times a number w; the product of 4 and a number w

33. a number b less than 15; 15 take away a number b

34. the difference of 14 and the product of 3 and a number z; 14 minus the product of 3 and a number z

35. $\frac{y}{4} - 3$; 2

36. $\frac{x+4}{3}$; 3

37. $8x + 6$; 46

38. $\frac{40}{y-16}$; 10

39. **a.** $2 + 3g$

b. \$26

40. **a.** $5a - 8$

b. $2f + 25$

c. Florida has 67 counties. Georgia has 159 counties.

41. **a.** $140 - 15n$

b. 20; There are $140 - 15(8) = 20$ people left after the eighth round.

42. $a + 8$

43. $\frac{x}{4}$

Learning Target

Identify equivalent expressions and apply properties to generate equivalent expressions.

Success Criteria

- Explain the meaning of equivalent expressions.
- Use properties of addition to generate equivalent expressions.
- Use properties of multiplication to generate equivalent expressions.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Discuss the meaning of the word *property*. In everyday language, property refers to things that people own. For example, your clothing and toys are your property. A house or land is also commonly referred to as property. In mathematics, a property is a rule.

Exploration 1

a–b. See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
6.EE.A.3, 6.EE.A.4

Preparing to Teach

- Students will identify expressions that are equivalent and then review the properties of addition and multiplication with numbers before applying them to algebraic expressions.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Mathematically proficient students justify their conclusions, communicate them to others, and respond to the arguments of others. In this lesson, students will make conjectures about which operations are commutative and associative. Expect clear communication of their thinking.

Motivate

- **Acting Time:** If there is a bit of an actor in you, start the class by pretending that you're brushing your teeth (about 10 seconds). Then take a tube of toothpaste out and apply some to the toothbrush. This will clearly evoke a few comments about the order in which you performed the two tasks.
- As students begin to comment, say, "Oh, does the order matter? Hmmm…"
- If brushing your teeth doesn't work for you, put a sock over a shoe or anything obviously in the wrong order. Do something that students will remember. Catch their attention!
- Have students think of other examples where order matters and ask volunteers to share.
- Then have students think of examples where order does *not* matter and ask volunteers to share. If students are struggling to think of ideas, say, "When you make a ham and cheese sandwich, does it matter if you put the ham on the bread and then the cheese?"

Exploration 1

- **Turn and Talk:** Ask students to discuss the meaning of the word *equivalent*. Students may say, "the same" or "equal." Refer to a balance, a visual that students used in prior courses. A balance shows that although each side looks different, they have the same "weight" when the sides are raised to the same height. Tell students to keep this in mind as they explore possible **equivalent expressions**.
- In part (a), tell students to read the directions carefully and skim the tables. Ask, "What kind of numbers do you want to choose as your four values for x?" They will probably mention "easy" numbers. Discuss what makes a number easy to compute with. Remind students that the four values they choose for the first table will be used for the other tables as well.
- After completing the tables, partners should discuss whether any expressions are equivalent and explain their reasoning.
- "What helps you remember the meaning of the Commutative and Associative Properties?" Listen for the root words *commute* and *associate*. "In part (b), how do you know the expressions in each example are equivalent?" The values of the expressions are the same on both sides of the equal sign.
- Discuss where the properties apply in part (a). "How do you know whether the algebraic expressions are equivalent?" The operations are the same in both expressions and the values of the expressions are the same for any value of x.

5.3 Properties of Addition and Multiplication

Learning Target: Identify equivalent expressions and apply properties to generate equivalent expressions.

Success Criteria:
- I can explain the meaning of equivalent expressions.
- I can use properties of addition to generate equivalent expressions.
- I can use properties of multiplication to generate equivalent expressions.

EXPLORATION 1 Identifying Equivalent Expressions

Work with a partner.

a. Choose four values for a variable x. Then evaluate each expression for each value of x. Are any of the expressions *equivalent*? Explain your reasoning.

x				
$4 + x + 4$				

x				
$16x$				

x				
$4 \cdot (x \cdot 4)$				

x				
$x + 4 + 4$				

x				
$x + 8$				

x				
$(4 \cdot x) \cdot 4$				

b. You have used the following properties in a previous course. Use the examples to explain the meaning of each property.

Commutative Property of Addition: $3 + 5 = 5 + 3$

Commutative Property of Multiplication: $9 \cdot 3 = 3 \cdot 9$

Associative Property of Addition: $8 + (3 + 1) = (8 + 3) + 1$

Associative Property of Multiplication: $12 \cdot (6 \cdot 2) = (12 \cdot 6) \cdot 2$

Are these properties true for algebraic expressions? Explain your reasoning.

Math Practice

Use Counterexamples

Use a counterexample to show that the Commutative Property is not true for division.

5.3 Lesson

Key Vocabulary
equivalent expressions, *p. 216*

Expressions that result in the same number for any value of each variable are **equivalent expressions**. You can use the Commutative and Associative Properties to write equivalent expressions.

Key Ideas

Commutative Properties

Words Changing the order of addends or factors does not change the sum or product.

Numbers $5 + 8 = 8 + 5$
$5 \cdot 8 = 8 \cdot 5$

Algebra $a + b = b + a$
$a \cdot b = b \cdot a$

Associative Properties

Words Changing the grouping of addends or factors does not change the sum or product.

Numbers $(7 + 4) + 2 = 7 + (4 + 2)$
$(7 \cdot 4) \cdot 2 = 7 \cdot (4 \cdot 2)$

Algebra $(a + b) + c = a + (b + c)$
$(a \cdot b) \cdot c = a \cdot (b \cdot c)$

EXAMPLE 1 Using Properties to Write Equivalent Expressions

One way to check whether expressions are equivalent is to evaluate each expression for any value of the variable. In Example 1(a), use $x = 2$.

$7 + (12 + x) \stackrel{?}{=} 19 + x$

$7 + (12 + 2) \stackrel{?}{=} 19 + 2$

$21 = 21$ ✓

a. Simplify the expression $7 + (12 + x)$.

$7 + (12 + x) = (7 + 12) + x$ Associative Property of Addition

$= 19 + x$ Add 7 and 12.

b. Simplify the expression $(6.1 + x) + 8.4$.

$(6.1 + x) + 8.4 = (x + 6.1) + 8.4$ Commutative Property of Addition

$= x + (6.1 + 8.4)$ Associative Property of Addition

$= x + 14.5$ Add 6.1 and 8.4.

c. Simplify the expression $5(11y)$.

$5(11y) = (5 \cdot 11)y$ Associative Property of Multiplication

$= 55y$ Multiply 5 and 11.

Try It **Simplify the expression. Explain each step.**

1. $10 + (a + 9)$ **2.** $\left(c + \frac{2}{3}\right) + \frac{1}{2}$ **3.** $5(4n)$

Laurie's Notes

Scaffolding Instruction

- Students have applied properties to numerical expressions and explored substituting values for variables in algebraic expressions. They will now use the Commutative and Associative Properties, as well as the Properties of Zero and One, to show that expressions are equivalent.
- Review the meanings of *simplify*, *evaluate*, and *solve* with all students.
- **Emerging:** Students can evaluate expressions for given values but need practice applying the properties of addition and multiplication to find equivalent algebraic expressions.
- **Proficient:** Students understand **equivalent expressions** and are confident in using all the properties, even when variables are present. After reviewing the Key Ideas, students can proceed to Self-Assessment Exercises 7–13.

Key Ideas

- Write the Commutative and Associative Properties. The key word for the Commutative Property is *order*. The key word for the Associative Property is *grouping*.
- For the Commutative Property, multiplication can be represented in different ways: $ab = ba$ or $a \cdot b = b \cdot a$. This is also true for the Associative Property: $(ab)c = a(bc)$ versus $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
- Discuss that the variables can represent all the types of numbers students have studied: whole numbers, fractions, and decimals. Tell students that as they encounter other types of numbers (integers), they will need to verify that the properties still apply.

EXAMPLE 1

- Students have used these properties to simplify numerical expressions, now they will apply them to algebraic expressions.
- **MP6 Attend to Precision:** The challenges students have with these problems are that the answers seem obvious. Students often say, "Just combine the numbers and then tag on the variable." Remind students that this is only true for addition and multiplication.
- Note how the expression in part (c) does not contain multiplication dots.

Try It

- **Neighbor Check:** Have students work independently, and then have their neighbors check their work. Have students discuss any discrepancies.

ELL Support

Have students work in pairs to discuss and complete the exercises.
Beginner: Write the simplified expression.
Intermediate: State the simplified expression. For example, "Nineteen plus *a*."
Advanced: Relate the original expression to the simplified expression using a complete sentence. For example, "Ten plus the quantity *a* plus nine equals nineteen plus *a*."

Extra Example 1

a. Simplify the expression $(r + 3.2) + 6.7$. $r + 9.9$

b. Simplify the expression $4(3p)$. $12p$

c. Simplify the expression $(6 \cdot b) \cdot 7$. $42b$

Try It

1. $10 + (a + 9) = 10 + (9 + a)$
Comm. Prop. of Add.
$= (10 + 9) + a$
Assoc. Prop. of Add.
$= 19 + a$
Add 10 and 9.

2. $\left(c + \frac{2}{3}\right) + \frac{1}{2} = c + \left(\frac{2}{3} + \frac{1}{2}\right)$
Assoc. Prop. of Add.
$= c + 1\frac{1}{6}$
Add $\frac{2}{3}$ and $\frac{1}{2}$.

3. $5(4n) = (5 \cdot 4)n$
Assoc. Prop. of Mult.
$= 20n$
Multiply 5 and 4.

Extra Example 2

a. Simplify the expression $0 + (s + 4)$.
$s + 4$

b. Simplify the expression $(h \cdot 1) \cdot 8$.
$8h$

Try It

4. $12 \cdot b \cdot 0 = 12 \cdot (b \cdot 0)$
Assoc. Prop. of Mult.
$= 12 \cdot 0$
Mult. Prop. of Zero
$= 0$
Mult. Prop. of Zero

5. $1 \cdot m \cdot 24 = (1 \cdot m) \cdot 24$
Assoc. Prop. of Mult.
$= m \cdot 24$
Mult. Prop. of One
$= 24m$
Comm. Prop. of Mult.

6. $(t + 15) + 0 = t + (15 + 0)$
Assoc. Prop. of Add.
$= t + 15$
Add. Prop. of Zero

ELL Support

Have students work independently on Self-Assessment for Concepts & Skills Exercises 7–9. Then have students compare their answers with a partner to check understanding and practice language. Discuss Exercises 10 and 11 with students to verify understanding.

Self-Assessment for Concepts & Skills

7–9. See Additional Answers.

10. Expressions are equivalent when they result in the same number for any values of their variables; *Sample answer:* $(4 + 3)x$, $7x$

11. *Sample answer:* $(5 \cdot x) \cdot 1$

Laurie's Notes

Key Ideas

- Write and discuss the three new properties. Students may say (or at least think) that these properties are obvious.
- Adding zero, in any form, does not change the value of the quantity.
- Multiplying by zero, in any form, produces a product of 0.
- Multiplying by 1, in any form, does not change the value of the quantity.

  Remind students that 1 can be represented in different ways: 1, $\left(\frac{1}{2} + \frac{1}{2}\right)$, or $\frac{3}{3}$.
- The Addition Property of Zero is also known as the Identity Property of Addition (or the Additive Identity Property). The Multiplication Property of One is also know as the Identity Property of Multiplication (or the Multiplicative Identity Property). Meaning you end up with the same value you started with, so the values are *identical*.

EXAMPLE 2

- Work through both parts and point out the properties as they are used. Mention to students that the first steps in both parts involve the Associative Property of Multiplication. In part (a), you get the same result if you group $(0 \cdot p)$ first.
- In part (b), students may apply the Commutative Property of Multiplication first: writing $r \cdot 1$ as $1 \cdot r$ to group $(4.5 \cdot 1)$ and then apply the Multiplication Property of One. Encourage students to share different methods with the class.

Try It

Think-Pair-Share: Students should read each exercise independently and then work in pairs to simplify the expressions. After completing the exercises, have each pair compare their answers with another pair and discuss any discrepancies. As students are working on the last two success criteria, listen to their conversations to identify any common errors or misconceptions.

Self-Assessment for Concepts & Skills

- Have students work independently.
- Exercises 7–9 are accessible to all students because they are similar to Examples 1 and 2.
- Exercises 10 and 11 provide an indication of students' understanding of the success criteria. Do students understand what *equivalent* means in terms of algebraic expressions? Can they write an expression that can be simplified?
- Allow time for students to test their expressions in Exercise 11 to ensure they meet the requirements.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Ask volunteers to write their expressions on the board for Exercise 11. Then ask the class if they agree or disagree with each expression.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Key Ideas

Addition Property of Zero

Words The sum of any number and 0 is that number.

Numbers $7 + 0 = 7$ **Algebra** $a + 0 = a$

Multiplication Properties of Zero and One

Words The product of any number and 0 is 0.

The product of any number and 1 is that number.

Numbers $9 \cdot 0 = 0$ **Algebra** $a \cdot 0 = 0$

$4 \cdot 1 = 4$ $a \cdot 1 = a$

EXAMPLE 2 Using Properties to Write Equivalent Expressions

a. Simplify the expression $9 \cdot 0 \cdot p$.

$9 \cdot 0 \cdot p = (9 \cdot 0) \cdot p$ Associative Property of Multiplication

$= 0 \cdot p$ Multiplication Property of Zero

$= 0$ Multiplication Property of Zero

b. Simplify the expression $4.5 \cdot r \cdot 1$.

$4.5 \cdot r \cdot 1 = 4.5 \cdot (r \cdot 1)$ Associative Property of Multiplication

$= 4.5 \cdot r$ Multiplication Property of One

$= 4.5r$ Rewrite.

Try It **Simplify the expression. Explain each step.**

4. $12 \cdot b \cdot 0$ **5.** $1 \cdot m \cdot 24$ **6.** $(t + 15) + 0$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

USING PROPERTIES **Simplify the expression. Explain each step.**

7. $(7 + c) + 4$ **8.** $4(b \cdot 6)$ **9.** $0 \cdot b \cdot 9$

10. WRITING Explain what it means for expressions to be equivalent. Then give an example of equivalent expressions.

11. OPEN-ENDED Write an algebraic expression that can be simplified using the Associative Property of Multiplication and the Multiplication Property of One.

EXAMPLE 3 Modeling Real Life

You and six friends play on a basketball team. A sponsor paid \$100 for the league fee, x dollars for each player's T-shirt, and \$68.25 for basketballs. Write an expression that represents the total amount (in dollars) the sponsor paid. Then find the total amount paid when each T-shirt costs \$14.50.

Use a verbal model to write an expression that represents the sum of the league fee, the cost of the T-shirts, and the cost of the basketballs. Then evaluate the expression when $x = 14.5$.

League fee (dollars)	+	Number of T-shirts	•	Cost per T-shirt (dollars)	+	Cost of basketballs (dollars)
\$100		7		x		\$68.25

$$\begin{aligned} 100 + 7x + 68.25 &= 7x + 100 + 68.25 && \text{Commutative Property of Addition} \\ &= 7x + (100 + 68.25) && \text{Associative Property of Addition} \\ &= 7x + 168.25 && \text{Add 100 and 68.25.} \end{aligned}$$

Evaluate $7x + 168.25$ when $x = 14.5$.

$$7x + 168.25 = 7(14.5) + 168.25 = 101.5 + 168.25 = 269.75$$

An expression that represents the total amount (in dollars) is $7x + 168.25$. When each T-shirt costs \$14.50, the sponsor pays \$269.75.

Common Error

You **and** six friends are on the team, so use 7, not 6, to represent the number of T-shirts.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

12. You and five friends form a team for an outdoor adventure race. Your team needs to raise money to pay for \$130 of travel fees, x dollars for each team member's entry fee, and \$85.50 for food. Use an algebraic expression to find the total amount your team needs to raise when the entry fee is \$25.50 per person.

13. You have \$50 and a \$15 gift card to spend online. You purchase a pair of headphones for \$34.99 and 8 songs for x dollars each. Use an algebraic expression to find the amount you have left when each song costs \$1.10.

Laurie's Notes

EXAMPLE 3

- Ask a student to read the problem aloud. Ask how many T-shirts were purchased and refer students to the Common Error note.
- Ask what information is needed to find the total amount and write a verbal model. Then use the verbal model to write an expression.

❓ "Can the expression be simplified using properties?"

Ask students to explain why $7x + 168.25$ is an equivalent expression and how the properties of addition helped solve this problem. Listen for students' understanding of the success criteria.

❓ **Extension:** "If the sponsor decides to purchase different T-shirts that cost $17.30 each, then what will the new total be?" $289.35 "How much more will the sponsor pay?" $19.60

Self-Assessment for Problem Solving

- Allow time in class for students to practice using the problem-solving plan. Students should work independently. Support students with probing questions and feedback. Encourage students who are struggling to begin by using a verbal model. Remember, some students may only be able to complete the first step.
- Remind students to think back to similar problems they have solved and ask, "How did I approach that problem? Will the same approach work for this problem?"

❓ As students complete the exercises, ask, "Do your answers seem reasonable? Is there a way to check your answers?"

- Writing an expression to represent a context is an essential skill for writing and solving equations.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Closure

- **Four Square:** Have students place $3.5 + (y + 1.1)$ in the ovals of their *Four Squares*. Then have students label each square as one of the four categories: writing an equivalent expression, identifying properties, evaluating the expression, and application. Students may choose a reasonable value for evaluating the expression and create a context for the application. *Sample answer:*

Writing an Equivalent Expression $3.5 + (y + 1.1) = 3.5 + (1.1 + y)$ $= (3.5 + 1.1) + y$ $= 4.6 + y$	**Identifying Properties** Commutative Property of Addition; Associative Property of Addition; Add 3.5 and 1.1.
$3.5 + (y + 1.1)$	
Evaluating the Expression Evaluate $3.5 + (y + 1.1)$ when $y = 4.4$. $3.5 + (y + 1.1) = 3.5 + (4.4 + 1.1)$ $= 3.5 + 5.5$ $= 9$	**Application** You purchase a stapler for $3.50, a binder for y dollars, and a pen for $1.10. Use an algebraic expression to find the total amount you paid when the binder costs $4.40. $9

Extra Example 3

You hand out 425 programs on the first night of your school's variety show, p programs on the second night, and 520 programs on the third night. Write an expression that represents the total number of programs you handed out. Then find the total number of programs that you handed out if you handed out 515 programs on the second night.

$p + 945$; 1460 programs

Self-Assessment for Problem Solving

12. $368.50 **13.** $21.21

Formative Assessment Tip

Four Square

A *Four Square* is often used as a study reference, however, it can be used to assess students' understanding of a concept. One way to use a *Four Square* is to write a problem in the oval and label each of the four squares surrounding the oval as a related category. Related categories may include: answer, meaning, application, algebra, numbers, model, graph, or equation. You can also ask students to illustrate each of the four depth of knowledge levels in the squares surrounding the problem. Consider hanging students' *Four Squares* around the classroom so they see a variety of ways to display information about a particular problem.

Learning Target

Identify equivalent expressions and apply properties to generate equivalent expressions.

Success Criteria

- Explain the meaning of equivalent expressions.
- Use properties of addition to generate equivalent expressions.
- Use properties of multiplication to generate equivalent expressions.

Review & Refresh

1. $10 + p$ **2.** $6m$

3. $b \div 15$ **4.** $s - 7$

5. $2^2 \times 3^2$ **6.** $2^4 \times 3^2$

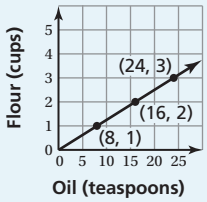
7. 3×7^2 **8.** 5×41

9. 11.592 **10.** 4.543

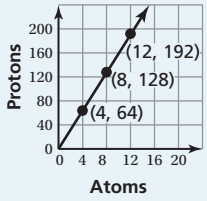
11. 13.641 **12.** 6.412

Concepts, Skills, & Problem Solving

13.



14.



15. B **16.** C

17. A **18.** D

19. Comm. Prop. of Mult.

20. Assoc. Prop. of Add.

21. Assoc. Prop. of Mult.

22. Comm. Prop. of Add.

23. Add. Prop. of Zero

24. Mult. Prop. of One

25. no; The statement illustrates the Commutative Property of Addition.

26. $6 + (5 + x) = (6 + 5) + x$
Assoc. Prop. of Add.
$= 11 + x$
Add 6 and 5.

27. $(14 + y) + 3 = (y + 14) + 3$
Comm. Prop. of Add.
$= y + (14 + 3)$
Assoc. Prop. of Add.
$= y + 17$
Add 14 and 3

28–37. See Additional Answers.

Assignment Guide and Concept Check

Check out the Dynamic Assessment System.
BigIdeasMath.com

Level	Assignment 1	Assignment 2
Emerging	4, 6, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24	25, 26, 28, 31, 33, 38, 39, 41, 47
Proficient	4, 6, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 43, 46	25, 27, 28, 31, 33, 39, 40, 41, 44
Advanced	4, 6, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 44, 46	25, 31, 32, 33, 34, 40, 42, 48, 49

- Assignment 1 is for use after students complete the Self-Assessment for Concepts & Skills.
- Assignment 2 is for use after students complete the Self-Assessment for Problem Solving.
- The red exercises can be used as a concept check.

Review & Refresh Prior Skills

Exercises 1–4 Writing Expressions
Exercises 5–8 Writing a Prime Factorization
Exercises 9–12 Adding and Subtracting Decimals
Exercises 13 and 14 Graphing Ratio Relationships

Common Errors

- **Exercises 26–37** Students are often confused by the differences between the Commutative and Associative Properties and may incorrectly label steps. Show students that the Associative Property moves the parentheses but does not change the position of the terms. The Commutative Property changes the positions of the terms.

5.3 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Write the phrase as an expression.

1. 10 added to a number p

2. the product of 6 and a number m

3. the quotient of a number b and 15

4. 7 fewer than a number s

Write the prime factorization of the number.

5. 36

6. 144

7. 147

8. 205

Evaluate the expression.

9. $8.092 + 3.5$

10. $16.78 - 12.237$

11. $9.17 + 1.83 + 2.641$

12. $8.43 - 6.218 + 4.2$

Represent the ratio relationship using a graph.

13.

Oil (teaspoons)	8	16	24
Flour (cups)	1	2	3

14.

Atoms	4	8	12
Protons	64	128	192

Concepts, Skills, & Problem Solving

MATCHING **Match the expression with an equivalent expression.**
(See Exploration 1, p. 215.)

15. $3 + 3 + y$

16. $(y \cdot y) \cdot 3$

17. $3 \cdot 1 \cdot y$

18. $(3 + 0) + (y + y)$

A. $y \cdot 3$

B. $y + 3 + 3$

C. $y(3 \cdot y)$

D. $(3 + y) + y$

IDENTIFYING PROPERTIES **Tell which property the statement illustrates.**

19. $5 \cdot p = p \cdot 5$

20. $2 + (12 + r) = (2 + 12) + r$

21. $4 \cdot (x \cdot 10) = (4 \cdot x) \cdot 10$

22. $x + 7.5 = 7.5 + x$

23. $(c + 2) + 0 = c + 2$

24. $a \cdot 1 = a$

25. **YOU BE THE TEACHER** Your friend states the property that the statement illustrates. Is your friend correct? Explain your reasoning.

$(7 + x) + 3 = (x + 7) + 3$

~~Associative Property of Addition~~

USING PROPERTIES **Simplify the expression. Explain each step.**

26. $6 + (5 + x)$

27. $(14 + y) + 3$

28. $6(2b)$

29. $7(9w)$

30. $3.2 + (x + 5.1)$

31. $(0 + a) + 8$

32. $9 \cdot c \cdot 4$

33. $(18.6 \cdot d) \cdot 1$

34. $\left(3k + 4\frac{1}{5}\right) + 8\frac{3}{5}$

35. $(2.4 + 4n) + 9$

36. $(3s) \cdot 8$

37. $z \cdot 0 \cdot 12$

38. GEOMETRY The expression $12 + x + 4$ represents the perimeter of a triangle. Simplify the expression.

39. MP PRECISION A case of scout cookies has 10 cartons. A carton has 12 boxes. The amount you earn on a whole case is $10(12x)$ dollars.

a. What does x represent?

b. Simplify the expression.

40. MODELING REAL LIFE A government estimates the cost to design new radar technology over a period of m months. The government estimates \$840,000 for equipment, \$15,000 for software, and \$40,000 per month for wages. Use an algebraic expression to find the total cost the government estimates when the project takes 16 months to complete.

WRITING EXPRESSIONS Write the phrase as an expression. Then simplify the expression.

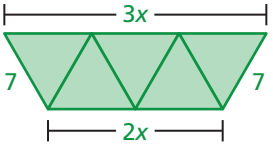
41. 7 plus the sum of a number x and 5

42. the product of 8 and a number y, multiplied by 9

USING PROPERTIES Copy and complete the statement using the specified property.

	Property	Statement
43.	Associative Property of Multiplication	$7(2y) =$
44.	Commutative Property of Multiplication	$13.2 \cdot (x \cdot 1) =$
45.	Associative Property of Addition	$17 + (6 + 2x) =$
46.	Addition Property of Zero	$2 + (c + 0) =$
47.	Multiplication Property of One	$1 \cdot w \cdot 16 =$

48. GEOMETRY Five identical triangles form the trapezoid shown.



a. What is the perimeter of the trapezoid?

b. How can you use some or all of the triangles to form a new trapezoid with a perimeter of $3x + 14$? Explain your reasoning.

49. DIG DEEPER! You and a friend sell hats at a fair booth. You sell 16 hats on the first shift and 21 hats on the third shift. Your friend sells x hats on the second shift.

a. The expression $37(14) + 10x$ represents the amount (in dollars) that you both earn. How can you tell that your friend is selling the hats for a lower price?

b. You earn more money than your friend. What can you say about the value of x?

Common Errors

- **Exercises 43–47** If students are struggling to identify properties, ask them to think about how they can complete the statement without looking at the property. Once they know how to simplify the expression, tell them to look at the definitions of the properties and identify which property they used.

Mini-Assessment

Simplify the expression. Explain each step.

1. $7 + (3 + y)$

$(7 + 3) + y$	Associative Property of Addition
$10 + y$	Add 7 and 3.

2. $4(8x)$

$(4 \cdot 8)x$	Associative Property of Multiplication
$32x$	Multiply 4 and 8.

3. $6 + (b + 4)$

$6 + (4 + b)$	Commutative Property of Addition
$(6 + 4) + b$	Associative Property of Addition
$10 + b$	Add 6 and 4.

4. $(7 \cdot d) \cdot 1$

$7 \cdot (d \cdot 1)$	Associative Property of Multiplication
$7 \cdot d$	Multiplication Property of One
$7d$	Rewrite.

5. The expression $5 + x + 7$ represents the perimeter of a triangle. Simplify the expression. $12 + x$

Section Resources

Surface Level	Deep Level
Resources by Chapter • Extra Practice • Reteach • Puzzle Time Student Journal • Self-Assessment • Practice Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension Graphic Organizers Dynamic Assessment System • Section Practice

Concepts, Skills, & Problem Solving

38. $x + 16$

39. **a.** the amount per box

b. $120x$

40. \$1,495,000

41. $7 + (x + 5)$; $x + 12$

42. $8 \cdot (y \cdot 9)$; $72y$

43. $(7 \cdot 2) \cdot y$

44. $13.2 \cdot (1 \cdot x)$

45. $(17 + 6) + 2x$

46. $2 + c$

47. $w \cdot 16$

48. **a.** $5x + 14$

b. *Sample answer:* Remove the two triangles at the right so the side lengths are x, 7, $2x$, and 7.

49. **a.** *Sample answer:* From the expression, the 37 hats you sold cost \$14 each and the x hats your friend sold cost \$10 each.

b. x is at most 51.

Learning Target

Apply the Distributive Property to generate equivalent expressions.

Success Criteria

- Explain how to apply the Distributive Property.
- Use the Distributive Property to simplify algebraic expressions.
- Use the Distributive Property to combine like terms.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Explain that the word *distributive* is related to the word *distribute*. When you distribute flyers to a class, you give a flyer to each person in the class. When using the Distributive Property to simplify an expression, you multiply each term in the parentheses by the number outside the parentheses before finding the sum or difference.

Exploration 1

a. $3x + 12$; $48 + 8y$; $9n$; n; Rewrite the area of the large rectangle as the sum of the areas of the smaller rectangles; There is a total of 9 parts and a difference of 1 part.

b. Answers will vary. Students should evaluate each expression for several values to show they are equivalent.

c. See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
6.EE.A.2b, 6.EE.A.3, 6.EE.A.4

Preparing to Teach

- In prior courses, students used the Distributive Property for numerical expressions. Now they will apply the Distributive Property to algebraic expressions. This property is very important to algebraic work in future courses. Allow students the time they need to be successful.
- In the exploration, students will use models and basic computation facts to find and verify equivalent algebraic expressions.
- **MP2 Reason Abstractly and Quantitatively:** Mathematically proficient students can identify quantities in a model and interpret the relationship. A model provides a foundation for the Distributive Property that is accessible to all students.

Motivate

- Draw two rectangles on the board that have the same width but different lengths. Label the dimensions.
- Have students write an expression for the total area of the rectangles: (_____ • _____) + (_____ • _____).
- Rearrange the rectangles by aligning the shortest sides to form one rectangle. Label the dimensions. Have students write an expression for the area: _____ • (_____ + _____).

? "Can the expressions be set equal to each other? Explain." Yes, both expressions represent the same area.

Exploration 1

? For the first two models, point out that there is one label for the width but two labels for the length. "Are the lengths represented by these two labels equal?" yes "If you consider the entire model as one large rectangle and then consider it as two smaller rectangles next to each other, are the total areas different? Explain." No, the two smaller rectangles together will have the same area as the larger rectangle because they represent the same area but in a different arrangement.

- Use the two different ways of identifying the length of each rectangle to create two different equivalent expressions to represent the area.

? Students will be able to count the boxes in the third model to find their sum and difference. This is similar to what students did with pictures models in prior courses. They should see $5n + 4n$ is $9n$. "What happens when the 4 n boxes are next to the 5 n boxes?" It forms a rectangle that is n wide and $(5 + 4)$ long. So the area is $n(5 + 4)$ square units. The Commutative Property allows students to write the expression as $9n$. Have students explain $5n - 4n$.

- In part (c), listen for understanding that the numerical expressions are equivalent because they have the same value. Students should also realize that the operations will be the same in both algebraic expressions and the values of the algebraic expressions will be the same for any value of x.
- **MP7 Look for and Make Use of Structure:** As students practice using both models, they will realize that the resulting expressions are equivalent and provide a structure for simplifying. Later in the lesson, this structure helps students reverse the Distributive Property to combine **like terms**.

5.4 The Distributive Property

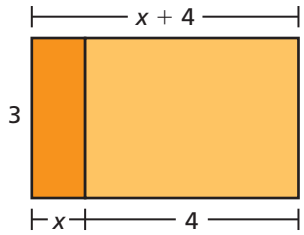
Learning Target: Apply the Distributive Property to generate equivalent expressions.

Success Criteria:
- I can explain how to apply the Distributive Property.
- I can use the Distributive Property to simplify algebraic expressions.
- I can use the Distributive Property to combine like terms.

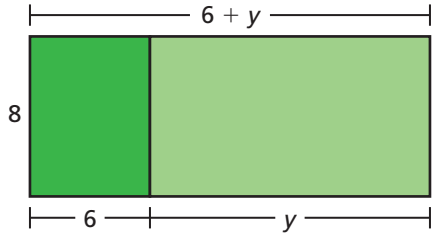
EXPLORATION 1 Using Models to Simplify Expressions

Work with a partner.

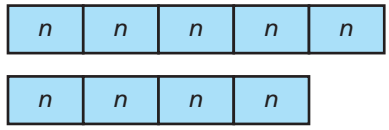
a. Use the models to simplify the expressions. Explain your reasoning.



$3(x + 4) =$ ____



$8(6 + y) =$ ____



$5n + 4n =$ ____

$5n - 4n =$ ____

b. In part (a), check that the original expressions are equivalent to the simplified expressions.

c. You used the Distributive Property in a previous course. Use the example to explain the meaning of the property.

Distributive Property: $6(20 + 3) = 6(20) + 6(3)$

Is this property true for algebraic expressions? Explain your reasoning.

Math Practice

Find Entry Points

How can the Distributive Property be used to find the product of 9 and 32?

5.4 Lesson

Key Vocabulary
like terms, *p. 223*

Key Idea

Distributive Property

Words To multiply a sum or difference by a number, multiply each term in the sum or difference by the number outside the parentheses. Then simplify.

Numbers $3(7 + 2) = 3 \times 7 + 3 \times 2$

$3(7 - 2) = 3 \times 7 - 3 \times 2$

Algebra $a(b + c) = ab + ac$

$a(b - c) = ab - ac$

EXAMPLE 1 Simplifying Algebraic Expressions

Use the Distributive Property to simplify each expression.

a. $4(n + 5)$

$4(n + 5) = 4(n) + 4(5)$ Distributive Property

$= 4n + 20$ Multiply.

b. $12(2y - 3)$

$12(2y - 3) = 12(2y) - 12(3)$ Distributive Property

$= 24y - 36$ Multiply.

c. $\frac{1}{2}(6y - 2z)$

$\frac{1}{2}(6y - 2z) = \frac{1}{2}(6y) - \frac{1}{2}(2z)$ Distributive Property

$= 3y - z$ Multiply.

d. $9(6 + x + 2)$

$9(6 + x + 2) = 9(6) + 9(x) + 9(2)$ Distributive Property

$= 54 + 9x + 18$ Multiply.

$= 9x + 54 + 18$ Commutative Property of Addition

$= 9x + (54 + 18)$ Associative Property of Addition

$= 9x + 72$ Add 54 and 18.

You can use the Distributive Property when there are more than two terms in the sum or difference.

Try It **Use the Distributive Property to simplify the expression.**

1. $7(a + 2)$
2. $3(d - 11)$
3. $12\left(a + \frac{2}{3}b\right)$
4. $7(2 + 6 - 4d)$

Laurie's Notes

Scaffolding Instruction

- In the exploration, students built upon experiences with the Distributive Property from prior courses and now have a sense of the usefulness of the Distributive Property.
- **Emerging:** Students understand that there is an order for distributing over the parentheses but make computational errors or neglect to multiply all the parts in the parentheses. They will benefit from guided instruction for the examples.
- **Proficient:** Students understand the models and are confident in applying the Distributive Property. Have students self-assess using the Try It exercises.

Key Idea

- Draw attention to how the multiplication is shown.
- Students may ask why you would want to write 3×9 as $3(7 + 2)$ and 3×5 as $3(7 - 2)$. The answer is that you wouldn't! Simple values are being used to demonstrate the property instead of using a problem such as 8×67.
- Spend more time discussing the algebraic representation. In the problem $a(b + c)$, ask what *operations* students see. Not all students will see two. Some may only see addition.
- Explain that the blue arrows are used to remind you to distribute 3 to all parts of the expression in the parentheses. The arrows can also be written above the expression.

❓ "Do you know where the arrows go in the other three examples?"

EXAMPLE 1

- Slowly work through each part. Introducing a variable expression is *not* a small change!
- Using the order of operations, you should complete the operation inside the parentheses first, however, n and 5 are not **like terms** so they cannot be combined. The next operation to be performed is multiplication, so distribute 4 to n and 5. Use arrows to show that 4 is being distributed to both n and 5.
- **Common Error:** Students may only multiply the first term in the parentheses when simplifying instead of all the terms. It is important to watch each student simplify at least one expression and offer guidance as needed. Guided practice now will prevent repeated mistakes later.

Try It

Think-Pair-Share: Have students read each exercise independently and then work in pairs to simplify the expressions. After completing the exercises, the pair should compare their answers with another pair and discuss any discrepancies. Listen for understanding of using the Distributive Property to simplify algebraic expressions.

- Look for students avoiding Exercise 3. Encourage students by asking probing questions that involve fractions.
- In Exercise 4, some students may recognize that $7(2 + 6 - 4d)$ is equivalent to $7(8 - 4d)$.

Extra Example 1

Use the Distributive Property to simplify each expression.

a. $11(x + 7)$ $11x + 77$

b. $8(c - 4)$ $8c - 32$

c. $\frac{1}{3}(9x - 3y)$ $3x - y$

d. $3(4 + r + 3)$ $3r + 21$

ELL Support

Have students work in pairs to complete Try It Exercises 1–4.
Beginner: Use the Distributive Property to simplify the expression and write out each step.
Intermediate: Verbally explain each step using complete sentences.
Advanced: Explain how the Distributive Property is used to simplify the expression.

Try It

1. $7a + 14$
2. $3d - 33$
3. $12a + 8b$
4. $56 - 28d$

Extra Example 2

Simplify each expression.

a. $7w + 11 + 3w - 4$ $10w + 7$

b. $b + b + b + b$ $4b$

c. $2c + 3(f + 5c)$ $17c + 3f$

Try It

5. $8 + 2z$

6. $4b + 17$

ELL Support

Provide support for Self-Assessment for Concepts & Skills Exercise 7 by reading the ELL Support on page T-221 to students. Allow students to work in pairs for Exercises 8–10. Have pairs display their answers on whiteboards for your review.

Self-Assessment for Concepts & Skills

7. *Sample answer:* You must distribute or give the number outside the parentheses to the terms inside the parentheses.

8. $3x + 30$

9. $60n - 30$

10. $15w + 5$

Laurie's Notes

Discuss

Write $5x + 2x + 2 + 19$ on the board. Ask, "How many terms are in the expression?" 4 "Why are 2 and 19 **like terms**?" They are both constants. "Why are $5x$ and $2x$ like terms?" They both have the same variable raised to the same exponent.

EXAMPLE 2

- Refer students to the push-pin note. The Distributive Property works in reverse. Remind them that this is similar to what they did with the n boxes in the exploration.
- Work through parts (a) and (b) as shown. These problems should make sense to students.
- In part (a), you may want to show combining like terms that contain variables using the visual shown.

$$3x + 2x = \overbrace{\underbrace{x + x + x}_{3x} + \underbrace{x + x}_{2x}}^{5x} = 5x$$

Part (c) is more involved. Write part (c) and ask, "How many operations do you see?" 5 operations "What are they?" multiply, add, multiply, subtract, multiply

- **MP7 Look for and Make Use of Structure:** Help students recognize the structure of this expression. I like to read it as, "7 times the variable z plus 2 times the quantity z minus $5y$." Before working with the $7z$, the Distributive Property must be used.
- Use arrows to help students see 2 is distributed to both z and $5y$.

Try It

- **Neighbor Check:** Have students work independently, and then have their neighbors check their work. Have students discuss any discrepancies.

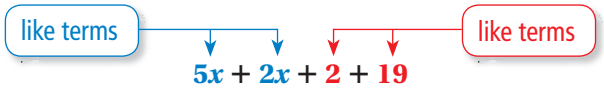
Thumbs Up: Ask students to assess their understanding of using the Distributive Property to combine like terms.

Self-Assessment for Concepts & Skills

- Exercise 7 asks students to explain their interpretations of the Distributive Property before applying the skill in Exercises 8–10. This will help you analyze your students' level of understanding.
- Identify the reasons for incorrect answers for Exercises 8–10. Are the errors computational? Can students complete Exercises 8 and 9 successfully but not 10, or vice versa? Make sure students are aware of the reasons for any mistakes.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

In an algebraic expression, **like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.



You can use the Distributive Property to *combine* like terms.

EXAMPLE 2 Combining Like Terms

Simplify each expression.

When you combine like terms, you are using the Distributive Property. You are applying the rules $ab + ac = a(b + c)$ and $ab - ac = a(b - c)$.

a. $3x + 9 + 2x - 5$

$3x + 9 + 2x - 5 = 3x + 2x + 9 - 5$ Commutative Property of Addition

$= (3 + 2)x + 9 - 5$ Distributive Property

$= 5x + 4$ Simplify.

b. $y + y + y$

$y + y + y = 1y + 1y + 1y$ Multiplication Property of One

$= (1 + 1 + 1)y$ Distributive Property

$= 3y$ Add coefficients.

c. $7z + 2(z - 5y)$

$7z + 2(z - 5y) = 7z + 2(z) - 2(5y)$ Distributive Property

$= 7z + 2z - 10y$ Multiply.

$= (7 + 2)z - 10y$ Distributive Property

$= 9z - 10y$ Add coefficients.

Try It **Simplify the expression.**

5. $8 + 3z - z$

6. $3(b + 5) + b + 2$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. WRITING One meaning of the word *distribute* is *to give something to each member of a group*. How can this help you remember the Distributive Property?

SIMPLIFYING EXPRESSIONS **Use the Distributive Property to simplify the expression.**

8. $3(x + 10)$

9. $15(4n - 2)$

10. $2w + 4 + 13w + 1$

EXAMPLE 3 Modeling Real Life

José is x years old. His brother, Felipe, is 2 years older than José. Their aunt, Maria, is three times as old as Felipe. Write and simplify an expression that represents Maria's age in years.

Use a table to organize the given information and write an expression that represents each person's age in years.

Name	Description	Expression
José	He is x years old.	x
Felipe	He is 2 years *older* than José. So, *add* 2 to x.	$x + 2$
Maria	She is three *times* as old as Felipe. So, *multiply* 3 and $(x + 2)$.	$3(x + 2)$

Look Back
If José is 10 years old, then Felipe is $10 + 2 = 12$ years old and Maria is $3(12) = 36$ years old. So, you should obtain 36 when you evaluate $3x + 6$ for $x = 10$.

$3x + 6 = 3(10) + 6$
$= 36$ ✓

Simplify the expression that represents Maria's age.

$3(x + 2) = 3(x) + 3(2)$ Distributive Property
$= 3x + 6$ Multiply.

Maria's age in years is represented by the expression $3x + 6$.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

11. You purchase a remote-controlled drone for d dollars. Your friend purchases a drone that costs \$35 more than your drone. Your brother purchases a drone that costs three times as much as your friend's drone. Write and simplify an expression that represents the cost (in dollars) of your brother's drone.

12. Write and simplify an expression that represents the total cost (in dollars) of buying the items shown for each member of a baseball team.

13. DIG DEEPER! One molecule of caffeine contains x oxygen atoms, twice as many nitrogen atoms as oxygen atoms, 4 more carbon atoms than nitrogen atoms, and 1.25 times as many hydrogen atoms as carbon atoms. Write and simplify an expression that represents the number of hydrogen atoms in one molecule of caffeine.

Laurie's Notes

EXAMPLE 3

- Write the table representing each person's age in words and algebraically. Connect this problem to writing expressions in Section 5.2.
- "Is there a way to check your expression?" Choose an age for José and use it to find Felipe's and Maria's ages. Substitute the age of José in the simplified expression to check that it is the same age you calculated for Maria. Refer students to the Look Back note.
- **Extension:** "If José is 12, find each person's age." Felipe: 14; Maria: 42

Self-Assessment for Problem Solving

- The goal for all students is to feel comfortable with the problem-solving plan. It is important for students to problem-solve in class, where they may receive support from you and their peers. Keep in mind that some students may only be ready to complete the first step.
- Each problem has an application of the Distributive Property. Remind students that making tables may help them organize their thoughts.
- Have students complete Exercises 11 and 12 independently, and then prepare to explain their methods.
- **Turn and Talk:** Have students review their methods and answers. Allow time for discussion and to ask questions that they are unable to resolve themselves.
- Have pairs work together on Exercise 13 to write equivalent expressions for the number of hydrogen atoms. Encourage pairs to substitute values to verify their expressions. Ask volunteers to explain their methods.
- Have students use *Fist of Five* to indicate their understanding of the success criteria.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Formative Assessment Tip

One-Minute Card

This technique provides a quick assessment of students' understanding of a concept. Write a short prompt on the board and allow 1 minute for students to consider the prompt. Give each student an index card and allow 1 minute for students to write their responses. When time is up, collect the cards and review the responses. The next day, spend a few minutes discussing any misconceptions or exceptional responses.

Closure

- **One-Minute Card:** Explain how to simplify the expression $5(n + 8)$ using the Distributive Property.

Extra Example 3

You are x years old. Your father is 25 years older than you. Your grandfather is two times older than your father. Write and simplify an expression that represents your grandfather's age in years. $2(x + 25)$; $2x + 50$

Self-Assessment for Problem Solving

11. $3(d + 35) = 3d + 105$

12. $(10 + x)y = 10y + xy$

13. $1.25(2x + 4) = 2.5x + 5$

Learning Target

Apply the Distributive Property to generate equivalent expressions.

Success Criteria

- Explain how to apply the Distributive Property.
- Use the Distributive Property to simplify algebraic expressions.
- Use the Distributive Property to combine like terms.

Review & Refresh

1. $(s + 4) + 8 = s + (4 + 8)$
Assoc. Prop. of Add.
$= s + 12$
Add 4 and 8.

2. $(12 + x) + 2 = (x + 12) + 2$
Comm. Prop. of Add.
$= x + (12 + 2)$
Assoc. Prop. of Add.
$= x + 14$
Add 12 and 2.

3. $3(4n) = (3 \cdot 4)n$
Assoc. Prop. of Mult.
$= 12n$
Multiply 3 and 4.

4. 15 boys, 12 girls

5. 12 boys, 20 girls

6. 9 boys, 13 girls

7. 14 boys, 8 girls

8. 43 **9.** 123

10. $52\frac{5}{12}$ **11.** $260\frac{8}{31}$

Concepts, Skills, & Problem Solving

12. $5z + 30$; Rewrite area of the large rectangle as the sum of the areas of the smaller rectangles.

13. $6s$; The sum of 4 parts and 2 parts is 6 parts.

14. $3x + 12$ **15.** $10b - 60$

16. $6s - 54$ **17.** $56 + 7y$

18. $96 + 8a$ **19.** $18n + 9$

20. $72 - 12k$ **21.** $90 - 54w$

22. $63 + 9c$ **23.** $3 + \frac{1}{4}x$

24. $40g + 24$ **25.** $78 + 6z$

26. $4x + 4y$ **27.** $25x - 25y$

28. $7p + 7q + 63$

29. $n + 2 + 3m$

30. C **31.** A

32. D **33.** B

34. $5(r + 15)$, $5r + 5 \cdot 15$;
$5(r + 15) = 5r + 5 \cdot 15$

Assignment Guide and Concept Check

Level	Assignment 1	Assignment 2
Emerging	3, 7, 11, 12, 13, 14, 16, 19, 25, 36, 37, 38	29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 49
Proficient	3, 7, 11, 12, 13, 16, 18, 19, 25, 36, 38, 43	29, 30, 31, 32, 33, 35, 42, 44, 48, 49, 50, 51
Advanced	3, 7, 11, 12, 13, 21, 23, 24, 29, 43, 45, 48	42, 46, 49, 50, 51, 52, 53

- Assignment 1 is for use after students complete the Self-Assessment for Concepts & Skills.
- Assignment 2 is for use after students complete the Self-Assessment for Problem Solving.
- The red exercises can be used as a concept check.

Review & Refresh Prior Skills

Exercises 1–3 Using Properties
Exercises 4–7 Using a Tape Diagram
Exercises 8–11 Dividing Whole Numbers

Common Errors

- **Exercises 14–29** Students may forget to distribute to each term in the parentheses, especially when there are more than two terms. Tell students to write the expression in the parentheses on their papers and draw arrows from the number being distributed to each term.

5.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Simplify the expression. Explain each step.

1. $(s + 4) + 8$ **2.** $(12 + x) + 2$ **3.** $3(4n)$

You are given the difference of the numbers of boys and girls in a class and the ratio of boys to girls. How many boys and how many girls are in the class?

4. 3 more boys; 5 for every 4

5. 8 more girls; 3 for every 5

6. 4 more girls; 9 for every 13

7. 6 more boys; 7 for every 4

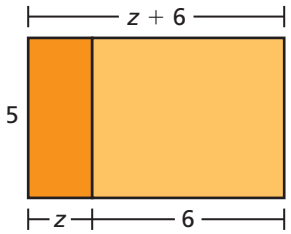
Divide.

8. $301 \div 7$ **9.** $1722 \div 14$ **10.** $629 \div 12$ **11.** $8068 \div 31$

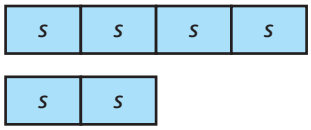
Concepts, Skills, & Problem Solving

USING MODELS **Use the model to simplify the expression. Explain your reasoning.** (See Exploration 1, p. 221.)

12. $5(z + 6) =$ ☐



13. $4s + 2s =$ ☐



SIMPLIFYING EXPRESSIONS **Use the Distributive Property to simplify the expression.**

14. $3(x + 4)$ **15.** $10(b - 6)$ **16.** $6(s - 9)$ **17.** $7(8 + y)$

18. $8(12 + a)$ **19.** $9(2n + 1)$ **20.** $12(6 - k)$ **21.** $18(5 - 3w)$

22. $9(3 + c + 4)$ **23.** $\frac{1}{4}(8 + x + 4)$ **24.** $8(5g + 5 - 2)$ **25.** $6(10 + z + 3)$

26. $4(x + y)$ **27.** $25(x - y)$ **28.** $7(p + q + 9)$ **29.** $\frac{1}{2}(2n + 4 + 6m)$

MATCHING **Match the expression with an equivalent expression.**

30. $6(n + 4)$ **31.** $2(3n + 9)$ **32.** $6(n + 2)$ **33.** $3(2n + 3)$

A. $3(2n + 6)$ **B.** $6n + 9$ **C.** $3(2n + 8)$ **D.** $6n + 12$

34. MP **STRUCTURE** Each day, you run on a treadmill for r minutes and lift weights for 15 minutes. Which expressions can you use to find how many minutes of exercise you do in 5 days? Explain your reasoning.

$5(r + 15)$ $5r + 5 \cdot 15$ $5r + 15$ $r(5 + 15)$

35. MODELING REAL LIFE A cheetah can run 103 feet per second. A zebra can run x feet per second. Write and simplify an expression that represents how many feet farther the cheetah can run in 10 seconds.

COMBINING LIKE TERMS **Simplify the expression.**

36. $6(x + 4) + 1$

37. $5 + 8(3 + x)$

38. $x + 3 + 5x$

39. $7y + 6 - 1 + 12y$

40. $4d + 9 - d - 8$

41. $n + 3(n - 1)$

42. $2v + 8v - 5v$

43. $5(z + 4) + 5(2 - z)$

44. $2.7(w - 5.2)$

45. $\frac{2}{3}y + \frac{1}{6}y + y$

46. $\frac{3}{4}\left(z + \frac{2}{5}\right) + 2z$

47. $7(x + y) - 7x$

48. $4x + 9y + 3(x + y)$

49. YOU BE THE TEACHER Your friend simplifies the expression. Is your friend correct? Explain your reasoning.

$$8x - 2x + 5x = 8x - 7x$$
$$= (8 - 7)x$$
$$= x$$

50. MP REASONING Evaluate each expression by (1) using the Distributive Property and (2) evaluating inside the parentheses first. Which method do you prefer? Is your preference the same for both expressions? Explain your reasoning.

a. $2(3.22 - 0.12)$

b. $12\left(\frac{1}{2} + \frac{2}{3}\right)$

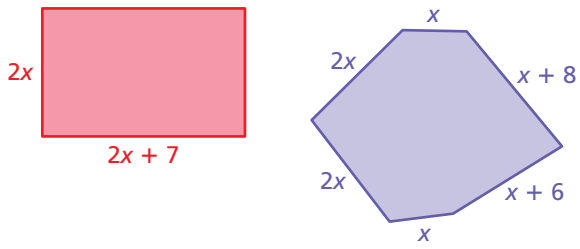
51. DIG DEEPER! An art club sells 42 large candles and 56 small candles.

a. Write and simplify an expression that represents the profit.

b. A large candle costs \$5, and a small candle costs \$3. What is the club's profit?

Profit = Price − Cost

52. MP REASONING Find the difference between the perimeters of the rectangle and the hexagon. Interpret your answer.



53. PUZZLE Add one set of parentheses to the expression $7 \cdot x + 3 + 8 \cdot x + 3 \cdot x + 8 - 9$ so that it is equivalent to $2(9x + 10)$.

Common Errors

- **Exercises 36–48** Students may struggle to combine like terms when using the Distributive Property. Remind them to bring the common factor outside the parentheses and place the terms that will be added or subtracted inside the parentheses.

Mini-Assessment

Use the Distributive Property to simplify the expression.

1. $6(x + 4)$ $6x + 24$
2. $14(g - 2)$ $14g - 28$
3. $3(a + 12)$ $3a + 36$
4. $8(n + 3 + 4)$ $8n + 56$
5. You are y years old. Your cousin is 3 years older than you. Your friend is two times as old as your cousin. Write and simplify an expression that represents your friend's age in years. $2(y + 3)$; $2y + 6$

Section Resources

Surface Level	Deep Level
Resources by Chapter • Extra Practice • Reteach • Puzzle Time Student Journal • Self-Assessment • Practice Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension Graphic Organizers Dynamic Assessment System • Section Practice

Concepts, Skills, & Problem Solving

35. $10(103 - x) = 1030 - 10x$

36. $6x + 25$ **37.** $29 + 8x$

38. $6x + 3$ **39.** $19y + 5$

40. $3d + 1$ **41.** $4n - 3$

42. $5v$ **43.** 30

44. $2.7w - 14.04$

45. $1\frac{5}{6}y$ **46.** $2\frac{3}{4}z + \frac{3}{10}$

47. $7y$ **48.** $7x + 12y$

49. no; $8x - 2x + 5x = 11x$

50. **a.** 6.2

b. 14

Sample answer: The preferred method is not the same for both expressions. For part (a), evaluating inside the parentheses first makes for easier and fewer calculations. For part (b), using the Distributive Property will eliminate the fractions.

51. **a.** $42(10 - x) + 56(5 - y)$
$= 700 - 42x - 56y$

b. $322

52. 0; The perimeters are equal.

53. $7(x + 3) + 8 \cdot x + 3 \cdot x + 8 - 9$
$= 2(9x + 10)$

Learning Target

Factor numerical and algebraic expressions.

Success Criteria

- Use the Distributive Property to factor numerical expressions.
- Identify the greatest common factor of terms including variables.
- Use the Distributive Property to factor algebraic expressions.
- Interpret factored expressions in real-life problems.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL Support

Remind students that the word *factor* has a special meaning in math. In everyday life, a factor is a circumstance that influences an outcome. In math, a factor is a number or quantity that when multiplied with another produces a given number or expression. Remind students that when finding the greatest common factor of two numbers, it will be the greatest of the factors that are shared by both numbers.

Exploration 1

a–c. See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
6.NS.B.4, 6.EE.A.2b, 6.EE.A.3, 6.EE.A.4

Preparing to Teach

- In this lesson, students will extend their understanding of the greatest common factor and the Distributive Property to factoring expressions. Recognizing common factors will allow students to represent expressions in different forms.
- The models in the exploration are similar to the models that were used to explore the Distributive Property in the previous section.
- This section links students' understanding of the number system to expressions and equations. Students should make connections between arithmetic and algebra. Common factors are important to both and provide flexibility in representing quantitative relationships.

Motivate

- Write this problem on the board: $\frac{2}{3} \cdot \frac{4}{5} \cdot \frac{5}{8} = ?$
- When simplifying the product, students should quickly see that there are many common factors. Tell students that they should look for the *greatest* common factor between the numerator and denominator to simplify.

Exploration 1

- Begin working through the first model as a class.
- Ask, "Which number can represent the width of the larger rectangle?" 1, 2, 4, or 8 "Is the width a factor of both 8 and 24?" yes
- **Note:** Tell students that when factoring, they should not use 1 as a common factor because it will not change the expression in the parentheses. Although 1 is a possible width of the larger rectangle, students should choose a number greater than 1 to use in their expressions.
- Once students choose the width, have them fill in the other question marks in the model. Do students know where to place the numbers in the expression?
- **MP8 Look for and Express Regularity in Repeated Reasoning:** Discuss the Math Practice note with students and have pairs check their answers.
- The first model has three possible factors greater than one, so you can ask each pair to find and check another expression. Have students share their answers with the class to see if all possible widths have been used.
- Have students work with their partners to finish part (a) and then share their answers.
- "Why is there only one expression for the $3x$ and 18 rectangle?" 3 and x are the only factors of $3x$ and x is not a factor of 18.
- Listen to conversations as they complete parts (b) and (c). Ask volunteers to share their strategies with the class.

5.5 Factoring Expressions

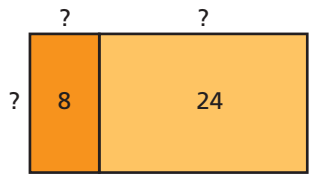
Learning Target: Factor numerical and algebraic expressions.

Success Criteria:
- I can use the Distributive Property to factor numerical expressions.
- I can identify the greatest common factor of terms including variables.
- I can use the Distributive Property to factor algebraic expressions.
- I can interpret factored expressions in real-life problems.

EXPLORATION 1 Finding Dimensions

Work with a partner.

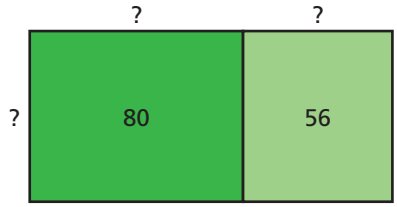
a. The models show the area (in square units) of each part of a rectangle. Use the models to find missing values that complete the expressions. Explain your reasoning.



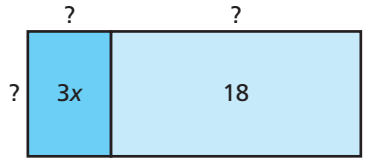
$8 + 24 = \square(\square + \square)$

Math Practice

Evaluate Results
Do your answers in the first two models seem reasonable? How can you check your answers?



$80 + 56 = \square(\square + \square)$



$3x + 18 = \square(\square + \square)$

b. In part (a), check that the original expressions are equivalent to the expressions you wrote. Explain your reasoning.

c. Explain how you can use the Distributive Property to rewrite a sum of two whole numbers with a common factor.

5.5 Lesson

Key Vocabulary
factoring an expression, *p. 228*

Key Idea

Factoring an Expression

Words Writing a numerical expression or algebraic expression as a product of factors is called **factoring the expression**. You can use the Distributive Property to factor expressions.

Numbers $3 \cdot 7 + 3 \cdot 2 = 3(7 + 2)$
$3 \cdot 7 - 3 \cdot 2 = 3(7 - 2)$

Algebra $ab + ac = a(b + c)$
$ab - ac = a(b - c)$

EXAMPLE 1 Factoring Numerical Expressions

When you factor an expression, you can *factor out* any common factor.

a. Factor 18 + 30 using the GCF.

One way to find the GCF of 18 and 30 is to list their factors.

Factors of 18: ①,②,③,⑥, 9, 18
Factors of 30: ①,②,③, 5,⑥, 10, 15, 30

Circle the common factors.

The GCF of 18 and 30 is 6.

Write each term of the expression as a product of the GCF and the remaining factor. Then use the Distributive Property to factor the expression.

$18 + 30 = 6(3) + 6(5)$ Rewrite using GCF.
$= 6(3 + 5)$ Distributive Property

b. Factor 20 − 12 using the GCF.

One way to find the GCF of 20 and 12 is to list their factors.

Factors of 20: ①,②,④, 5, 10, 20
Factors of 12: ①,②, 3,④, 6, 12

Circle the common factors.

The GCF of 20 and 12 is 4.

Write each term of the expression as a product of the GCF and the remaining factor. Then use the Distributive Property to factor the expression.

$20 - 12 = 4(5) - 4(3)$ Rewrite using GCF.
$= 4(5 - 3)$ Distributive Property

Try It **Factor the expression using the GCF.**

1. $9 + 15$ **2.** $60 + 45$ **3.** $30 - 20$

Laurie's Notes

Scaffolding Instruction

- Students can use the Distributive Property to simplify numerical and algebraic expressions. They will now connect the greatest common factor to factoring expressions.
- **Emerging:** Students may be able to find common factors but struggle to identify the GCF of numerical and algebraic expressions. Students will benefit from guided instruction for the examples.
- **Proficient:** Students can successfully apply the Distributive Property and find the GCF of two numbers. They can apply these skills to factor both numerical and algebraic expressions. Review the Key Idea and have students self-assess using the Try It exercises.

Key Idea

"What does the Distributive Property state?" $a(b + c) = ab + ac$

- **MP7 Look for and Make Use of Structure:** Discuss equality. Students often view the equal sign as computation on the left side that results in the answer on the right. Equality means that the quantities on each side of the equal sign have the same value. The Distributive Property can be written as (the numerical or algebraic expression) = (the factored form of the expression).
- Students may think of factoring out the GCF as the reverse of the Distributive Property.
- **MP6 Attend to Precision:** Use precise language. When an expression is rewritten as the product of factors, it is called **factoring the expression**.
- Explain that when students combined like terms in the last section, they were essentially factoring out the variable.

EXAMPLE 1

"How can you find the GCF of 18 and 30?" *Sample answer:* List the factors of each number and identify the greatest number that is common to both lists.

- In part (a), discuss that you can factor 2 out of the expression. The resulting factors are 2 and a sum of 9 and 15. Then you can factor 3 out of the sum, so the GCF is still 6.
- **MP7 Look for and Make Use of Structure:** While $18 + 30 = 48$ is a true statement, it does not answer the question. The GCF of 18 and 30 is 6, so there will be a factor of 6 in the expression. Students will apply this structure in part (b).

Try It

Have students work independently on the exercises, which are related to the first success criterion. Then have their neighbors check their work.

Extra Example 1

a. Factor $20 + 8$ using the GCF. $4(5 + 2)$

b. Factor $32 - 20$ using the GCF. $4(8 - 5)$

ELL Support

Have students work in pairs to complete Try It Exercises 1–3. Remind students that to find factors, they can use the strategy of starting with 1 and working their way up the whole numbers. "Can the number be divided by 1? 2? 3? 4? 5? and so on." Each partner should write the list of factors for one of the given numbers. Then partners should compare their lists, circle the GCF, and use the Distributive Property to factor the expression.

Beginner: Factor the expression using the GCF and show work.

Intermediate: Verbally describe each step of factoring the expression.

Advanced: Explain the process.

Try It

1. $3(3 + 5)$
2. $15(4 + 3)$
3. $10(3 - 2)$

Extra Example 2

a. Factor $33 - 121x$ using the GCF.
$11(3 - 11x)$

b. Factor $18a + 42b$ using the GCF.
$6(3a + 7b)$

Try It

4. $7(x + 7)$

5. $4(2y - 11)$

6. $5(5a + 2b)$

Self-Assessment for Concepts & Skills

7. $8(2 + 3)$ **8.** $7(7 - 4)$

9. $2(4y + 7)$

10. $6(4n + 3)$;
$6(4n + 3) = 24n + 18$,
the other expressions are equivalent to $24n + 36$.

11. The GCF of 18, 30, and 9 is 3, so factor out a 3;
$3(6x + 10y + 3z)$

12. x; Both terms have x as a factor; $x(x + 4)$

Laurie's Notes

EXAMPLE 2

- Students often have less difficulty factoring algebraic expressions.
- Factor the GCF out of each term in the expression and then use the Distributive Property.

? "How can you check your answers?" Use the Distributive Property to multiply the factors and verify that the product is the same as the original expression.

- Tell students that they are following the same procedure as in Example 1. Example 1 is factoring *numerical* expressions, while Example 2 is factoring *algebraic* expressions. In both examples, students need to factor out the GCF and use the Distributive Property to write the expression as a product of its factors. Help students realize that the methods are the same and both examples result in factored forms of the expressions.

Try It

Have students factor the expressions independently and then share with a partner. Discuss any discrepancies and correct any errors. Students are working on the second and third success criteria.

Self-Assessment for Concepts & Skills

- Have students complete the exercises independently and then compare their answers with a partner.

? Ask, "Did anyone answer Exercise 10 correctly without distributing?" Ask students who did not distribute to share their methods.

- Solicit volunteers to write their work for Exercises 11 and 12 on the board and explain their reasoning.
- **MP6 Attend to Precision:** There are several mathematical terms in these exercises that students are expected to know and understand. There are opportunities for students to communicate precisely and mathematically when explaining their reasoning.

ELL Support

Allow students extra support and the opportunity to practice language by working in pairs. For all exercises that require factoring, have partners each factor a different term and compare factors to find the GCF. Have pairs write their answers on whiteboards for your review. Discuss Exercises 10–12 with students to check for understanding.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

EXAMPLE 2 Factoring Algebraic Expressions

a. Factor $3x + 42$ using the GCF.

You can find the GCF of $3x$ and 42 by writing their prime factorizations.

$3x = 3 \cdot x$
$42 = 2 \cdot 3 \cdot 7$ Circle the common prime factor.

The GCF of $3x$ and 42 is 3. Use the GCF to factor the expression.

$3x + 42 = 3(x) + 3(14)$ Rewrite using GCF.
$= 3(x + 14)$ Distributive Property

b. Factor $63z - 27y$ using the GCF.

You can find the GCF of $63z$ and $27y$ by writing their prime factorizations.

$63z = 3 \cdot 3 \cdot 7 \cdot z$
$27y = 3 \cdot 3 \cdot 3 \cdot y$ Circle the common prime factors.

The GCF of $63z$ and $27y$ is $3 \cdot 3 = 9$. Use the GCF to factor the expression.

$63z - 27y = 9(7z) - 9(3y)$ Rewrite using GCF.
$= 9(7z - 3y)$ Distributive Property

Try It Factor the expression using the GCF.

4. $7x + 49$ **5.** $8y - 44$ **6.** $25a + 10b$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FACTORING EXPRESSIONS Factor the expression using the GCF.

7. $16 + 24$ **8.** $49 - 28$ **9.** $8y + 14$

10. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$3(8n + 12)$ $4(6n + 9)$ $6(4n + 3)$ $12(2n + 3)$

11. MP REASONING Use what you know about factoring to explain how you can factor the expression $18x + 30y + 9z$. Then factor the expression.

12. CRITICAL THINKING Identify the GCF of the terms $(x \cdot x)$ and $(4 \cdot x)$. Explain your reasoning. Then use the GCF to factor the expression $x^2 + 4x$.

EXAMPLE 3 Modeling Real Life

You receive a discount on each book you buy for your electronic reader. The original price of each book is x dollars. You buy 5 books for a total of $(5x - 15)$ dollars. Factor the expression. What can you conclude about the discount?

To factor $5x - 15$, you can find the GCF of $5x$ and 15 by writing their prime factorizations.

$5x = 5 \cdot x$

$15 = 5 \cdot 3$ Circle the common prime factor.

So, the GCF of $5x$ and 15 is 5. Use the GCF to factor the expression.

$5x - 15 = 5(x) - 5(3)$ Rewrite using GCF.

$= 5(x - 3)$ Distributive Property

The factor 5 represents the number of books purchased. The factor $(x - 3)$ represents the discounted price of each book. This factor is a difference of two terms, showing that the original price, $\$x$, of each book is decreased by \$3.

So, the factored expression shows a \$3 discount for every book you buy. The original expression shows a total savings of \$15.

Check Suppose that the original price of each book is \$6. Verify that each expression has the same value when $x = 6$.

$5x - 15 = 5(6) - 15 = 15$

$5(x - 3) = 5(6 - 3) = 15$ ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

13. A youth club receives a discount on each pizza purchased for a party. The original price of each pizza is x dollars. The club leader purchases 8 pizzas for a total of $(8x - 32)$ dollars. Factor the expression. What can you conclude about the discount?

14. Three crates of food are packed on a shuttle departing for the Moon. Each crate weighs x pounds. On the Moon, the combined weight of the crates is $(3x - 81)$ pounds. What can you conclude about the weight of each crate on the Moon?

Laurie's Notes

EXAMPLE 3

- Ask a volunteer to read the problem.

❓ "How many books do you buy and what is the cost of each book?" You buy 5 books and each book costs x dollars.

❓ "Do $5x$ and 15 have any common factors other than 1?" yes "What are they?" 5 Write the expression $5x - 15$ in factored form by factoring out the common factor of 5.

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Looking at the factored form (the answer) and asking what it would be equivalent to if they distribute the 5, helps students make sense of the problem. In other words, they are checking that the answer is correct.

❓ "Because $5x - 15 = 5(x - 3)$, what does the factored form mean in the context of the problem?" There is a \$3 discount on each book you buy. "What is the total savings?" \$15

- Have students read the Check note and verify the expression using another value for the original price of the book.

Self-Assessment for Problem Solving

- Encourage students to use a Four Square to complete the exercises. Until students become comfortable with the problem-solving plan, they may only be ready to complete the first square.
- These problems have a lot of information for students to digest. Allow plenty of time for students to read and re-read the problems.
- In both exercises, students need to see how factoring the expression helps to answer the question. If students are struggling to find an entry point, ask probing questions or refer back to Example 3.
- Have students share answers and discuss the information that the factored form provides.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Closure

- **Example and Non-Example Chart:** Have students work in pairs to create an Example and Non-Example Chart for algebraic expressions with two terms that can be factored. Then have pairs compare their charts with another pair and discuss any discrepancies. Solicit volunteers to share their charts and reasoning. *Sample answer:*

Algebraic Expressions that Can Be Factored

Examples	Non-Examples
$3x + 12$	$4x + 3$
$22y - 11$	$91 - 37y$
$48y + 24z$	$32 + 17c$

Extra Example 3

You receive a discount on each song you purchase for your MP3 player. The original price of each song is x dollars. You purchase 3 songs for a total of $(3x - 6)$ dollars. Factor the expression. What can you conclude about the discount?
$3(x - 2)$; The discount is \$2 per song.

Self-Assessment for Problem Solving

13. $8(x - 4)$; The discount is \$4 per pizza.

14. Each crate weighs 27 pounds less on the Moon than on Earth.

Formative Assessment Tip

Example and Non-Example Chart
This technique allows students to demonstrate their understanding of a concept by comparing examples and non-examples. Students write examples of the concept in the left column and non-examples in the right column. Students should be able to explain their choices and reasoning. Allow time for students to receive feedback from you and their peers.

Learning Target

Factor numerical and algebraic expressions.

Success Criteria

- Use the Distributive Property to factor numerical expressions.
- Identify the greatest common factor of terms including variables.
- Use the Distributive Property to factor algebraic expressions.
- Interpret factored expressions in real-life problems.

Review & Refresh

1. $2n + 16$
2. $12 + 3m$
3. $7b - 21$
4. $40 - 10w$
5. $5 + p$
6. $r - 18$
7. $11d$
8. $c \div 25$
9. equivalent
10. equivalent
11. C
12. B
13. A
14. D

Concepts, Skills, & Problem Solving

15. *Sample answer:* $4(3 + 4)$; Use a common factor as the height and find the lengths.
16. *Sample answer:* $8(6 + 4)$; Use a common factor as the height and find the lengths.
17. $7(1 + 2)$
18. $6(2 + 7)$
19. $11(2 + 1)$
20. $5(14 + 19)$
21. $12(5 - 3)$
22. $20(5 - 4)$
23. $28(3 + 1)$
24. $16(3 + 5)$
25. $19(1 + 5)$
26. $11(4 - 1)$
27. $6(3 - 2)$
28. $16(3 + 1)$
29. $14(7 - 5)$
30. $2(29 + 14)$
31. $3(24 - 13)$
32. $3(23 + 28)$
33. yes; yes; you can factor c out of each expression.
34. D

Assignment Guide and Concept Check

Check out the Dynamic Assessment System.
BigIdeasMath.com

Level	Assignment 1	Assignment 2
Emerging	4, 6, 10, 11, 12, 13, 14, 16, 17, 21, 35, 37	26, 30, 34, 42, 49, 51, 52, 53, 54, 55, 56, 57
Proficient	4, 6, 10, 11, 12, 13, 14, 16, 19, 21, 36, 37	32, 34, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59
Advanced	4, 6, 10, 11, 12, 13, 14, 16, 26, 32, 42, 49	33, 46, 50, 51, 56, 57, 58, 59, 60

- Assignment 1 is for use after students complete the Self-Assessment for Concepts & Skills.
- Assignment 2 is for use after students complete the Self-Assessment for Problem Solving.
- The red exercises can be used as a concept check.

Review & Refresh Prior Skills

Exercises 1–4 Simplifying Expressions
Exercises 5–8 Writing Expressions
Exercises 9 and 10 Comparing Rates
Exercises 11–14 Writing Decimals as Percents

Common Errors

- **Exercises 17–32** Students may factor out a common factor but not the *greatest* common factor. For example, students might say that $12 + 42 = 2(6 + 21)$. While this is a true statement, it does not use the GCF. Students should realize that 6 and 21 still have the common factor of 3.

5.5 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Use the Distributive Property to simplify the expression.

1. $2(n + 8)$ **2.** $3(4 + m)$ **3.** $7(b - 3)$ **4.** $10(4 - w)$

Write the phrase as an expression.

5. 5 plus a number p

6. 18 less than a number r

7. 11 times a number d

8. a number c divided by 25

Decide whether the rates are equivalent.

9. 84 feet in 12 seconds
217 feet in 31 seconds

10. 12 cups of soda for every 54 cups of juice
8 cups of soda for every 36 cups of juice

Match the decimal with its equivalent percent.

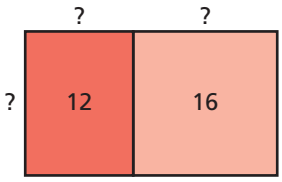
11. 0.36 **12.** 3.6 **13.** 0.0036 **14.** 0.036

A. 0.36% **B.** 360% **C.** 36% **D.** 3.6%

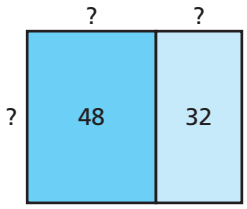
Concepts, Skills, & Problem Solving

FINDING DIMENSIONS **The model shows the area (in square units) of each part of a rectangle. Use the model to find missing values that complete the expression. Explain your reasoning.** (See Exploration 1, p. 227.)

15. $12 + 16 = \square(\square + \square)$



16. $48 + 32 = \square(\square + \square)$



FACTORING NUMERICAL EXPRESSIONS **Factor the expression using the GCF.**

17. $7 + 14$ **18.** $12 + 42$ **19.** $22 + 11$ **20.** $70 + 95$

21. $60 - 36$ **22.** $100 - 80$ **23.** $84 + 28$ **24.** $48 + 80$

25. $19 + 95$ **26.** $44 - 11$ **27.** $18 - 12$ **28.** $48 + 16$

29. $98 - 70$ **30.** $58 + 28$ **31.** $72 - 39$ **32.** $69 + 84$

33. MP **REASONING** The whole numbers a and b are divisible by c, where b is greater than a. Is $a + b$ divisible by c? Is $b - a$ divisible by c? Explain your reasoning.

34. **MULTIPLE CHOICE** Which expression is *not* equivalent to $81x + 54$?

A. $27(3x + 2)$ **B.** $3(27x + 18)$ **C.** $9(9x + 6)$ **D.** $6(13x + 9)$

FACTORING ALGEBRAIC EXPRESSIONS **Factor the expression using the GCF.**

35. $2x + 10$ **36.** $15x + 6$ **37.** $26x - 13$ **38.** $50x - 60$

39. $36x + 9$ **40.** $14x - 98$ **41.** $18p + 26$ **42.** $16m + 40$

43. $24 + 72n$ **44.** $50 + 65h$ **45.** $76d - 24$ **46.** $27 - 45c$

47. $18t + 38x$ **48.** $90y + 65z$ **49.** $10x - 25y$ **50.** $24y + 88x$

51. **OPEN-ENDED** Use the Distributive Property to write two expressions that are equivalent to $8x + 16$.

MATCHING **Match the expression with an equivalent expression.**

52. $8x + 16y$ **53.** $4x + 8y$ **54.** $16x + 8y$ **55.** $8x + 4y$

A. $4(2x + y)$ **B.** $2(4y + 2x)$ **C.** $4(2x + 4y)$ **D.** $8(y + 2x)$

56. **YOU BE THE TEACHER** Your friend factors the expression $24x + 56$. Is your friend correct? Explain your reasoning.

$$24x + 56 = 8(3x) + 8(7)$$
$$= (8 + 8) \cdot (3x + 7)$$
$$= 16(3x + 7)$$

57. **MODELING REAL LIFE** You sell soup mixes for a fundraiser. For each soup mix you sell, the company that makes the soup receives x dollars, and you receive the remaining amount. You sell 16 soup mixes for a total of $(16x + 96)$ dollars. How much money do you receive for each soup mix that you sell?

58. **MP PROBLEM SOLVING** A clothing store is having a sale on holiday socks. Each pair of socks costs x dollars. You leave the store with 6 pairs of socks and spend a total of $(6x - 14)$ dollars. You pay with \$40. How much change do you receive? Explain your reasoning.

59. **MP STRUCTURE** You buy 37 concert tickets for \$8 each, and then sell all 37 tickets for \$11 each. The work below shows two ways you can determine your profit. Describe each solution method. Which do you prefer? Explain your reasoning.

$$\text{Profit} = 37(11) - 37(8)$$
$$= 407 - 296$$
$$= \$111$$

$$\text{Profit} = 37(11) - 37(8)$$
$$= 37(11 - 8)$$
$$= 37(3)$$
$$= \$111$$

60. **MP NUMBER SENSE** The prime factorizations of two numbers are shown, where a and b represent prime numbers. Write the sum of the two numbers as an expression of the form $14(\square + \square)$. Explain your reasoning.

Number 1: $2 \cdot 11 \cdot 5 \cdot a$ **Number 2:** $7 \cdot b \cdot 3 \cdot 3$

Common Errors

- **Exercises 35–50** As students factor out the GCF, they may forget to include the variables in their answers. Remind students they can check their answers by using the Distributive Property to simplify the expressions.

Mini-Assessment

Factor the expression using the GCF.

1. $24 - 18$ $6(4 - 3)$
2. $32 + 16$ $16(2 + 1)$
3. $6x + 42$ $6(x + 7)$
4. $36m - 54n$ $18(2m - 3n)$
5. You sell T-shirts for a fundraiser. For each T-shirt you sell, the company that makes the shirts receives x dollars, and you receive the remaining amount. You sell 12 T-shirts for a total of $(12x + 72)$ dollars. How much money do you receive for each T-shirt that you sell? $6

Section Resources

Surface Level	Deep Level
Resources by Chapter • Extra Practice • Reteach • Puzzle Time Student Journal • Self-Assessment • Practice Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension Graphic Organizers Dynamic Assessment System • Section Practice
Transfer Level	
Dynamic Assessment System • End-of-Chapter Quiz	Assessment Book • End-of-Chapter Quiz

Concepts, Skills, & Problem Solving

35. $2(x + 5)$ **36.** $3(5x + 2)$

37. $13(2x - 1)$ **38.** $10(5x - 6)$

39. $9(4x + 1)$ **40.** $14(x - 7)$

41. $2(9p + 13)$ **42.** $8(2m + 5)$

43. $24(1 + 3n)$ **44.** $5(10 + 13h)$

45. $4(19d - 6)$ **46.** $9(3 - 5c)$

47. $2(9t + 19x)$ **48.** $5(18y + 13z)$

49. $5(2x - 5y)$ **50.** $8(3y + 11x)$

51. *Sample answer:* $8(x + 2)$, $4(2x + 4)$

52. C **53.** B

54. D **55.** A

56. no; $24x + 56 = 8(3x + 7)$

57. $6

58. $12; You save $14 by getting two pairs free, so each pair costs $7 and $40 - (6 \cdot 7 - 14) = 12$.

59. The first solution calculates the total spent and the total earned, then subtracts. The second solution uses the Distributive Property first. *Sample answer:* second; There are fewer calculations.

60. $14(55 + 9)$; *Sample answer:* For each number to have a factor of 14, the missing primes must be $a = 7$ and $b = 2$.

Skills Needed

Exercise 1

- Factoring Algebraic Expressions
- Finding the Whole
- Writing Expressions

Exercise 2

- Converting Measures
- Evaluating Expressions
- Multiplying Decimals
- Writing Expressions

Exercise 3

- Evaluating Expressions
- Writing Equivalent Ratios
- Writing Expressions

ELL Support

Remind students that pounds are used to measure weight and the U.S. customary system is considerably different than the metric system. A pound is approximately 0.45 kilogram. Ounces are often used to measure lighter quantities and tons are used to measure heavier quantities.
A pound is equivalent to 16 ounces and 2000 pounds equal one ton.

Using the Problem-Solving Plan

1. \$140
2. $2.2z$ or $z \div 0.45$; 44.88 lb or $45\frac{1}{3}$ lb
3. a. $\frac{7}{2}n$; *Sample answer:* 2 : 7 is equivalent to $1 : \frac{7}{2}$.
   b. 42

Performance Task

The *STEAM Video Performance Task* provides the opportunity for additional enrichment and greater depth of knowledge as students explore the mathematics of the chapter within a context tied to the chapter STEAM Video. The performance task and a detailed scoring rubric are provided at *BigIdeasMath.com*.

Laurie's Notes

Scaffolding Instruction

- The goal of this lesson is to help students become more comfortable with problem solving. These exercises combine numerical and algebraic expressions with prior skills from other chapters. The solution for Exercise 1 is worked out below, to help you guide students through the problem-solving plan. Use the remaining class time to have students work on the other exercises.
- **Emerging:** The goal for these students is to feel comfortable with the problem-solving plan. Allow students to work in pairs to write the beginning steps of the problem-solving plan for Exercise 2. Keep in mind that some students may only be ready to do the first step.
- **Proficient:** Students may be able to work independently or in pairs to complete Exercises 2 and 3.
- Visit each pair to review their plan for each problem. Ask students to describe their plans.

Using the Problem-Solving Plan

Exercise 1

Understand the problem. You know the percent discount on a pair of wireless earbuds, the number of pairs of earbuds sold, and the total amount of money that customers saved. You are asked to find the original price of the earbuds.

Make a plan. First, write an expression that represents the total amount of money that customers pay for the earbuds. Then factor the expression to find the discount (in dollars) on each pair of earbuds. Finally, solve a percent problem to find the original price.

Solve and check. Use the plan to solve the problem. Then check your solution.

- The store sells 18 pairs of earbuds for an original price of \$$x$ each. Customers saved a total of \$882. The expression that represents the total amount of money that customers pay for the earbuds is $18x - 882$.
- Write the prime factorizations of $18x$ and 882 to find the GCF.

$$18x = 2 \cdot 2 \cdot 3 \cdot x$$
$$882 = 2 \cdot 3 \cdot 3 \cdot 7 \cdot 7$$

Circle the common prime factors.

The GCF of $18x$ and 882 is $2 \cdot 3 \cdot 3 = 18$.

- Use the GCF to factor the expression.

$$18x - 882 = 18(x) - 18(49)$$ Rewrite using GCF.
$$= 18(x - 49)$$ Distributive Property

The factor $(x - 49)$ represents the discounted price of each pair of earbuds. Showing the original price, \$$x$, of each pair of earbuds is decreased by \$49.

- Solve the percent problem: 35% of what number is \$49?

$$49 \div 35\% = 49 \div \frac{7}{20}$$ Write the percent as a fraction.
$$= 140$$ Multiply 49 by the reciprocal of $\frac{7}{20}$.

So, the original price of the earbuds is \$140.

Check: Verify the total amount of money that customers saved.
The total amount of money that customers would have spent is $18 \cdot \$140 = \$2520$. So, 35% of $\$2520 = \frac{7}{20} \cdot \$2520 = \frac{\$17{,}640}{20} = \$882$, which is the total amount of money that customers saved. ✓

5 Connecting Concepts

Using the Problem-Solving Plan

1. A store sells 18 pairs of the wireless earbuds shown. Customers saved a total of $882 on the earbuds. Find the original price of the earbuds.

Understand the problem.

You know the percent discount on a pair of wireless earbuds, the number of pairs of earbuds sold, and the total amount of money that customers saved. You are asked to find the original price of the earbuds.

Make a plan.

First, write an expression that represents the total amount of money that customers pay for the earbuds. Then factor the expression to find the discount (in dollars) on each pair of earbuds. Finally, solve a percent problem to find the original price.

Solve and check.

Use the plan to solve the problem. Then check your solution.

2. All of the weight plates in a gym are labeled in kilograms. You want to convert the weights to pounds. Write an expression to find the number of pounds in z kilograms. Then find the weight in pounds of a plate that weighs 20.4 kilograms.

3. You buy apple chips and banana chips in the ratio of 2 : 7.

a. How many ounces of banana chips do you buy when you buy n ounces of apple chips? Explain.

b. You buy 12 ounces of apple chips. How many ounces of banana chips do you buy?

Performance Task

Describing Change

At the beginning of this chapter, you watched a STEAM video called "Shadow Drawings." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

5 Chapter Review

Go to *BigIdeasMath.com* to download blank graphic organizers.

Review Vocabulary

Write the definition and give an example of each vocabulary term.

algebraic expression, *p. 202*
variable, *p. 202*
term, *p. 202*
coefficient, *p. 202*
constant, *p. 202*
equivalent expressions, *p. 216*
like terms, *p. 223*
factoring an expression, *p. 228*

Graphic Organizers

You can use an **Example and Non-Example Chart** to list examples and non-examples of a concept. Here is an Example and Non-Example Chart for the ***Commutative Property of Addition***.

Commutative Property of Addition

Examples	Non-Examples
$a + b = b + a$	$a \cdot b = b \cdot a$
$2.1 + 9 = 9 + 2.1$	$(7 + 4) + 2 = 7 + (4 + 2)$
$17 + 34 = 34 + 17$	$b \cdot 0 = 0$
$(6 + x) + 8 = (x + 6) + 8$	$46 \cdot 1 = 46$
$(3 + y) + 1 = 1 + (3 + y)$	$2(12 + x) = 2(12) + 2(x)$

Choose and complete a graphic organizer to help you study the concept.

1. algebraic expressions
2. variable
3. Commutative Property of Multiplication
4. Associative Property of Addition
5. Associative Property of Multiplication
6. Addition Property of Zero
7. Multiplication Property of Zero
8. Multiplication Property of One
9. Distributive Property

"I finished my Example and Non-Example Chart about things we need on the moon."

Review Vocabulary

- As a review of the chapter vocabulary, have students revisit the vocabulary section in their *Student Journals* to fill in any missing definitions and record examples of each term.

Graphic Organizers

Sample answers:

1. Algebraic Expressions

Examples	Non-Examples
$4x + 3$	9 fewer than 28
$2y$	$91 - 37$
$5z^2 + w + 1$	$x + 13 = 15$
$\frac{m}{10} - 6$	75%

2. Variable

Examples	Non-Examples
m	2
x	%
y	+
z	78.5

3. Commutative Property of Multiplication

Examples	Non-Examples
$a \cdot b = b \cdot a$	$a + b = b + a$
$7 \cdot 5 = 5 \cdot 7$	$a(b + c) = ab + ac$
$2 \cdot x \cdot 4 = 2 \cdot 4 \cdot x$	$21 + 0 = 21$
$x(4 + 5) = (4 + 5)x$	$78 \cdot 1 = 78$

4. Associative Property of Addition

Examples	Non-Examples
$a + (b + c) = (a + b) + c$	$x + y = y + x$
$3 + (5 + m) = (3 + 5) + m$	$a(b + c) = ab + ac$
$6 + (4 + 11) = (6 + 4) + 11$	$m \cdot n = n \cdot m$
$x + (y + 5) = (x + y) + 5$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

5–9. Available at *BigIdeasMath.com*.

List of Organizers

Available at *BigIdeasMath.com*

Definition and Example Chart
Example and Non-Example Chart
Four Square
Information Frame
Summary Triangle

About this Organizer

An **Example and Non-Example Chart** can be used to list examples and non-examples of a concept. Students write examples of the concept in the left column and non-examples in the right column. Blank Example and Non-Example Charts can be included on tests or quizzes for this purpose.

1. Terms: $9x$, 2, $8y$
   Coefficients: 9, 8
   Constant: 2
2. Terms: $3x^2$, x, 7
   Coefficients: 3, 1
   Constant: 7
3. Terms: 1, $\frac{q}{4}$, $7q$
   Coefficients: $\frac{1}{4}$, 7
   Constant: 1
4. 4
5. 5
6. 16
7. 24
8. 140
9. 16
10. 29
11. 12
12. 100
13. $56
14. 305 points
15. $70
16. $2(x^2 + 4) - 5$

Chapter Self-Assessment

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

ELL Support

Allow students to work in pairs to complete the Chapter Self-Assessment. After students complete the first section, check for understanding of Exercises 1–3 by having students indicate whether each term is a constant using a thumbs up for *yes* or a thumbs down for *no*. Check for understanding of Exercises 13–16 by having each pair write their answers on a whiteboard to display for your review. You should be able to quickly assess which students understand the concepts and who may need additional practice. Use these techniques for the remaining sections.

Common Errors

- **Exercises 4–12** Students may substitute the wrong value(s) for the variable(s). Tell students to write out the expression and then write the value(s) of the variable(s) underneath the variable(s) before substituting the value(s).

Chapter Self-Assessment

As you complete the exercises, use the scale below to rate your understanding of the success criteria in your journal.

1	2	3	4
I do not understand.	I can do it with help.	I can do it on my own.	I can teach someone else.

5.1 Algebraic Expressions *(pp. 201–208)*

Learning Target: Evaluate algebraic expressions given values of their variables.

Identify the terms, coefficients, and constants in the expression.

1. $9x + 2 + 8y$

2. $3x^2 + x + 7$

3. $1 + \frac{q}{4} + 7q$

Evaluate the expression when $x = 20$, $y = 4$, and $z = 7$.

4. $x \div 5$

5. $12 - z$

6. $4y$

7. $y + x$

8. $x \cdot z$

9. $x - y$

10. $3z + 8$

11. $8y - x$

12. $\frac{x^2}{y}$

13. The amount earned (in dollars) for recycling p pounds of copper is $2p$. How much do you earn for recycling 28 pounds of copper?

14. While playing a video game, you score p game points and b triple bonus points. An expression for your score is $p + 3b$. What is your score when you earn 245 game points and 20 triple bonus points?

15. Tickets for a baseball game cost a dollars for adults and c dollars for children. The expression $2a + 3c$ represents the cost (in dollars) for a family to go to the game. What is the cost for the family when an adult ticket is \$17 and a child ticket is \$12?

16. Add one set of parentheses to the expression $2x^2 + 4 - 5$ so that the value of the expression is 75 when $x = 6$.

5.2 Writing Expressions *(pp. 209–214)*

Learning Target: Write algebraic expressions and solve problems involving algebraic expressions.

Write the phrase as an expression.

17. 9 fewer than 23

18. 6 more than the quotient of 15 and 3

19. the product of a number d and 32

20. a number t decreased by 17

21. Your basketball team scored 4 fewer than twice as many points as the other team.

a. Write an expression that represents the number of points your team scored.

b. The other team scored 24 points. How many points did your team score?

22. The boiling temperature (in degrees Celsius) of platinum is 199 more than four times the boiling temperature (in degrees Celsius) of zinc.

a. Write an expression that represents the boiling temperature (in degrees Celsius) of platinum.

b. The boiling temperature of zinc is 907 degrees Celsius. What is the boiling temperature of platinum?

23. Write an algebraic expression with two variables, x and y, that has a value of 50 when $x = 3$ and $y = 5$.

5.3 Properties of Addition and Multiplication *(pp. 215–220)*

Learning Target: Identify equivalent expressions and apply properties to generate equivalent expressions.

Simplify the expression. Explain each step.

24. $10 + (2 + y)$

25. $(21 + b) + 1$

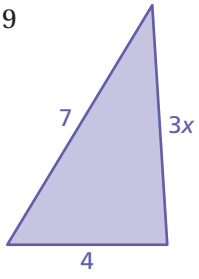
26. $3(7x)$

27. $1(3.2w)$

28. $5.3 + (w + 1.2)$

29. $(0 + t) + 9$

30. The expression $7 + 3x + 4$ represents the perimeter of the triangle. Simplify the expression.



31. Write an algebraic expression that can be simplified using the Associative Property of Addition.

Common Errors

- **Exercises 17–20** Students may write the subtraction or division problems in the wrong order. For subtraction problems, have students look for the key words *from* and *than* to know that the order of the numbers must be switched from the way the phrase is written.
- **Exercises 24–29** Students are often confused by the differences between the Commutative and Associative Properties and may incorrectly label steps. Show students that the Associative Property moves the parentheses but does not change the positions of the terms. The Commutative Property changes the positions of the terms.

Chapter Self-Assessment

17. $23 - 9$ **18.** $15 \div 3 + 6$

19. $32d$ **20.** $t - 17$

21. **a.** $2p - 4$

b. 44

22. **a.** $4z + 199$

b. 3827°C

23. *Sample answer:* $10x + 4y$

24. $10 + (2 + y) = (10 + 2) + y$
Assoc. Prop. of Add.
$= 12 + y$
Add 10 and 2.

25. $(21 + b) + 1 = (b + 21) + 1$
Comm. Prop. of Add.
$= b + (21 + 1)$
Assoc. Prop. of Add.
$= b + 22$
Add 21 and 1.

26. $3(7x) = (3 \cdot 7)x$
Assoc. Prop. of Mult.
$= 21x$
Multiply 3 and 7.

27. $1(3.2w) = (1 \cdot 3.2)w$
Assoc. Prop. of Mult.
$= 3.2\,w$
Mult. Prop. of One

28. $5.3 + (w + 1.2)$
$= 5.3 + (1.2 + w)$
Comm. Prop. of Add.
$= (5.3 + 1.2) + w$
Assoc. Prop. of Add.
$= 6.5 + w$
Add 5.3 and 1.2.

29. $(0 + t) + 9 = t + 9$
Add. Prop. of Zero

30. $3x + 11$

31. *Sample answer:* $5 + (3 + x)$

Chapter Self-Assessment

32. $2x + 24$

33. $44b - 33$

34. $8s - 8$

35. $36 + 6y$

36. $9n + 15$

37. $7t + 2$

38. $8z + 5$

39. $3(15 + x) = 45 + 3x$

40. $4(v + 30) = 4v + 120$

41. $6(7 - 2)$

42. $5(3 + 7)$

43. $4(9x - 7)$

44. $8(3 + 8x)$

45. $30(2 - 5x)$

46. $8(2x + 7y)$

47. The discount is \$2 per jersey.

48. \$3

Common Errors

- **Exercises 32–35** Students may forget to distribute to each term in the parentheses. Tell students to write the expression on their papers and draw arrows from the number being distributed to each term.
- **Exercises 36–38** Students may try to add a constant to a term with a variable. For example, they may try to add $4n$ to 15 instead of adding $4n$ to $5n$. Students may not be confident with *like terms*. Remind them that because $4n$ has a variable and you do not know the value of that variable, you cannot add $4n$ to 15.

Chapter Resources

Surface Level	Deep Level
Resources by Chapter • Extra Practice • Reteach • Puzzle Time Student Journal • Practice • Chapter Self-Assessment Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer Game Library	Resources by Chapter • Enrichment and Extension Graphic Organizers Game Library
Transfer Level	
STEAM Video Dynamic Assessment System • Chapter Test	Assessment Book • Chapter Tests A and B • Alternative Assessment • STEAM Performance Task

5.4 The Distributive Property *(pp. 221–226)*

Learning Target: Apply the Distributive Property to generate equivalent expressions.

Use the Distributive Property to simplify the expression.

32. $2(x + 12)$ **33.** $11(4b - 3)$ **34.** $8(s - 1)$ **35.** $6(6 + y)$

Simplify the expression.

36. $5(n + 3) + 4n$ **37.** $t + 2 + 6t$ **38.** $3z + 14 + 5z - 9$

39. A family of three goes to a salon. Each person gets a haircut and highlights. The cost of each haircut is \$15, and the cost per person for highlights is x dollars. Write and simplify an expression that represents the total cost (in dollars) for the family at the salon.

40. Each day, you take vocal lessons for v minutes and trumpet lessons for 30 minutes. Write and simplify an expression to find how many minutes of lessons you take in 4 days.

5.5 Factoring Expressions *(pp. 227–232)*

Learning Target: Factor numerical and algebraic expressions.

Factor the expression using the GCF.

41. $42 - 12$ **42.** $15 + 35$ **43.** $36x - 28$

44. $24 + 64x$ **45.** $60 - 150x$ **46.** $16x + 56y$

47. A soccer team receives a discount on each jersey purchased. The original price of each jersey is x dollars. The team buys 18 jerseys for a total of $(18x - 36)$ dollars. What can you conclude about the discount?

48. You sell apple cider for a fundraiser. For each gallon of cider you sell, the company that makes the cider receives x dollars, and you receive the remaining amount. You sell 15 gallons of cider for $(15x + 45)$ dollars. How much money do you receive for each gallon of cider that you sell?

5 Practice Test

1. Identify the terms, coefficients, and constants of $\frac{q}{3} + 6 + 9q$.

2. Evaluate $4b - a$ when $a = 12$ and $b = 7$.

Write the phrase as an expression.

3. 25 more than 50

4. 6 less than the quotient of 32 and a number y

Simplify the expression. Explain each step.

5. $3.1 + (8.6 + m)$

6. $\left(\frac{2}{3} \cdot t\right) \cdot 1\frac{1}{2}$

7. $4(x + 8)$

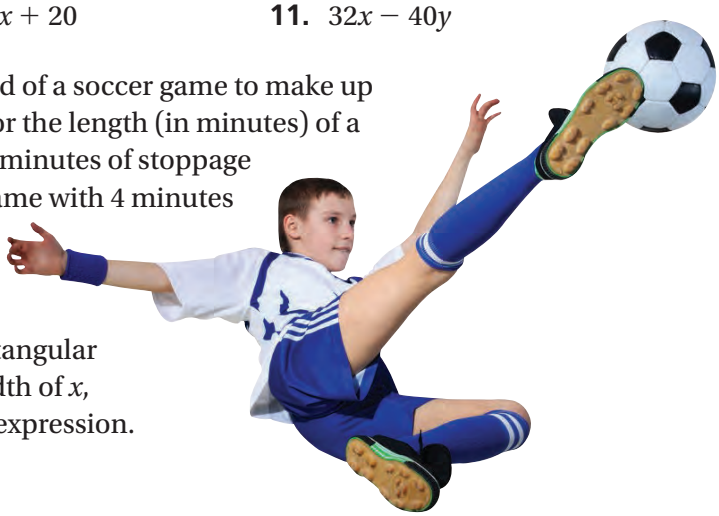
8. $4t + 7 + 2t - 2$

Factor the expression using the GCF.

9. $18 + 24$

10. $15x + 20$

11. $32x - 40y$



12. Playing time is added at the end of a soccer game to make up for stoppages. An expression for the length (in minutes) of a 90-minute soccer game with x minutes of stoppage time is $90 + x$. How long is a game with 4 minutes of stoppage time?

13. The expression $15 \cdot x \cdot 6$ represents the volume of a rectangular prism with a length of 15, a width of x, and a height of 6. Simplify the expression.

14. The Coiling Dragon Cliff Skywalk in China is 128 feet longer than the length x (in feet) of the Tianmen Skywalk in China. The world's longest glass-bottom bridge, located in China's Zhangjiajie National Park, is about 4.3 times longer than the Coiling Dragon Cliff Skywalk. Write and simplify an expression that represents the length (in feet) of the world's longest glass-bottom bridge.

15. A youth group is making and selling sandwiches to raise money. The cost to make each sandwich is h dollars. The group sells 150 sandwiches for a total of $(150h + 450)$ dollars. How much profit does the group earn for each sandwich sold?

16. You make party favors for an event. You tie 9 inches of ribbon around each party favor. Write an expression for the number of inches of ribbon needed for n party favors. The ribbon costs \$3 for each *yard*. Write an expression for the total cost (in dollars) of the ribbon.

Practice Test Item References

Practice Test Questions	Section to Review
1, 2, 12	5.1
3, 4, 16	5.2
5, 6, 13	5.3
7, 8, 14	5.4
9–11, 15	5.5

Test-Taking Strategies

Remind students to quickly look over the entire test before they start so that they can budget their time. When writing expressions, students need to pay attention to the order of the terms. Remind students to **Stop** and **Think** before they write their answers.

Common Errors

- **Exercise 2** Students may not replace the variables correctly. Tell students to rewrite the expression with parentheses in place of the variables and then place the values of the variables in the parentheses.

Practice Test

1. Terms: $\frac{q}{3}$, 6, $9q$
Coefficients: $\frac{1}{3}$, 9
Constant: 6

2. 16 **3.** $50 + 25$

4. $32 \div y - 6$

5. $3.1 + (8.6 + m)$
$= (3.1 + 8.6) + m$
Assoc. Prop. of Add.
$= 11.7 + m$
Add 3.1 and 8.6.

6. $\left(\frac{2}{3} \cdot t\right) \cdot 1\frac{1}{2} = \left(t \cdot \frac{2}{3}\right) \cdot 1\frac{1}{2}$
Comm. Prop. of Mult.
$= t \cdot \left(\frac{2}{3} \cdot 1\frac{1}{2}\right)$
Assoc. Prop. of Mult.
$= t$
Multiply $\frac{2}{3}$ and $1\frac{1}{2}$.

7. $4(x + 8) = 4(x) + 4(8)$
Distributive Property
$= 4x + 32$
Multiply.

8. $4t + 7 + 2t - 2$
$= 4t + 2t + 7 - 2$
Comm. Prop. of Add.
$= (4 + 2)t + 7 - 2$
Distributive Property
$= 6t + 5$
Simplify.

9. $6(3 + 4)$

10. $5(3x + 4)$

11. $8(4x - 5y)$

12. 94 min

13. $90x$

14. $4.3(x + 128) = 4.3x + 550.4$

15. \$3

16. $9n$; $3\left(\frac{1}{4}n\right)$

Test-Taking Strategies

Available at *BigIdeasMath.com*

After Answering Easy Questions, Relax

Answer Easy Questions First

Estimate the Answer

Read All Choices before Answering

Read Question before Answering

Solve Directly or Eliminate Choices

Solve Problem before Looking at Choices

Use Intelligent Guessing

Work Backwards

About this Strategy

When taking a multiple-choice test, be sure to read each question carefully and thoroughly. After skimming the test and answering the easy questions, stop for a few seconds, take a deep breath, and relax. Work through the remaining questions carefully, using your knowledge and test-taking strategies. Remember, you already completed many of the questions on the test!

Cumulative Practice

1. C
2. G
3. 26

Item Analysis

1. **A.** The student does not consider that there is a cost for *each* game. The student finds the sum of the admission price and the price per game, and does not use a variable to represent the number of games.

   B. The student multiplies the number of games by the sum of the admission price and the price per game. This expression multiplies the admission price by the number of games, not just the price per game.

   C. Correct answer

   D. The student uses x to represent the number of admissions, not the number of games.

2. **F.** The student does not identify the correct ratio of flour to sugar.

   G. Correct answer

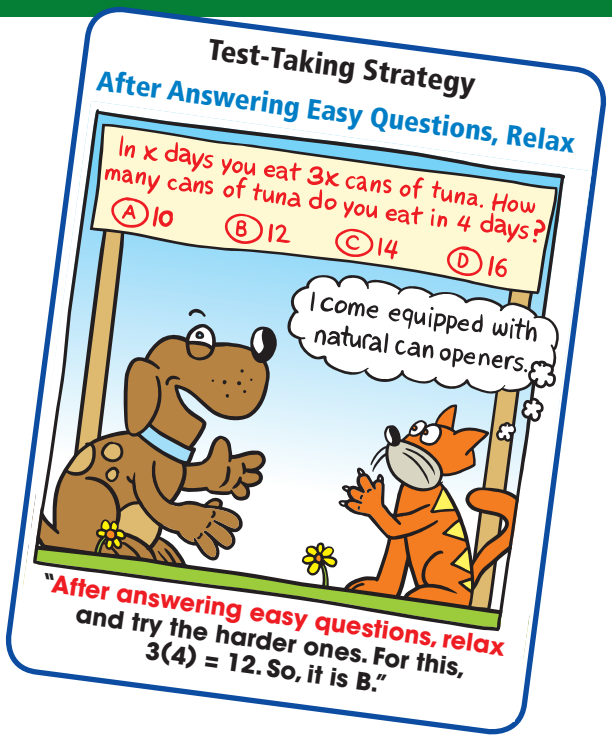
   H. The student does not identify the correct ratio of flour to sugar.

   I. The student reverses the order of the numbers in the ratio.

3. **Gridded Response:** Correct answer: 26

   Common error: The student multiplies the cost of a hardcover book by the number of paperback books and the cost of a paperback book by the number of hardcover books to get an answer of $24.

5 Cumulative Practice



1. The student council is organizing a school fair. Council members are making signs to show the prices for admission and for each game a person can play.

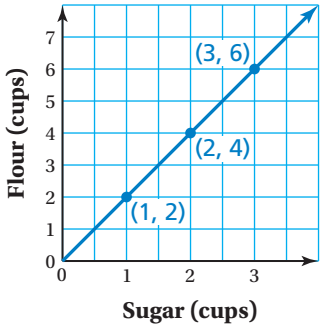
SCHOOL FAIR	
Admission	$2.00
Price per game	$0.25

Let x represent the number of games. Which expression can you use to determine the total amount (in dollars) a person pays for admission and playing x games?

A. 2.25 **B.** $2.25x$

C. $2 + 0.25x$ **D.** $2x + 0.25$

2. Which ratio relationship is represented in the graph?



F. 2 cups of flour for every $\frac{1}{2}$ cup of sugar

G. 6 cups of flour for every 3 cups of sugar

H. 1 cup of flour for every 4 cups of sugar

I. $\frac{1}{2}$ cup of flour for every 1 cup of sugar

3. At a used bookstore, you can purchase two types of books.

You can use the expression $3h + 2p$ to find the total cost (in dollars) for h hardcover books and p paperback books. What is the total cost (in dollars) for 6 hardcover books and 4 paperback books?

4. Your friend divided two decimal numbers. Her work is shown in the box below. What should your friend change in order to divide the two decimal numbers correctly?

$$0.07\overline{)14.56} \rightarrow 7\overline{)14.56}$$
$$2.08$$

A. Rewrite the problem as $0.07\overline{)0.1456}$.

B. Rewrite the problem as $0.07\overline{)1456}$.

C. Rewrite the problem as $7\overline{)0.1456}$.

D. Rewrite the problem as $7\overline{)1456}$.

5. What is the value of $4.391 + 5.954$?

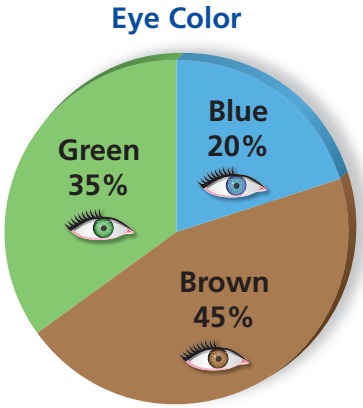
F. 9.12145

G. 9.245

H. 9.345

I. 10.345

6. The circle graph shows the eye color of students in a sixth-grade class. Nine students in the class have brown eyes. How many students are in the class?



A. 4 students

B. 18 students

C. 20 students

D. 405 students

7. Properties of Addition and Multiplication are used to simplify an expression.

$$\begin{aligned} 36 \cdot 23 + 33 \cdot 64 &= 36 \cdot 23 + 64 \cdot 33 \\ &= 36 \cdot 23 + 64 \cdot (23 + 10) \\ &= 36 \cdot 23 + 64 \cdot 23 + 64 \cdot 10 \\ &= x \cdot 23 + 64 \cdot 10 \end{aligned}$$

What number belongs in place of the x?

Item Analysis (continued)

4. **A.** The student moves the decimal point in the dividend two places but in the wrong direction and does not move the decimal point in the divisor.

   B. The student moves the decimal point in the dividend correctly but does not move the decimal point in the divisor.

   C. The student moves the decimal point in the divisor correctly but moves the decimal point in the dividend two places in the wrong direction.

   D. Correct answer

5. **F.** The student adds each place value and inserts the 1s instead of carrying them.

   G. The student does not carry any of the 1s.

   H. The student does not carry the 1 from the tenths place to the ones place.

   I. Correct answer

6. **A.** The student multiplies 9 by 45% instead of dividing 9 by 45%.

   B. The student uses a benchmark of 50%.

   C. Correct answer

   D. The student multiplies 9 by 45 instead of dividing 9 by 45%.

7. **Gridded Response:** Correct answer: 100

   Common error: The student does not recognize the application of the Distributive Property and instead tries to work backwards from 2300. The student makes a place value error while dividing 2300 by 23 to get 1000.

Cumulative Practice

4. D
5. I
6. C
7. 100

8. I
9. B
10. G
11. C
12. F
13. 1800
14. C

Item Analysis (continued)

8. **F.** The student identifies a factor pair of 1350 instead of the prime factorization of 1350.

   G. The student lists the prime factors but does not include the correct exponent for each factor.

   H. The student identifies a factor pair of 1350 instead of the prime factorization of 1350.

   I. Correct answer

9. **A.** The student finds the value of miles per minute instead of miles per hour.

   B. Correct answer

   C. The student reverses the conversions; identifying 3600 hours as 1 second and 5280 miles as 1 foot, instead of 1 hour as 3600 seconds and 1 mile as 5280 feet.

   D. The student finds the value of feet per hour instead of miles per hour.

10. **F.** The student does not distribute a to c.

    G. Correct answer

    H. The student incorrectly distributes the addition of a.

    I. The student incorrectly distributes the addition of a and then multiplies the resulting sums.

11. **A.** The student divides by $1\frac{2}{7}$ instead of multiplying by $1\frac{2}{7}$.

    B. The student finds the product of the whole numbers and then adds it to the product of the fractional parts of the mixed numbers.

    C. Correct answer

    D. The student adds the mixed numbers instead of multiplying.

12. **F.** Correct answer

    G. The student does not find the LCM correctly; the LCM of 3 and 8 is 24.

    H. The student does not find the LCM correctly; the LCM of 6 and 8 is 24.

    I. The student does not find the LCM correctly; the LCM of 12 and 24 is 24.

13. **2 points** The students' explanation demonstrates a thorough understanding of the Associative and Commutative Properties of Multiplication. For instance, the student explains that the Commutative Property can be used to rewrite the expression as $(18 \times 25) \times 4$ and then the Associative Property can be used to rewrite the expression as $18 \times (25 \times 4)$. Then the student uses compatible numbers to get 18×100 and a final answer of 1800.

    1 point The student's explanation demonstrates a partial but limited understanding of the Associative and Commutative Properties of Multiplication. For instance, the student shows correct, efficient steps but provides no explanation.

    0 points The student provides no response, a completely incorrect or incomprehensible response, or a response that demonstrates insufficient understanding of the Associative and Commutative Properties of Multiplication.

14. **A.** The student does not recognize that 64 is the perfect square of 8.

    B. The student does not recognize that 81 is the perfect square of 9.

    C. Correct answer

    D. The student does not recognize that 100 is the perfect square of 10.

8. What is the prime factorization of 1350?

F. $10 \cdot 135$

G. $2 \cdot 3 \cdot 5$

H. $6 \cdot 225$

I. $2 \cdot 3^3 \cdot 5^2$

9. A horse gallops at a speed of 44 feet per second. What is the speed of the horse in miles per hour?

A. $\frac{1}{2}$ mile per hour

B. 30 miles per hour

C. $64\frac{8}{15}$ miles per hour

D. 158,400 miles per hour

10. Which equation correctly demonstrates the Distributive Property?

F. $a(b + c) = ab + c$

G. $a(b + c) = ab + ac$

H. $a + (b + c) = (a + b) + (a + c)$

I. $a + (b + c) = (a + b) \cdot (a + c)$

11. Which number is equivalent to $2\frac{4}{5} \cdot 1\frac{2}{7}$?

A. $2\frac{8}{45}$

B. $2\frac{8}{35}$

C. $3\frac{3}{5}$

D. $4\frac{3}{35}$

12. Which pair of numbers does *not* have a least common multiple of 24?

F. 2, 12

G. 3, 8

H. 6, 8

I. 12, 24

13. Use the Properties of Multiplication to simplify the expression in an efficient way. Show your work and explain how you used the Properties of Multiplication.

Think
Solve
Explain

$$(25 \times 18) \times 4$$

14. Which number is *not* a perfect square?

A. 64

B. 81

C. 96

D. 100