Chapter 2 Pacing Guide

Chapter Opener/ Mathematical Practices	1 Day
Section 1	1 Day
Section 2	1 Day
Section 3	1 Day
Section 4	1 Day
Quiz	1 Day
Section 5	2 Days
Section 6	2 Days
Chapter Review/ Chapter Tests	2 Days
Total Chapter 2	12 Days
Year-to-Date	24 Days

2 Solving Linear Inequalities

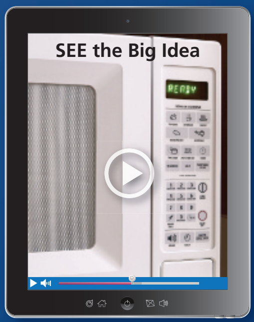
Chapter Learning Target:
Understand solving linear inequalities.

Chapter Success Criteria:
- I can graph inequalities.
- I can solve one-step inequalities.
- I can solve multi-step inequalities.
- I can solve compound and absolute value inequalities.

Camel Physiology *(p. 91)*

Mountain Plant Life *(p. 85)*

Digital Camera *(p. 70)*



Microwave Electricity *(p. 64)*

Natural Arch *(p. 59)*

ALABAMA EDITION

Laurie's Notes

Chapter Summary

- Students have just finished a chapter on solving linear equations—simple and multi-step equations along with absolute value equations. The techniques used in solving linear equations are applied to linear inequalities in this chapter. In middle school, students solved and graphed linear inequalities, so many of the topics in this chapter should be familiar to them.
- The chapter begins with an introduction to writing and graphing inequalities. Color coding and verbal models are used to help students develop confidence in writing inequalities, a necessary skill for the chapter. The graphs are used to display and check solutions.
- The next three lessons focus on solving increasingly complex inequalities. Tools used in developing facility with these problems include symbolic manipulation, tables, and spreadsheets. Practice with real number operations is integrated throughout.
- The last two lessons of the chapter introduce compound inequalities, which are necessary in solving absolute value inequalities. Look for the helpful teaching strategies offered in these lessons.
- New formative assessment tips are offered in many of the lessons, and tips from the previous chapter are referenced throughout the notes at point of use.

What Your Students Have Learned

Middle School

- Use substitution to determine whether numbers are solutions to linear inequalities.
- Write and graph inequalities of the form $x > c$ or $x < c$.
- Recognize that inequalities have infinitely many solutions.
- Solve linear inequalities with positive coefficients and graph the solution set.

What Your Students Will Learn

Algebra 1

- Write linear inequalities and sketch graphs of linear inequalities.
- Solve multi-step linear inequalities in one variable using inverse operations, including multiplying and dividing by negative numbers.
- Write compound linear inequalities in one variable joined by the word *and* or the word *or*.
- Graph and solve compound linear inequalities in one variable.
- Solve absolute value inequalities.

Dynamic Teaching Tools

- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom
- Real-Life STEM Videos

Scaffolding in the Classroom

Graphic Organizers: Information Frame

An Information Frame can be used to help students organize and remember concepts. Students write the topic in the middle rectangle. Then students write related concepts in the spaces around the rectangle. Related concepts can include *Words, Numbers, Algebra, Definition, Example, Non-Example, Visual, Procedure, Details,* and *Vocabulary*. Students can place their Information Frames on note cards to use as a quick study reference.

STANDARDS SUMMARY

Section	Alabama Standards
2.1	A1.11, A1.13
2.2	A1.11, A1.13
2.3	A1.11, A1.13
2.4	A1.11, A1.13
2.5	A1.11, A1.13
2.6	A1.11, A1.13

Questioning in the Classroom

Be open to multiple answers.
Questions can frequently be interpreted differently. Allow students time to discuss and explain their point of view.

Laurie's Notes

Maintaining Mathematical Proficiency

Graphing Numbers on a Number Line

- Remind students that positive numbers are greater than 0 and are to the right of 0 on a number line. Negative numbers are less than 0 and are to the left of 0 on a number line.
- Remind students that the absolute value of a number is the distance between the number and 0 on a number line.

COMMON ERROR Students may ignore the absolute value symbols when they graph the numbers or they may find the sum or difference without first finding the absolute value.

Comparing Real Numbers

- Remind students to graph each pair of integers on a number line. On a horizontal number line, the number on the right is the greater number.

COMMON ERROR Students may ignore the negative signs when they compare the numbers.

Mathematical Practices *(continued on page 52)*

- The *Mathematical Practices* page focuses attention on how mathematics is learned—process versus content. This page demonstrates the use of graphing calculators as a tool in learning important mathematics.
- Use the *Mathematical Practices* page to help students develop mathematical habits of mind—how mathematics can be explored and how mathematics is thought about.
- Refer to the *Core Concept* and explain that the graphing calculators can be used to graph solutions to single-variable inequalities.
- Example 1 demonstrates the syntax needed when entering an inequality. The example shown has variables on both sides of the inequality symbol. The same technique is used when one side involves only a constant.

Review Resources

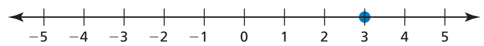
Surface Level	Deep Level
Tutorial Videos Skills Review Handbook Skills Trainer Game Library	Game Library

Maintaining Mathematical Proficiency

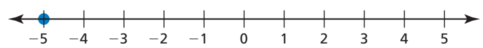
Graphing Numbers on a Number Line

Example 1 **Graph each number.**

a. 3

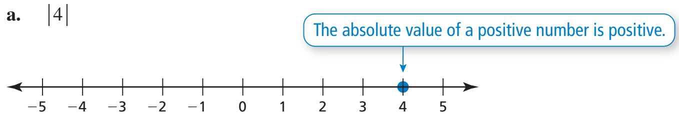


b. -5

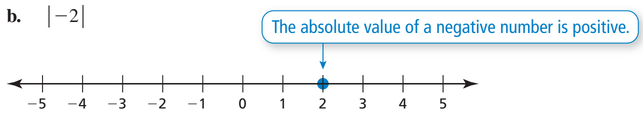


Example 2 **Graph each number.**

a. $|4|$



b. $|-2|$



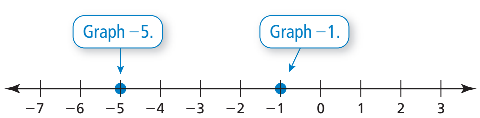
Graph the number.

1. 6 **2.** $|2|$ **3.** $|-1|$

4. $2 + |-2|$ **5.** $1 - |-4|$ **6.** $-5 + |3|$

Comparing Real Numbers

Example 3 **Complete the statement -1 ☐ -5 with <, >, or =.**



▶ -1 is to the right of -5. So, $-1 > -5$.

Complete the statement with <, >, or =.

7. 2 ☐ 9 **8.** -6 ☐ 5 **9.** -12 ☐ -4

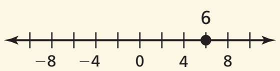
10. -7 ☐ -13 **11.** $|-8|$ ☐ $|8|$ **12.** -10 ☐ $|-18|$

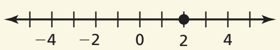
13. **ABSTRACT REASONING** A number a is to the left of a number b on the number line. How do the numbers $-a$ and $-b$ compare?

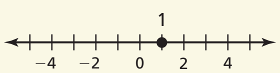
What Your Students Have Learned

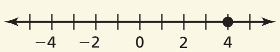
- Graph numbers and numerical expressions on a number line.
- Compare numbers and numerical expressions and write statements with <, >, or =.

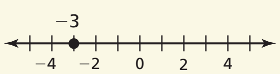
ANSWERS

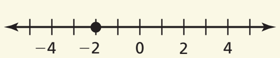
1. 

2. 

3. 

4. 

5. 

6. 

7. <

8. <

9. <

10. >

11. =

12. <

13. $-b < -a$

Vocabulary Review

Have students make an Idea and Examples Chart for the following words.

- Absolute value
- Negative number
- Positive number

MONITORING PROGRESS ANSWERS

1. $x < -4$
2. $x > 3$
3. $x \geq -2$

Mathematical Practices

Mathematically proficient students use technology tools to explore concepts.

Using a Graphing Calculator

Core Concept

Solving an Inequality in One Variable

You can use a graphing calculator to solve an inequality.

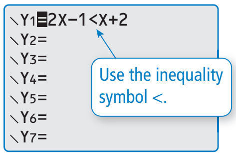
1. Enter the inequality into a graphing calculator.
2. Graph the inequality.
3. Use the graph to write the solution.

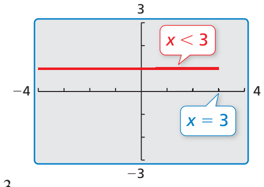
EXAMPLE 1 Using a Graphing Calculator

Use a graphing calculator to solve (a) $2x - 1 < x + 2$ and (b) $2x - 1 \leq x + 2$.

SOLUTION

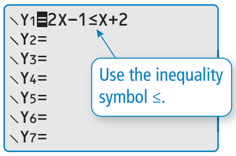
a. Enter the inequality $2x - 1 < x + 2$ into a graphing calculator. Press *graph*.

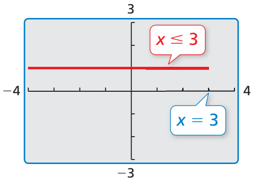




▶ The solution of the inequality is $x < 3$.

b. Enter the inequality $2x - 1 \leq x + 2$ into a graphing calculator. Press *graph*.





▶ The solution of the inequality is $x \leq 3$.

Notice that the graphing calculator does not distinguish between the solutions $x < 3$ and $x \leq 3$. You must distinguish between these yourself, based on the inequality symbol used in the original inequality.

Monitoring Progress

Use a graphing calculator to solve the inequality.

1. $2x + 3 < x - 1$
2. $-x - 1 > -2x + 2$
3. $\frac{1}{2}x + 1 \leq \frac{3}{2}x + 3$

Laurie's Notes — Mathematical Practices (continued from page T-51)

- Explain why the graphs given in Example 1 are horizontal and not vertical: the graphing calculator graphs one dimension horizontally, similar to how the graph would be on a number line. It offsets the solution from the *x*-axis so that the solution is visible.
- Be sure to point out that the graph does not distinguish $x < 3$ and $x \leq 3$, meaning that students will not see the open circle or closed circle on the calculator screen! They will need to interpret any endpoints based on the inequality symbols used in the original problem.
- The three questions in *Monitoring Progress* should not take long. Students may enter the fractions in Question 3 as fractions using the division symbol or as decimals.

Laurie's Notes

Overview of Section 2.1

Introduction

- Students have written and graphed inequalities in middle school. The domain may have been restricted to whole numbers, followed by integers. In this lesson, students will work with real numbers, and they will also decide whether a number is a solution of an inequality.
- When students translate a verbal phrase or context into symbols, the variable may end up on the right side of the inequality symbol. *Example:* 80 points was the lowest score on the test and students write $80 < p$. Suggest to students that they rewrite the inequality so that the variable is read first (left side of inequality). In doing so, the direction of the inequality symbol must be switched; $p > 80$.
- Solutions of an inequality are represented in symbols and by a graph. An alternate way of representing solutions is to use set-builder notation.

Common Misconceptions

- Students will often make the mistake of thinking $-1 < -2$, forgetting that relationships are reversed on the negative side of 0; $-1 > -2$.

Formative Assessment Tips

- **Learning Goals Inventory:** At the beginning of a chapter or unit of study, it is important for students to identify their level of understanding or knowledge of the learning intentions (goals) for the chapter. Taking an inventory of the explicit goals for the chapter informs students what you want them to learn—they know the target. On one side of a sheet of paper, I type the assignment guide for the chapter. On the other side, I list the learning intentions for the chapter and ask students to do a self-assessment for each learning goal.

  0—no knowledge, understanding, or ability 2—feeling okay
  1—beginning to get it 3—very confident
- Students return to the *Learning Goals Inventory* midway through the chapter and after the chapter assessment. Students recognize their own growth in completing the inventory multiple times.

Applications

- **Model with Mathematics:** In Exercise 46, maximum load limits on roads are explored. Have students research the load limits for state and federal roads in your state. The number of axles on the vehicle will be a factor.

Pacing Suggestion

- Given the review nature of the content in this lesson, you should only spend one day on it. You could do the *Motivate* and explorations or simply present the formal lesson. Students would prefer the more active version!

Dynamic Teaching Tools

- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ALABAMA STANDARDS

A1.11 Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.

A1.13 Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.

Learning Target

Write inequalities and represent solutions of inequalities on number lines.

Success Criteria

- Write word sentences as inequalities.
- Determine whether a value is a solution of an inequality.
- Graph and interpret inequalities.

Laurie's Notes

Exploration

Motivate

- **Preparation:** Write eight inequalities on index cards. Draw the matching graphs on eight strips of paper large enough to be seen by students across your room. Tape the eight graphs in different locations around your room.
- Examples of inequalities to explore: $x > 4$; $x \leq -4$; $x > -4$; $x \leq 4$; $x < -2.5$; $x \leq -2.5$; $x > 3.5$; $x \geq 3.5$
- Explain to students that they are starting a new chapter today. Express your confidence in them, knowing that they will have little difficulty with graphing inequalities.
- Select eight students at random and hand each an index card. Ask students to find their graphs and to go stand next to the graphs.

? After students have matched their cards to the graphs, ask each student to explain how he or she knows the match is correct. What features of the graph did they look for? Listen for: open circle versus closed circle, shading the correct side of the number line.

- After all of the students have made their explanations, collect their cards. Next, ask eight different students to go to one of the graphs and say aloud the inequality that is shown by the graph.

Exploration 1

? "The variable *t* is suggested to represent temperatures. Could any variable be used?" yes

? "Could the temperature in Sweden be −10°C? Explain." yes; The phrase "at least" would include −10°C and anything warmer.

- Discuss the real-life physical limitations. For instance, although the graph displays solutions to infinity, the temperatures in Sweden are unlikely to approach infinity.
- **Reason Abstractly and Quantitatively:** Learning to interpret solutions in the context of the problem is a practice you want students to develop.

Exploration 2

- Students may have forgotten the difference in notation between the closed and open circle. When both partners have forgotten what the notation means, I encourage them to check in with another group.
- Teachers should not be the only source of knowledge and information in the class. Develop the culture in your classroom of students listening to others and offering information to others.
- There may be different words students use to describe the inequalities. For part (a) students may say, "all values of *x* greater than or equal to 1" or "all *x*-values that are at least 1."
- **Attend to Precision:** Solutions should refer to a variable, meaning it would not be acceptable to give "greater than or equal to 1" as an answer to part (a).

Communicate Your Answer

- Students sometimes have difficulty thinking of a context much different from the one they have just completed. Suggest that they think of units that might be associated with each number. In Question 4(a), 3.5 might refer to inches, pieces of pizza, pounds, weeks, hours, sheets of plywood, and so on.

Connecting to Next Step

- If students are confident of their understanding of writing and graphing inequalities, you might have them record the *Core Vocabulary* and *Concept Summary* in the formal lesson and perhaps complete Example 4, an application. Students would then be ready for homework exercises.

2.1 Writing and Graphing Inequalities

ALABAMA STANDARDS
A1.11, A1.13

Essential Question

How can you use an inequality to describe a real-life statement?

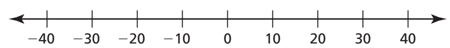
EXPLORATION 1 Writing and Graphing Inequalities

Work with a partner. Write an inequality for each statement. Then sketch the graph of the numbers that make each inequality true.

a. Statement The temperature *t* in Sweden is at least −10°C.

Inequality

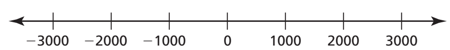
Graph



b. Statement The elevation *e* of Alabama is at most 2407 feet.

Inequality

Graph

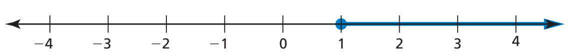


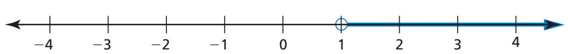
ATTENDING TO PRECISION

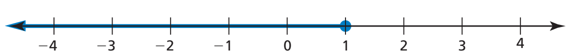
To be proficient in math, you need to communicate precisely. You also need to state the meanings of the symbols you use.

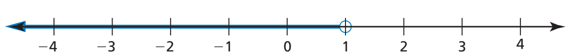
EXPLORATION 2 Writing Inequalities

Work with a partner. Write an inequality for each graph. Then, in words, describe all the values of *x* that make each inequality true.

a.


b.


c.


d.


Communicate Your Answer

3. How can you use an inequality to describe a real-life statement?

4. Write a real-life statement that involves each inequality.

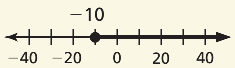
a. $x < 3.5$
b. $x \leq 6$
c. $x > -2$
d. $x \geq 10$

Dynamic Teaching Tools

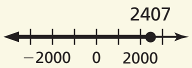
- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ANSWERS

1. a. $t \geq -10$°C



b. $e \leq 2407$ ft



2. a. $x \geq 1$; one and all numbers greater than one

b. $x > 1$; all numbers greater than one

c. $x \leq 1$; one and all numbers less than one

d. $x < 1$; all numbers less than one

3. An inequality can indicate the largest or smallest value for a quantity.

4. a. *Sample answer:* The judge's score is less than 3.5.

b. *Sample answer:* The time in the pool was at most 6 hours.

c. *Sample answer:* The temperature is greater than −2°C.

d. *Sample answer:* The profit was at least $10.

English Language Learners

Vocabulary and Symbols

Have students create an index card for each of the symbols $<$, $>$, $\leq$, and $\geq$. Have them write the symbol on the front of the card and the descriptive phrase on the back. Students can use the cards to help them choose the correct symbol when they write inequalities for word sentences.

Extra Example 1

Write each sentence as an inequality.

a. A number c plus 9.4 is greater than or equal to -6. $c + 9.4 \geq -6$

b. Six is greater than a number p minus 8. $6 > p - 8$

c. Ten is no more than the sum of 3 times a number y and 4. $10 \leq 3y + 4$

MONITORING PROGRESS ANSWERS

1. $b < 30.4$
2. $-\frac{7}{10} \geq 2k - 4$

2.1 Lesson

Learning Target: Write inequalities and represent solutions of inequalities on number lines.

Success Criteria:
- I can write word sentences as inequalities.
- I can determine whether a value is a solution of an inequality.
- I can graph and interpret inequalities.

Core Vocabulary

inequality, *p. 54*
solution of an inequality, *p. 55*
solution set, *p. 55*
graph of an inequality, *p. 56*

Previous
expression

Writing Linear Inequalities

An **inequality** is a mathematical sentence that compares expressions. An inequality contains the symbol $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine what inequality symbol to use.

Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Key Phrases	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most • is no more than	• is greater than or equal to • is at least • is no less than

EXAMPLE 1 **Writing Inequalities**

Write each sentence as an inequality.

a. A number w minus 3.5 is less than or equal to -2.

b. Three is less than a number n plus 5.

c. Zero is greater than or equal to twice a number x plus 1.

SOLUTION

a. A number w minus 3.5 | is less than or equal to | -2.

$w - 3.5$ $\leq$ -2

▶ An inequality is $w - 3.5 \leq -2$.

READING

The inequality $3 < n + 5$ is the same as $n + 5 > 3$.

b. Three | is less than | a number n plus 5.

3 $<$ $n + 5$

▶ An inequality is $3 < n + 5$.

c. Zero | is greater than or equal to | twice a number x plus 1.

0 $\geq$ $2x + 1$

▶ An inequality is $0 \geq 2x + 1$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Write the sentence as an inequality.

1. A number b is fewer than 30.4.
2. $-\frac{7}{10}$ is at least twice a number k minus 4.

Laurie's Notes Teacher Actions

- To review the vocabulary associated with inequalities, you can do a matching activity. Write expressions (key phrases) on cards and hand them to students. Include cards that have the inequality symbols written on them. Have students sort themselves into four groups in different regions of the room.
- **Use Appropriate Tools Strategically:** Using color can be a helpful tool as students are learning to translate words into symbols.

Sketching the Graphs of Linear Inequalities

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 5 \geq -2$	Is the inequality true?
-6	$-6 + 5 \overset{?}{\geq} -2$ $-1 \geq -2$ ✓	yes
-7	$-7 + 5 \overset{?}{\geq} -2$ $-2 \geq -2$ ✓	yes
-8	$-8 + 5 \overset{?}{\geq} -2$ $-3 \ngeq -2$ ✗	no

Recall that a diagonal line through an inequality symbol means the inequality is *not* true. For instance, the symbol $\ngeq$ means "is not greater than or equal to."

EXAMPLE 2 Checking Solutions

Tell whether -4 is a solution of each inequality.

a. $x + 8 < -3$

b. $-4.5x > -21$

SOLUTION

a. $x + 8 < -3$ — Write the inequality.

$-4 + 8 \overset{?}{<} -3$ — Substitute -4 for x.

$4 \nless -3$ ✗ — Simplify.

4 is *not* less than -3.

▶ So, -4 is *not* a solution of the inequality.

b. $-4.5x > -21$ — Write the inequality.

$-4.5(-4) \overset{?}{>} -21$ — Substitute -4 for x.

$18 > -21$ ✓ — Simplify.

18 is greater than -21.

▶ So, -4 is a solution of the inequality.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Tell whether -6 is a solution of the inequality.

3. $c + 4 < -1$

4. $10 \leq 3 - m$

5. $21 \div x \geq -3.5$

6. $4x - 25 > -2$

Extra Example 2

Tell whether 3 is a solution of each inequality.

a. $x - 7 > -5$ 3 is a solution.

b. $-3x \leq -12.5$ 3 is *not* a solution.

MONITORING PROGRESS ANSWERS

3. yes

4. no

5. yes

6. no

Laurie's Notes Teacher Actions

- **? Assessing Question:** Write a simple inequality on the board, such as $x - 5 > -2$, and ask for a solution. "How do you know 10 is a solution?" Substitute 10 in place of x and do the arithmetic to see whether the inequality is true. "How many solutions are there?" infinitely many
- Work through Example 2 as shown, noting the use of the diagonal line through the inequality symbol.
- **? Probing Question:** "Is there a symbol equivalent to $<$?" yes, $\ngeq$
- When working with inequalities in real life, there are some cases where negative values do not apply in the real-life context or where only certain values like whole numbers do apply. Students need to be aware that an inequality like $x < 4$ represents *all possible* solutions less than 4 and *not all numbers* less than 4.

Differentiated Instruction

Visual

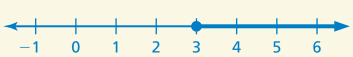
Some students may have difficulty graphing inequalities. Have students complete this table to describe the type of circle to use and the direction to draw the arrow in the graph. Then have students graph the inequalities and exchange graphs with a classmate to check their work.

Inequality	Circle	Direction
$x < 6$	open	left
$x > -4$	open	right
$x \le -1$	closed	left
$x \ge 1$	closed	right

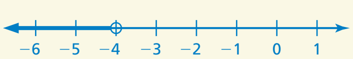
Extra Example 3

Graph each inequality.

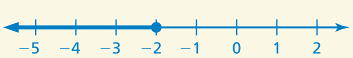
a. $x \ge 3$



b. $-4 > y$

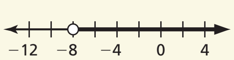


c. $x \le -2$

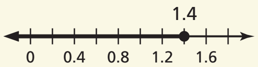


MONITORING PROGRESS ANSWERS

7.



8.



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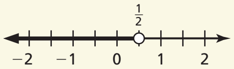

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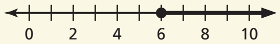


The **graph of an inequality** shows the solution set of the inequality on a number line. An open circle, ○, is used when a number is *not* a solution. A closed circle, ●, is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 **Graphing Inequalities**

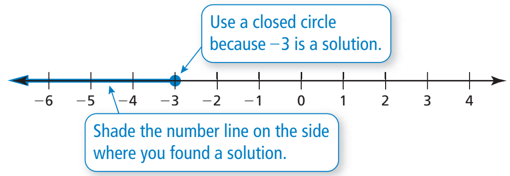
Graph each inequality.

a. $y \le -3$ **b.** $2 < x$ **c.** $x > 0$

SOLUTION

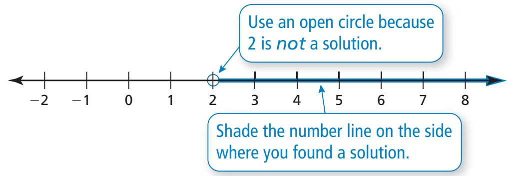
a. Test a number to the left of -3. $y = -4$ is a solution.

Test a number to the right of -3. $y = 0$ is *not* a solution.

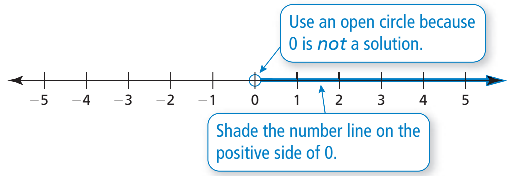


b. Test a number to the left of 2. $x = 0$ is *not* a solution.

Test a number to the right of 2. $x = 4$ is a solution.



c. Just by looking at the inequality, you can see that it represents the set of all positive numbers.



ANOTHER WAY

Another way to represent the solutions of an inequality is to use *set-builder notation*. In Example 3b, the solutions can be written as $\{x \mid x > 2\}$, which is read as "the set of all numbers x such that x is greater than 2."

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

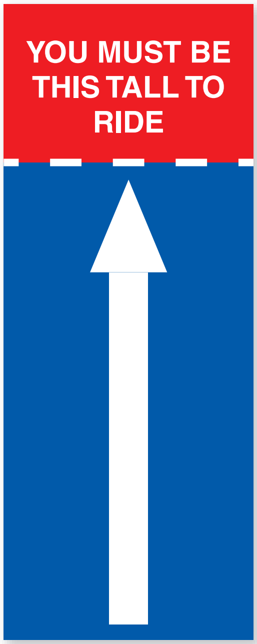
Graph the inequality.

7. $b > -8$ **8.** $1.4 \ge g$

9. $r < \frac{1}{2}$ **10.** $v \ge \sqrt{36}$

Laurie's Notes Teacher Actions

- Students may well remember how to graph inequalities. A quick reminder of the open circle versus closed circle may be all that is necessary.
- The inequality symbol points in the direction of the graph when the variable is on the left side of the symbol. This is not true when the variable is on the right side.
- **Big Idea:** When looking at a graph we can read the values of the variable that make the inequality true and the values of the variable that make the inequality *not* true. Numbers *not* shaded are *not* solutions.
- Ask students to show a *Thumbs Up* indicating their understanding of graphing inequalities.
- **Monitoring Progress:** Check to make sure that students have shaded graphs in the correct direction.



Writing Linear Inequalities from Graphs

EXAMPLE 4 **Writing Inequalities from Graphs**

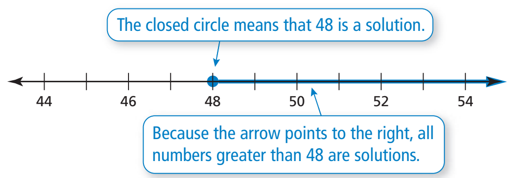
The graphs show the height restrictions h (in inches) for two rides at an amusement park. Write an inequality that represents the height restriction of each ride.

Ride A

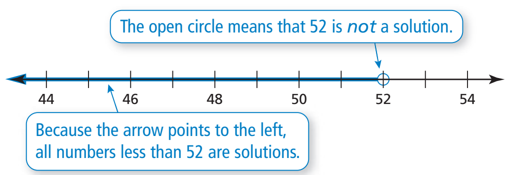
Ride B

SOLUTION

Ride A



Ride B



▶ So, $h \geq 48$ represents the height restriction for Ride A, and $h < 52$ represents the height restriction for Ride B.

Monitoring Progress

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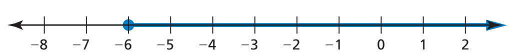
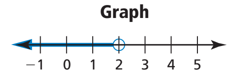
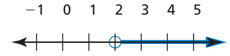
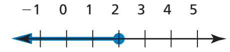
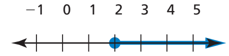
11. Write an inequality that represents the graph.



Concept Summary

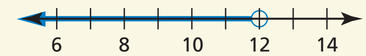
Representing Linear Inequalities

Words	Algebra	Graph
x is less than 2	$x < 2$	
x is greater than 2	$x > 2$	
x is less than or equal to 2	$x \leq 2$	
x is greater than or equal to 2	$x \geq 2$	

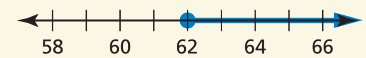
Extra Example 4

The graphs show the age restrictions (in years) for two types of movie tickets. Write an inequality that represents the age restriction of each ticket.

Ticket A



Ticket B



Ticket A: $a < 12$; Ticket B: $a \geq 62$

MONITORING PROGRESS ANSWER

11. $x \geq -6$

Laurie's Notes — Teacher Actions

? "How many of you have been to an amusement park where there was a sign indicating a height limit or height requirement?" Answers will vary.

- **Whiteboarding:** Pose Example 4 and have students work with partners to complete it. Have students hold the whiteboards to reveal their answers.
- Have students write the *Concept Summary* in their notebooks or journals.

Closure

- Explain how to graph the inequality $x > -2$ to someone you are speaking to on the phone. *Sample answer:* On a number line, draw an open circle on −2 and shade the line to the right of −2.

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–4, 5–23 odd, 27–35 odd, 41, 45, 50, 52, 60–67

Average: 1–4, 6–46 even, 50–54 even, 60–67

Advanced: 1–4, 10–14, 22–26, 28, 34–58 even, 59–67

HOMEWORK CHECK

Basic: 9, 17, 33, 41

Average: 10, 16, 32, 44

Advanced: 12, 24, 36, 44

ANSWERS

1. inequality
2. no; If x is 5, then the value on the left simplifies to 8, which is not greater than 8.
3. Draw an open circle when a number is not part of the solution. Draw a closed circle when a number is part of the solution. Draw an arrow to the left or right to show that the graph continues in that direction.
4. w is no more than -7; $w \leq -7$; $w \geq -7$
5. $x > 3$
6. $n + 7 \leq 9$
7. $15 \leq \frac{t}{5}$
8. $3w < 18$
9. $\frac{1}{2}y > 22$
10. $3 < s + 4$
11. $13 \geq v - 1$
12. $4 \geq \frac{x}{2.1}$
13. $w \geq 1.7$
14. $x \leq 170$
15. no
16. no
17. yes
18. yes
19. yes
20. no
21. yes
22. no
23. yes
24. no
25. **a.** $h < 107$

    b. no; A height of 9 feet is equal to 108 inches, which is not less than 107 inches.

2.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A mathematical sentence using the symbols <, >, ≤, or ≥ is called a(n)_______.
2. **VOCABULARY** Is 5 in the solution set of $x + 3 > 8$? Explain.
3. **ATTENDING TO PRECISION** Describe how to graph an inequality.
4. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

w is greater than or equal to -7.	w is no less than -7.
w is no more than -7.	w is at least -7.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, write the sentence as an inequality. *(See Example 1.)*

5. A number x is greater than 3.
6. A number n plus 7 is less than or equal to 9.
7. Fifteen is no more than a number t divided by 5.
8. Three times a number w is less than 18.
9. One-half of a number y is more than 22.
10. Three is less than the sum of a number s and 4.
11. Thirteen is at least the difference of a number v and 1.
12. Four is no less than the quotient of a number x and 2.1.
13. **MODELING WITH MATHEMATICS** On a fishing trip, you catch two fish. The weight of the first fish is shown. The second fish weighs at least 0.5 pound more than the first fish. Write an inequality that represents the possible weights of the second fish.

14. **MODELING WITH MATHEMATICS** There are 430 people in a wave pool. Write an inequality that represents how many more people can enter the pool.

In Exercises 15–24, tell whether the value is a solution of the inequality. *(See Example 2.)*

15. $r + 4 > 8$; $r = 2$
16. $5 - x < 8$; $x = -3$
17. $3s \leq 19$; $s = -6$
18. $17 \geq 2y$; $y = 7$
19. $-1 > -\frac{x}{2}$; $x = 3$
20. $\frac{4}{z} \geq 3$; $z = 2$
21. $14 \geq -2n + 4$; $n = -5$
22. $-5 \div (2s) < -1$; $s = 10$
23. $20 \leq \frac{10}{2z} + 20$; $z = 5$
24. $\frac{3m}{6} - 2 > 3$; $m = 8$
25. **MODELING WITH MATHEMATICS** The tallest person who ever lived was approximately 8 feet 11 inches tall.

    a. Write an inequality that represents the heights of every other person who has ever lived.

    b. Is 9 feet a solution of the inequality? Explain.

26. **DRAWING CONCLUSIONS** The winner of a weight-lifting competition bench-pressed 400 pounds. The other competitors all bench-pressed at least 23 pounds less.

a. Write an inequality that represents the weights that the other competitors bench-pressed.

b. Was one of the other competitors able to bench-press 379 pounds? Explain.

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in determining whether 8 is in the solution set of the inequality.**

27. ✗

$-y + 7 < -4$

$-8 + 7 \overset{?}{<} -4$

$-1 < -4$

8 is in the solution set.

28. ✗

$\frac{1}{2}x + 2 \le 6$

$\frac{1}{2}(8) + 2 \overset{?}{\le} 6$

$4 + 2 \overset{?}{\le} 6$

$6 \le 6$

8 is *not* in the solution set.

In Exercises 29–36, graph the inequality. *(See Example 3.)*

29. $x \ge 2$ **30.** $z \le 5$

31. $-1 > t$ **32.** $-2 < w$

33. $v \le -4$ **34.** $s < 1$

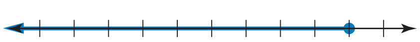
35. $\frac{1}{4} < p$ **36.** $r \ge -|5|$

In Exercises 37–40, write and graph an inequality for the given solution set.

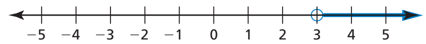
37. $\{x \mid x < 7\}$ **38.** $\{n \mid n \ge -2\}$

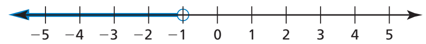
39. $\{z \mid 1.3 \le z\}$ **40.** $\{w \mid 5.2 > w\}$

In Exercises 41–44, write an inequality that represents the graph. *(See Example 4.)*

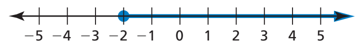
41.


42.


43.


44.


45. **ANALYZING RELATIONSHIPS** The water temperature of a swimming pool must be no less than 76°F. The temperature is currently 74°F. Which graph correctly shows how much the temperature needs to increase? Explain your reasoning.

Ⓐ


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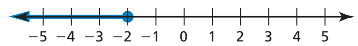

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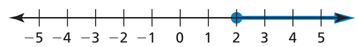

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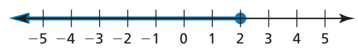
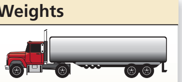


46. **MODELING WITH MATHEMATICS** According to a state law for vehicles traveling on state roads, the maximum total weight of a vehicle and its contents depends on the number of axles on the vehicle. For each type of vehicle, write and graph an inequality that represents the possible total weights w (in pounds) of the vehicle and its contents.

Maximum Total Weights		
 2 axles, 40,000 lb	 3 axles, 60,000 lb	 4 axles, 80,000 lb

47. **PROBLEM SOLVING** The Xianren Bridge is located in Guangxi Province, China. This arch is the world's longest natural arch, with a length of 400 feet. Write and graph an inequality that represents the lengths ℓ (in *inches*) of all other natural arches.

Dynamic Teaching Tools

Dynamic Assessment System

Dynamic Classroom

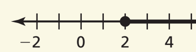
ANSWERS

26. a. $x \le 377$

b. no; A weight of 379 pounds is not a solution of the inequality in part (a).

27. Because -1 is not less than -4, the final result is not true; $-1 \nless -4$; 8 is not in the solution set.

28. Because 6 equals 6, the final inequality is true; 8 is in the solution set.

29.


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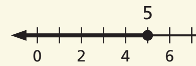

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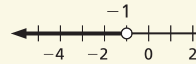

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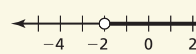

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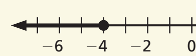

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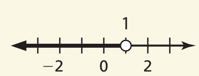

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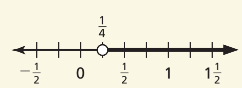
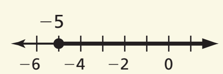
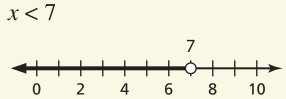
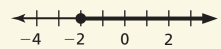
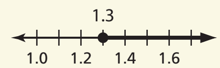
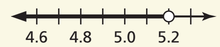


36.


37. $x < 7$


38. $n \ge -2$


39. $1.3 \le z$


40. $5.2 > w$


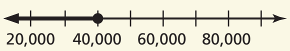
41. $x \le 4$

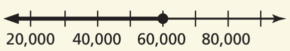
42. $x \ge -2$

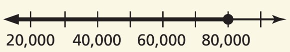
43. $x > 3$

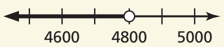
44. $x < -1$

45. C; The temperature must be at least 2°F warmer, so the increase is represented by $x \ge 2$.

46. 2 axles: $w \le 40{,}000$


3 axles: $w \le 60{,}000$


4 axles: $w \le 80{,}000$


47. $\ell < 4800$


ANSWERS

48. *Sample answer:* $h \leq \frac{25}{7}$

49. *Sample answer:* You spend $23 on admission and x dollars on snacks, and you can spend no more than $31 total.

50. **a.** $T \geq -38.87$

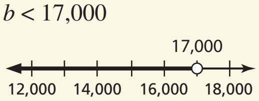
b. yes; The graph shows −38.87°C as the lowest possible melting point.

51. $0.90x \leq 24$; yes; Because $0.9(25) = \$22.50$, which is less than $24, the inequality is true.

52. cousin; Because the inequality is "less than or equal to," a weight equal to the given amount is possible.

53. *Sample answer:* A temperature above the freezing point of water can be represented by $T > 0$ if the temperature is in degrees Celsius, or by $T > 32$ if the temperature is in degrees Fahrenheit.

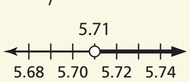
54. $b < 17{,}000$



55. $x < 14$ **56.** $x \geq 1.6$

57. $x < 3$ **58.** $x > 6$

59. **a.** $r > \frac{40}{7}$ (about 5.71)



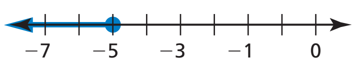
b. no; The graph includes speeds beyond the maximum speed a human can run.

60–67. See Additional Answers.

Mini-Assessment

1. Write the sentence as an inequality.
   One-half is at least 3 times a number p. $\frac{1}{2} \geq 3p$
2. Tell whether −2 is a solution of the inequality $n - 5 < 8$. −2 is a solution.
3. Graph the inequality $s > -3$.

4. Write an inequality that represents the graph.

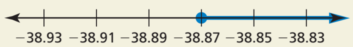


$x \leq -5$

48. THOUGHT PROVOKING A student works no more than 25 hours each week at a part-time job. Write an inequality that represents how many hours the student can work each day.

49. WRITING Describe a real-life situation modeled by the inequality $23 + x \leq 31$.

50. HOW DO YOU SEE IT? The graph represents the known melting points of all metallic elements (in degrees Celsius).



a. Write an inequality represented by the graph.

b. Is it possible for a metallic element to have a melting point of −38.87°C? Explain.

51. DRAWING CONCLUSIONS A one-way ride on a subway costs $0.90. A monthly pass costs $24. Write an inequality that represents how many one-way rides you can buy before it is cheaper to buy the monthly pass. Is it cheaper to pay the one-way fare for 25 rides? Explain.

Subway Prices	
One-way ride	$0.90
Monthly pass	$24.00

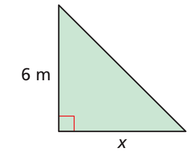
52. MAKING AN ARGUMENT The inequality $x \leq 1324$ represents the weights (in pounds) of all mako sharks ever caught using a rod and reel. Your friend says this means no one using a rod and reel has ever caught a mako shark that weighs 1324 pounds. Your cousin says this means someone using a rod and reel *has* caught a mako shark that weighs 1324 pounds. Who is correct? Explain your reasoning.

53. CRITICAL THINKING Describe a real-life situation that can be modeled by more than one inequality.

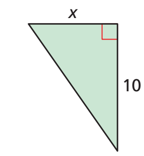
54. MODELING WITH MATHEMATICS In 1997, Superman's cape from the 1978 movie *Superman* was sold at an auction. The winning bid was $17,000. Write and graph an inequality that represents the amounts all the losing bids.

MATHEMATICAL CONNECTIONS **In Exercises 55–58, write an inequality that represents the missing dimension x.**

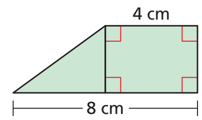
55. The area is less than 42 square meters.



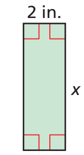
56. The area is greater than or equal to 8 square feet.



57. The area is less than 18 square centimeters.



58. The area is greater than 12 square inches.



59. WRITING A runner finishes a 200-meter dash in 35 seconds. Let r represent any speed (in meters per second) faster than the runner's speed.

a. Write an inequality that represents r. Then graph the inequality.

b. Every point on the graph represents a speed faster than the runner's speed. Do you think every point could represent the speed of a runner? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.1)*

60. $x + 2 = 3$

61. $y - 9 = 5$

62. $6 = 4 + y$

63. $-12 = y - 11$

Solve the literal equation for x. *(Section 1.5)*

64. $v = x \cdot y \cdot z$

65. $s = 2r + 3x$

66. $w = 5 + 3(x - 1)$

67. $n = \frac{2x + 1}{2}$

Section Resources

Surface Level	Deep Level
Resources by Chapter • Practice A and Practice B • Puzzle Time Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension • Cumulative Review Dynamic Assessment System • Section Practice

Laurie's Notes

Overview of Section 2.2

Introduction
- In this lesson, students will write, solve, and graph inequalities that involve addition and subtraction. Students solved equations in the last chapter and inequalities in middle school. Conceptually the process is the same.
- **Look For and Make Use of Structure:** Mathematically proficient students will recognize the similarity between solving $x + 7 = -12$ and $x + 7 < -12$.

Common Misconceptions
- When students encounter an inequality involving subtraction, they may immediately think, "I need to add in order to solve this inequality." In this example: $x - (-4) > 10$, they should first simplify the inequality and write $x + 4 > 10$. To solve, they subtract 4 from each side of the inequality. Show the following examples in parallel and have students discuss the approaches.

$$\begin{aligned} x - (-4) &> 10 \\ +(-4) & \quad +(-4) \\ \hline x &> 6 \end{aligned} \qquad \begin{aligned} x + 4 &> 10 \\ -4 & \quad -4 \\ \hline x &> 6 \end{aligned}$$

Formative Assessment Tips
- **Think-Pair-Share:** This technique allows students to share their thinking about a problem with their partners after they have had time to consider the problem alone. Once partners have discussed the problem, small groups or the whole class should discuss the problem.
- The initial time working alone is important for students to develop their own understanding of the mathematics. "Private think time" is what I call it. Once students have engaged in the problem, sharing with partners helps confirm their understanding or perhaps the need to modify their thinking. Sharing your thinking with the whole class is more comfortable for students when they have had the chance to discuss their thinking with partners.
- **Construct Viable Arguments and Critique the Reasoning of Others:** Using this formative assessment technique allows you to check students' conceptual knowledge and their ability to construct a viable argument.

Applications
- Inequalities are more common than students might guess. Applications often involve measurement (weight, height, length). For instance, airlines have guidelines for the maximum weight of luggage, depending upon whether it is carry-on or checked luggage, or whether the travel is domestic or international.

Another Way
- Introduce the Addition or Subtraction Properties of Inequalities with a personal example. I summarize these two properties in the following way:
  I am older than you. In two years, I will still be older than you.
  Laurie's age > Student's age — if $a > b$,
  Laurie's age + 2 > Student's age + 2 — then $a + c > b + c$
  Two years ago, I was older than you.
  Laurie's age > Student's age — if $a > b$,
  Laurie's age − 2 > Student's age − 2 — then $a - c > b - c$

Pacing Suggestion
- Once students have worked the explorations, continue with the formal lesson.

Dynamic Teaching Tools
- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ALABAMA STANDARDS

A1.11 Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.

A1.13 Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.

Learning Target

Write and solve inequalities using addition or subtraction.

Success Criteria

- Apply the Addition and Subtraction Properties of Inequality to produce equivalent inequalities.
- Solve inequalities using addition or subtraction.
- Use inequalities to model real-life problems.

Laurie's Notes

Exploration

Motivate

- Wear a football-related piece of clothing today if you own one.
- Set the tone by tossing a few passes in class with a small foam football. Ask a statistician to record your efforts in a table at the board. Use three columns: **C**ompleted, I**N**tercepted, and Inco**M**plete.
- I recommend **A**ttempting 10 short passes to students nearby. You may need to give permission to have a pass intercepted.

? Ask the following questions.
  - "How many passes did I attempt?" 10 Record this next to the table.
  - "Can I complete more passes than I attempt?" no
  - "Are completed passes + incomplete passes always less than or equal to attempted passes?" yes
  - "Are completed passes + incomplete passes always less than attempted passes?" No, they could be equal.

Exploration 1

- The tree diagram should be a helpful aid to students.
- Discuss students' answers and their reasoning when they have finished.
- For parts (c) and (d), point out the need to pay attention to the inequality symbol. It is possible, though unlikely, that $T = C$. In that case, the inequality $T < C$ may *not* be true, while the inequality $T \leq C$ is true.
- **Reason Abstractly and Quantitatively:** There will be some heated discussion about the inequalities, but remember to ask, "is it *possible*" versus "is it *probable*." Take time for students to explain their reasoning.

Exploration 2

- In this exploration, two of the inequalities force students to think about solving the inequality before they begin. In part (b), if $P + 100 > 250$, then it means $P > 150$. Students should decontextualize the situation and manipulate the symbols to solve the inequality.
- Answers will vary for this exploration, but suggest to students that they keep the numbers as simple as possible. For instance, one possible answer for the first question is (1, 0, 0, 0, 1), where only one pass is attempted and it is intercepted. The result is $P = -200$, and so $P < 0$.
- **Reason Abstractly and Quantitatively:** Students will need to do a little trial and error with these problems. They should ask themselves, "What happens when I increase this variable; does another variable decrease?" Students need the opportunity to practice this type of reasoning.

Communicate Your Answer

- Students should recognize the similarity between solving $x + 3 < 4$ and $x + 3 = 4$.

Connecting to Next Step

- The inequalities students have worked in the explorations have been done using mental math. In the formal lesson, they will solve inequalities symbolically and graph the solutions.

2.2 Solving Inequalities Using Addition or Subtraction

ALABAMA STANDARDS
A1.11, A1.13

Essential Question

How can you use addition or subtraction to solve an inequality?

EXPLORATION 1 Quarterback Passing Efficiency

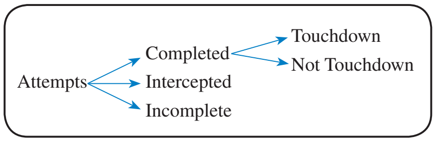
Work with a partner. The National Collegiate Athletic Association (NCAA) uses the following formula to rank the passing efficiencies P of quarterbacks.

$$P = \frac{8.4Y + 100C + 330T - 200N}{A}$$

$Y =$ total length of all completed passes (in Yards) $\quad C =$ Completed passes

$T =$ passes resulting in a Touchdown $\quad N =$ iNtercepted passes

$A =$ Attempted passes $\quad M =$ incoMplete passes



MODELING WITH MATHEMATICS

To be proficient in math, you need to identify and analyze important relationships and then draw conclusions, using tools such as diagrams, flowcharts, and formulas.

Determine whether each inequality must be true. Explain your reasoning.

a. $T < C$ **b.** $C + N \leq A$ **c.** $N < A$ **d.** $A - C \geq M$

EXPLORATION 2 Finding Solutions of Inequalities

Work with a partner. Use the passing efficiency formula to create a passing record that makes each inequality true. Record your results in the table. Then describe the values of P that make each inequality true.

Attempts	Completions	Yards	Touchdowns	Interceptions

a. $P < 0$

b. $P + 100 \geq 250$

c. $P - 250 > -80$

Communicate Your Answer

3. How can you use addition or subtraction to solve an inequality?

4. Solve each inequality.

a. $x + 3 < 4$ **b.** $x - 3 \geq 5$

c. $4 > x - 2$ **d.** $-2 \leq x + 1$

Dynamic Teaching Tools

- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ANSWERS

1. a. no; It is also possible for the number of touchdowns to equal the number of completed passes.
   b. yes; Completed and intercepted passes are both included in attempts.
   c. no; It is also possible for the number of intercepted passes to equal the number of attempts.
   d. yes; After subtracting completed passes from attempts, all remaining passes could be incomplete.
2. a. *Sample answer:* 25; 5; 100; 0; 10; negative numbers
   b. *Sample answer:* 10; 9; 600; 4; 0; numbers greater than or equal to 150
   c. *Sample answer:* 10; 7; 300; 2; 1; numbers greater than 170
3. Add or subtract the same number to or from each side of the inequality to create an equivalent inequality.
4. a. $x < 1$
   b. $x \geq 8$
   c. $x < 6$
   d. $x \geq -3$

Differentiated Instruction

Kinesthetic

To help students understand the difference between the solutions of equations and inequalities, write the equation $x + 5 = 10$ and the inequality $x + 5 \geq 10$ on the board. Ask two students to assist you at the board. Assign the equation to one student to solve and graph on a number line. Assign the inequality to the second student to solve and graph on a number line. Have students discuss how and why the solutions are different.

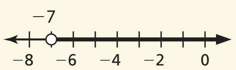
Extra Example 1

Solve $x - 5 < -3$. Graph the solution.

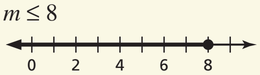
$x < 2$

MONITORING PROGRESS ANSWERS

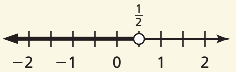
1. $b > -7$



2. $m \leq 8$



3. $y < \frac{1}{2}$



2.2 Lesson

Learning Target: Write and solve inequalities using addition or subtraction.

Success Criteria:
- I can apply the Addition and Subtraction Properties of Inequality to produce equivalent inequalities.
- I can solve inequalities using addition or subtraction.
- I can use inequalities to model real-life problems.

Core Vocabulary

equivalent inequalities, *p. 62*

Previous

inequality

Solving Inequalities Using Addition

Just as you used the properties of equality to produce equivalent equations, you can use the properties of inequality to produce equivalent inequalities. **Equivalent inequalities** are inequalities that have the same solutions.

Core Concept

Addition Property of Inequality

Words Adding the same number to each side of an inequality produces an equivalent inequality.

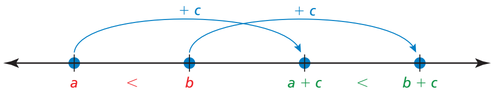
Numbers

$$\begin{array}{rcl} -3 & < & 2 \\ +4 & & +4 \\ 1 & < & 6 \end{array} \qquad \begin{array}{rcl} -3 & \geq & -10 \\ +3 & & +3 \\ 0 & \geq & -7 \end{array}$$

Algebra If $a > b$, then $a + c > b + c$. If $a \geq b$, then $a + c \geq b + c$.

If $a < b$, then $a + c < b + c$. If $a \leq b$, then $a + c \leq b + c$.

The diagram shows one way to visualize the Addition Property of Inequality when $c > 0$.



EXAMPLE 1 Solving an Inequality Using Addition

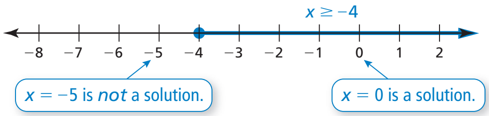
Solve $x - 6 \geq -10$. Graph the solution.

SOLUTION

$x - 6 \geq -10$	Write the inequality.
$+6 \quad +6$	Add 6 to each side.
$x \geq -4$	Simplify.

Addition Property of Inequality

▶ The solution is $x \geq -4$.



REMEMBER

To check this solution, substitute a few numbers to the left and right of −4 into the original inequality.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the inequality. Graph the solution.

1. $b - 2 > -9$ **2.** $m - 3 \leq 5$ **3.** $\frac{1}{4} > y - \frac{1}{4}$

Laurie's Notes Teacher Actions

- Define *equivalent inequalities* and write the *Core Concept*.
- **Look For and Make Use of Structure:** This property should look very familiar, as it is similar to the Addition Property of Equality. Demonstrating the property using just numbers helps students make sense of the property.
- Discuss the visual representation of the property.
- **Attend to Precision:** In Example 1, remind students that they are using the Addition Property of Inequality.

COMMON ERROR In graphing the solution $x \geq -4$, students may shade in the wrong direction by thinking $-5 > -4$. Remind them that the further to the right you are on a number line, the greater the number.

Solving Inequalities Using Subtraction

Core Concept

Subtraction Property of Inequality

Words Subtracting the same number from each side of an inequality produces an equivalent inequality.

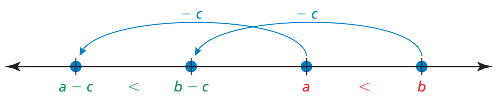
Numbers

$$\begin{array}{rcl} -3 & \le & 1 \\ \underline{-5} & & \underline{-5} \\ -8 & \le & -4 \end{array} \qquad \begin{array}{rcl} 7 & > & -20 \\ \underline{-7} & & \underline{-7} \\ 0 & > & -27 \end{array}$$

Algebra If $a > b$, then $a - c > b - c$. If $a \ge b$, then $a - c \ge b - c$.

If $a < b$, then $a - c < b - c$. If $a \le b$, then $a - c \le b - c$.

The diagram shows one way to visualize the Subtraction Property of Inequality when $c > 0$.



EXAMPLE 2 **Solving an Inequality Using Subtraction**

Solve each inequality. Graph the solution.

a. $y + 8 \le 5$ **b.** $-8 < 1.4 + m$

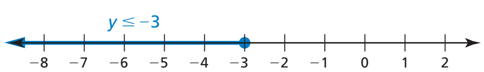
SOLUTION

a. $y + 8 \le 5$ Write the inequality.

Subtraction Property of Inequality → $\underline{-8} \quad \underline{-8}$ Subtract 8 from each side.

$y \le -3$ Simplify.

▶ The solution is $y \le -3$.

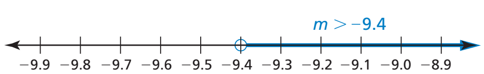


b. $-8 < 1.4 + m$ Write the inequality.

Subtraction Property of Inequality → $\underline{-1.4} \quad \underline{-1.4}$ Subtract 1.4 from each side.

$-9.4 < m$ Simplify.

▶ The solution is $m > -9.4$.



Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

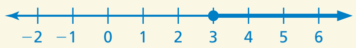
Solve the inequality. Graph the solution.

4. $k + 5 \le -3$ **5.** $\frac{5}{6} \le z + \frac{1}{6}$ **6.** $p + 0.7 > -2.3$

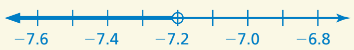
Extra Example 2

Solve each inequality. Graph the solution.

a. $s + 18 \ge 21$ $s \ge 3$

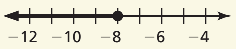


b. $-5.2 > x + 2$ $x < -7.2$

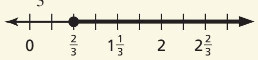


MONITORING PROGRESS ANSWERS

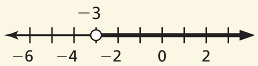
4. $k \le -8$



5. $z \ge \frac{2}{3}$



6. $p > -3$



Laurie's Notes Teacher Actions

- The Subtraction Property of Inequality is first demonstrated with numbers. In the example shown, students may need to be reminded that $-8 \le -4$.
- ? "Can the inequality in Example 2(b) be written as $-8 < m + 1.4$? Explain." yes; Commutative Property
- Encourage students to check their solution by selecting a value in the solution set and substituting it into the original inequality.
- **Attend to Precision:** When graphing the solution $m > -9.4$, be sure students scale the number line correctly.

Extra Example 3

An electric grill that uses 1600 watts of electricity is plugged into the circuit described in Example 3.

a. Write and solve an inequality that represents how many watts you can add to the circuit without overloading the circuit. $1600 + w < 1800$; $w < 200$; You can add up to 200 watts.

b. In addition to the electric grill, which of the appliances listed in the table in Example 3 can you plug into the circuit at the same time without overloading the circuit? clock radio

MONITORING PROGRESS ANSWER

7. no; The additional electricity must be less than 800 watts, and the toaster uses 800 watts.

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

A circuit overloads at 1800 watts of electricity. You plug a microwave oven that uses 1100 watts of electricity into the circuit.

a. Write and solve an inequality that represents how many watts you can add to the circuit without overloading the circuit.

b. In addition to the microwave oven, which of the following appliances can you plug into the circuit at the same time without overloading the circuit?

Appliance	Watts
Clock radio	50
Blender	300
Hot plate	1200
Toaster	800

SOLUTION

1. **Understand the Problem** You know that the microwave oven uses 1100 watts out of a possible 1800 watts. You are asked to write and solve an inequality that represents how many watts you can add without overloading the circuit. You also know the numbers of watts used by four other appliances. You are asked to identify the appliances you can plug in at the same time without overloading the circuit.

2. **Make a Plan** Use a verbal model to write an inequality. Then solve the inequality and identify other appliances that you can plug into the circuit at the same time without overloading the circuit.

3. **Solve the Problem**

Words Watts used by microwave oven + Additional watts < Overload wattage

Variable Let w be the additional watts you can add to the circuit.

Inequality $1100 \quad + \quad w \quad < \quad 1800$

$1100 + w < 1800$ — Write the inequality.

Subtraction Property of Inequality → $\underline{-1100} \quad \underline{-1100}$ — Subtract 1100 from each side.

$w < 700$ — Simplify.

▶ You can add up to 700 watts to the circuit, which means that you can also plug in the clock radio and the blender.

4. **Look Back** You can check that your answer is correct by adding the numbers of watts used by the microwave oven, clock radio, and blender.

$1100 + 50 + 300 = 1450$

The circuit will not overload because the total wattage is less than 1800 watts.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. The microwave oven uses only 1000 watts of electricity. Does this allow you to have both the microwave oven and the toaster plugged into the circuit at the same time? Explain your reasoning.

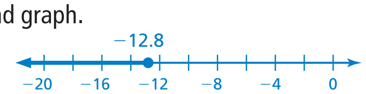
Laurie's Notes — Teacher Actions

- The problem posed is not difficult to solve with mental math. The point is not that you can solve the problem mentally. The practice you want to provide students is the opportunity to *read* a problem, *make sense* of the context, and *translate* words into symbols. This is a building block for future problems. I ask a volunteer to read Steps 1 and 2 in the problem-solving plan.
- **FYI:** Most modern residential circuits are 15 or 20 amps, so there is a maximum load of either (15A × 120V =) 1800 watts or (20A × 120V =) 2400 watts before the breaker trips.
- **Use Appropriate Tools Strategically:** The color coding is a tool to help students connect words and symbols.
- **Think-Pair-Share:** Partners should work Question 7 together.

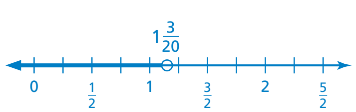
Closure

- **Think-Pair-Share:** Solve and graph.

$x + 3.8 \le -9$ $\quad x \le -12.8$



$\frac{2}{5} > x - \frac{3}{4}$ $\quad x < 1\frac{3}{20}$



2.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Why is the inequality $x \leq 6$ equivalent to the inequality $x - 5 \leq 6 - 5$?
2. **WRITING** Compare solving equations using addition with solving inequalities using addition.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, tell which number you would add to or subtract from each side of the inequality to solve it.

3. $k + 11 < -3$
4. $v - 2 > 14$
5. $-1 \geq b - 9$
6. $-6 \leq 17 + p$

In Exercises 7–20, solve the inequality. Graph the solution. *(See Examples 1 and 2.)*

7. $x - 4 < -5$
8. $1 \leq s - 8$
9. $6 \geq m - 1$
10. $c - 12 > -4$
11. $r + 4 < 5$
12. $-8 \leq 8 + y$
13. $9 + w > 7$
14. $15 \geq q + 3$
15. $h - (-2) \geq 10$
16. $-6 > t - (-13)$
17. $j + 9 - 3 < 8$
18. $1 - 12 + y \geq -5$
19. $10 \geq 3p - 2p - 7$
20. $18 - 5z + 6z > 3 + 6$

In Exercises 21–24, write the sentence as an inequality. Then solve the inequality.

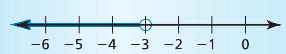
21. A number plus 8 is greater than 11.
22. A number minus 3 is at least -5.
23. The difference of a number and 9 is fewer than 4.
24. Six is less than or equal to the sum of a number and 15.

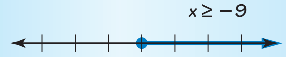
25. **MODELING WITH MATHEMATICS** You are riding a train. Your carry-on bag can weigh no more than 50 pounds. Your bag weighs 38 pounds. *(See Example 3.)*
    a. Write and solve an inequality that represents how much weight you can add to your bag.
    b. Can you add both a 9-pound laptop and a 5-pound pair of boots to your bag without going over the weight limit? Explain.

26. **MODELING WITH MATHEMATICS** You order the hardcover book shown from a website that offers free shipping on orders of \$25 or more. Write and solve an inequality that represents how much more you must spend to get free shipping.

Price: \$19.76

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in solving the inequality or graphing the solution.**

27. ✗
$$-17 < x - 14$$
$$-17 + 14 < x - 14 + 14$$
$$-3 < x$$


28. ✗
$$-10 + x \geq -9$$
$$-10 + 10 + x \geq -9$$
$$x \geq -9$$


29. **PROBLEM SOLVING** An NHL hockey player has 59 goals so far in a season. What are the possible numbers of additional goals the player can score to match or break the NHL record of 92 goals in a season?

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1–6, 7–29 odd, 30, 36, 39–46

Average: 1, 2–32 even, 36–46

Advanced: 1–2, 16–20, 22, 24–26, 28–38 even, 39–46

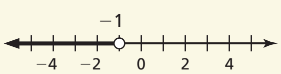
HOMEWORK CHECK

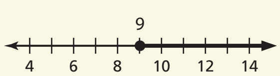
Basic: 5, 7, 13, 25

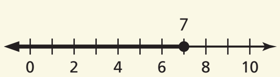
Average: 8, 18, 22, 26

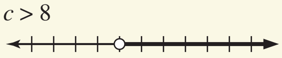
Advanced: 16, 17, 24, 25

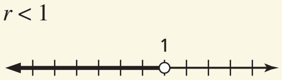
ANSWERS

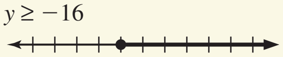
1. Subtraction Property of Inequality
2. *Sample answer:* In both cases the same number is added to each side to obtain an equivalent mathematical sentence.
3. subtract 11
4. add 2
5. add 9
6. subtract 17
7. $x < -1$


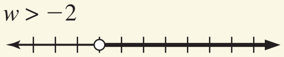
8. $s \geq 9$


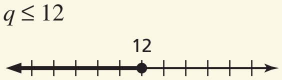
9. $m \leq 7$


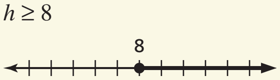
10. $c > 8$


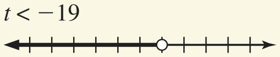
11. $r < 1$


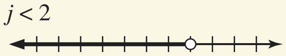
12. $y \geq -16$


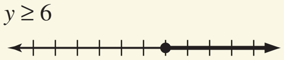
13. $w > -2$


14. $q \leq 12$


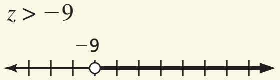
15. $h \geq 8$


16. $t < -19$


17. $j < 2$


18. $y \geq 6$


19. $p \leq 17$


20. $z > -9$


21. $n + 8 > 11$; $n > 3$
22. $n - 3 \geq -5$; $n \geq -2$
23. $n - 9 < 4$; $n < 13$
24. $6 \leq n + 15$; $n \geq -9$

25–29. See Additional Answers.

Dynamic Teaching Tools

Dynamic Assessment System

Dynamic Classroom

ANSWERS

30. **a.** score greater than 117.4 points

b. they both are; Both scores are greater than 117.4 points.

31. A; Subtract 3 from each side; D; order of inequality reverses for opposites

32. $14.2 + 15.5 + x < 51.3$; $x < 21.6$

33. $6.4 + 4.9 + 4.1 + x \leq 18.7$; $x \leq 3.3$

34. *Sample answer:* $x + 34 \leq 50$; A shelf can safely support 50 pounds. The combined weight of items on the shelf is 34 pounds. x represents the possible weights of items that can be put on the shelf safely.

35. no; *Sample answer:* 3, 7, 8, 9, and 12; There are infinitely many solutions. Check 8 and a few numbers greater than and less than 8.

36–46. See Additional Answers.

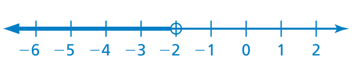
Mini-Assessment

Solve the inequality. Graph the solution.

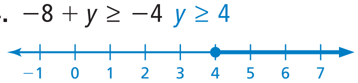
1. $x - 2 > -3$ $x > -1$

2. $4 \geq m - 3$ $m \leq 7$

3. $r + 5 < 3$ $r < -2$



4. $-8 + y \geq -4$ $y \geq 4$



5. A shirt that you want to buy costs \$15.50. You can use a \$5 discount coupon if you spend at least \$20. Write and solve an inequality that represents how much more you must spend to use the \$5 coupon. $x + 15.5 \geq 20$; $x \geq 4.5$; at least \$4.50 more

30. **MAKING AN ARGUMENT** In an aerial ski competition, you perform two acrobatic ski jumps. The scores on the two jumps are then added together.

Ski jump	Competitor's score	Your score
1	117.1	119.5
2	119.8	

a. Describe the score that you must earn on your second jump to beat your competitor.

b. Your coach says that you will beat your competitor if you score 118.4 points. A teammate says that you only need 117.5 points. Who is correct? Explain.

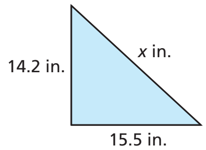
31. **REASONING** Which of the following inequalities are equivalent to the inequality $x - b < 3$, where b is a constant? Justify your answer.

Ⓐ $x - b - 3 < 0$ Ⓑ $0 > b - x + 3$

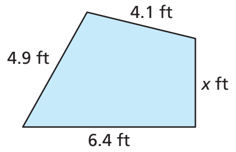
Ⓒ $x < 3 - b$ Ⓓ $-3 < b - x$

MATHEMATICAL CONNECTIONS **In Exercises 32 and 33, write and solve an inequality to find the possible values of x.**

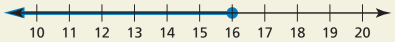
32. Perimeter < 51.3 inches



33. Perimeter ≤ 18.7 feet

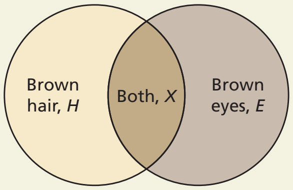


34. **THOUGHT PROVOKING** Write an inequality that has the solution shown in the graph. Describe a real-life situation that can be modeled by the inequality.



35. **WRITING** Is it possible to check all the numbers in the solution set of an inequality? When you solve the inequality $x - 11 \geq -3$, which numbers can you check to verify your solution? Explain your reasoning.

36. **HOW DO YOU SEE IT?** The diagram represents the numbers of students in a school with brown eyes, brown hair, or both.



Determine whether each inequality must be true. Explain your reasoning.

a. $H \geq E$ **b.** $H + 10 \geq E$

c. $H \geq X$ **d.** $H + 10 \geq X$

e. $H > X$ **f.** $H + 10 > X$

37. **REASONING** Write and graph an inequality that represents the numbers that are *not* solutions of each inequality.

a. $x + 8 < 14$

b. $x - 12 \geq 5.7$

38. **PROBLEM SOLVING** Use the inequalities $c - 3 \geq d$, $b + 4 < a + 1$, and $a - 2 \leq d - 7$ to order a, b, c, and d from least to greatest.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the product or quotient. *(Skills Review Handbook)*

39. $7 \cdot (-9)$ **40.** $-11 \cdot (-12)$ **41.** $-27 \div (-3)$ **42.** $20 \div (-5)$

Solve the equation. Check your solution. *(Section 1.1)*

43. $6x = 24$ **44.** $-3y = -18$ **45.** $\frac{s}{-8} = 13$ **46.** $\frac{n}{4} = -7.3$

Section Resources

Surface Level	Deep Level
Resources by Chapter • Practice A and Practice B • Puzzle Time Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension • Cumulative Review Dynamic Assessment System • Section Practice

Laurie's Notes

Overview of Section 2.3

Introduction

- In this lesson, students will write, solve, and graph inequalities that involve multiplication and division. Students solved equations in the last chapter and inequalities in middle school. Conceptually the process is the same.
- **Look For and Make Use of Structure:** Mathematically proficient students will recognize the similarity between solving $3x = -12$ and $3x < -12$.
- The direction of the inequality sign can change for two reasons: (1) you have multiplied or divided by a negative number or (2) you want to make the final solution more readable. Before the lesson ends, students need to identify each case. Here they both are shown in the solution of an inequality:

$12 < -4x$	
$-3 > x$	Divide by -4 and change direction of inequality symbol.
$x < -3$	Change direction of inequality symbol to read solution.

Formative Assessment Tips

- **Exit Ticket:** This technique asks students to respond to a question at the end of the lesson, activity, or learning experience. The *Exit Ticket* allows you to collect evidence of student learning. I cut scrap paper into smaller pieces so that "exit tickets" can be distributed quickly to students.
- The *Exit Ticket* is helpful in planning instruction. During the class there may be students you have not heard from. They may not have raised their hands, or they may have been less vocal when working with partners. The *Exit Ticket* helps you gauge the ability of all students to answer a particular type of question.
- Students write their names on the exit tickets, which are then collected.
- When a subset of students has difficulty with the skill addressed by the *Exit Ticket*, instruction for the following day should address this problem.

Pacing Suggestion

- There are four inequalities in part (b) of each exploration. Divide the class into four different groups, have each group do one of the four inequalities, and then share as a class.

Dynamic Teaching Tools

- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ALABAMA STANDARDS

A1.11 Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.

A1.13 Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.

Learning Target

Write and solve inequalities using multiplication or division.

Success Criteria

- Apply the Multiplication and Division Properties of Inequality to produce equivalent inequalities.
- Solve inequalities using multiplication or division.
- Recognize when to reverse an inequality symbol while solving an inequality.

Laurie's Notes

Exploration

Motivate

- Ask a series of questions and record students' solutions.
  - ? "What integers are solutions of $x > 4$?" 5, 6, 7, ...
  - ? "What integers are solutions of $-x > 4$, meaning what numbers have an opposite that is greater than 4?" $-5, -6, -7, \ldots$
  - ? "What integers are solutions of $x < -4$?" $-5, -6, -7, \ldots$
- Leave these three problems on the board and refer to them at the end of class.

Exploration Note

- **Look For and Express Regularity in Repeated Reasoning:** The approach in this investigation is to use a table of values to see what numbers satisfy the inequality. The inequalities involving positive coefficients behave as expected. It is the inequalities involving negative coefficients that seem not to work as expected—from the student perspective. Through repeated trials students should recognize that when the coefficient is negative, the inequality symbol is reversed.

Exploration 1

- Using the information in the table, students decide which graph represents the solution. Finally, they write the solution.
- Discuss how the solution is written: $2 < x$ or $x > 2$.
- Discuss the results with students:
  - ? "What did you find as the solution for part (a)?" $x > 2$
  - ? "Is this what you would have expected?" Likely, they will say "yes."
  - Repeat this line of questions for the inequalities in part (b).
- Do not tell students a rule at this point. Simply say that perhaps they need to try a few more inequalities to help figure out what is going on.

Exploration 2

- The format of this exploration is the same as the first.
- Discuss the results with students:
  - ? "What did you find as the solution for part (a)?" $x < -2$
  - ? "Is this what you would have expected?" Likely, they will say "no."
  - Repeat this line of questions for the inequalities in part (b).
- ? "What was different about these inequalities from those in Exploration 1?" The coefficients were negative.

Communicate Your Answer

- *Actively listen* as students describe how to use division to solve an inequality.
- ? If you began with the *Motivate*, ask "Which of the inequalities had the same solution?" $-x > 4$ and $x < -4$
- ? "Is this consistent with what you discovered in the explorations?" yes; Listen for comments about the negative coefficient of x and the switching of the inequality symbol.

Connecting to Next Step

- The rule for changing the direction of the inequality symbol when dividing by a negative should make sense to students. Connect the operations of multiplication and division when stating the *Core Concepts*.

2.3 Solving Inequalities Using Multiplication or Division

ALABAMA STANDARDS
A1.11, A1.13

Essential Question

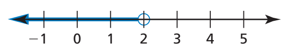
How can you use division to solve an inequality?

EXPLORATION 1 Writing a Rule

Work with a partner.

a. Copy and complete the table. Decide which graph represents the solution of the inequality $6 < 3x$. Write the solution of the inequality.

x	-1	0	1	2	3	4	5
$3x$	-3						
$6 \overset{?}{<} 3x$	No						



b. Use a table to solve each inequality. Then write a rule that describes how to use division to solve the inequalities.

i. $2x < 4$ **ii.** $3 \geq 3x$ **iii.** $2x < 8$ **iv.** $6 \geq 3x$

LOOKING FOR A PATTERN

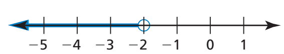
To be proficient in math, you need to investigate relationships, observe patterns, and use your observations to write general rules.

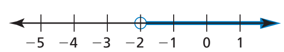
EXPLORATION 2 Writing a Rule

Work with a partner.

a. Copy and complete the table. Decide which graph represents the solution of the inequality $6 < -3x$. Write the solution of the inequality.

x	-5	-4	-3	-2	-1	0	1
$-3x$							
$6 \overset{?}{<} -3x$							





b. Use a table to solve each inequality. Then write a rule that describes how to use division to solve the inequalities.

i. $-2x < 4$ **ii.** $3 \geq -3x$ **iii.** $-2x < 8$ **iv.** $6 \geq -3x$

Communicate Your Answer

3. How can you use division to solve an inequality?

4. Use the rules you wrote in Explorations 1(b) and 2(b) to solve each inequality.

a. $7x < -21$ **b.** $12 \leq 4x$ **c.** $10 < -5x$ **d.** $-3x \leq 0$

Dynamic Teaching Tools
- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ANSWERS

1. a. 0; 3; 6; 9; 12; 15; no; no; no; yes; yes; yes; right graph; $x > 2$

b. i. $x < 2$

ii. $x \leq 1$

iii. $x < 4$

iv. $x \leq 2$

Dividing each side of an inequality by the same positive number produces an equivalent inequality.

2. a. 15; 12; 9; 6; 3; 0; -3; yes; yes; yes; no; no; no; no; left graph; $x < -2$

b. i. $x > -2$

ii. $x \geq -1$

iii. $x > -4$

iv. $x \geq -2$

When dividing each side of an inequality by the same negative number, the direction of the inequality must be reversed to produce an equivalent inequality.

3. Divide each side of the inequality by the same number. If the number is positive, this produces an equivalent inequality. If the number is negative, the inequality must be reversed to be equivalent.

4. a. $x < -3$

b. $x \geq 3$

c. $x < -2$

d. $x \geq 0$

English Language Learners

Pair Activity

Have students work in pairs to solve inequalities. Each student solves a different inequality. Students share their work and explain their reasoning while their partners follow along.

Extra Example 1

a. Solve $-4 > \frac{x}{4}$. Graph the solution.

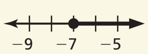
$x < -16$

b. Solve $2n \geq -14$. Graph the solution.

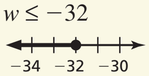
$n \geq -7$

MONITORING PROGRESS ANSWERS

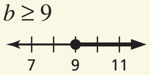
1. $n \geq -7$



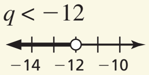
2. $w \leq -32$



3. $b \geq 9$



4. $q < -12$



2.3 Lesson

Learning Target: Write and solve inequalities using multiplication or division.

Success Criteria:
- I can apply the Multiplication and Division Properties of Inequality to produce equivalent inequalities.
- I can solve inequalities using multiplication or division.
- I can recognize when to reverse an inequality symbol while solving an inequality.

Multiplying or Dividing by Positive Numbers

Core Concept

Multiplication and Division Properties of Inequality ($c > 0$)

Words Multiplying or dividing each side of an inequality by the same *positive* number produces an equivalent inequality.

Numbers

$-6 < 8$ $\qquad$ $6 > -8$

$2 \cdot (-6) < 2 \cdot 8$ $\qquad$ $\frac{6}{2} > \frac{-8}{2}$

$-12 < 16$ $\qquad$ $3 > -4$

Algebra If $a > b$ and $c > 0$, then $ac > bc$. If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$.

If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$.

These properties are also true for ≤ and ≥.

EXAMPLE 1 Multiplying or Dividing by Positive Numbers

Solve (a) $\frac{x}{8} > -5$ and (b) $-24 \geq 3x$. Graph each solution.

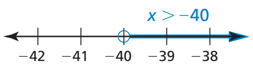
SOLUTION

a. $\frac{x}{8} > -5$ — Write the inequality.

Multiplication Property of Inequality → $8 \cdot \frac{x}{8} > 8 \cdot (-5)$ — Multiply each side by 8.

$x > -40$ — Simplify.

▶ The solution is $x > -40$.

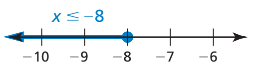


b. $-24 \geq 3x$ — Write the inequality.

Division Property of Inequality → $\frac{-24}{3} \geq \frac{3x}{3}$ — Divide each side by 3.

$-8 \geq x$ — Simplify.

▶ The solution is $x \leq -8$.



Monitoring Progress

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Solve the inequality. Graph the solution.

1. $\frac{n}{7} \geq -1$ **2.** $-6.4 \geq \frac{1}{5}w$ **3.** $4b \geq 36$ **4.** $-18 > 1.5q$

Laurie's Notes Teacher Actions

- **Look For and Make Use of Structure:** These properties should look very familiar, as they are similar to the Multiplication and Division Properties of Equality. Demonstrating the property using just numbers helps students make sense of it.
- Note that the properties are restricted to multiplying and dividing by a positive number. This is very important.
- **Representation:** Multiplication is represented by the dot notation, and −5 is enclosed in parentheses for clarity only. Otherwise, students might become confused and think 5 is being subtracted.
- Division is represented differently in Questions 1 and 2. The first representation is more common in algebra. In Question 2, check to see that students multiply by the reciprocal of the coefficient (most efficient) versus dividing both sides by $\frac{1}{5}$.

Multiplying or Dividing by Negative Numbers

Core Concept

Multiplication and Division Properties of Inequality ($c < 0$)

Words When multiplying or dividing each side of an inequality by the same *negative* number, the direction of the inequality symbol must be reversed to produce an equivalent inequality.

Numbers

$-6 < 8$ $6 > -8$

$-2 \cdot (-6) > -2 \cdot 8$ $\frac{6}{-2} < \frac{-8}{-2}$

$12 > -16$ $-3 < 4$

Algebra If $a > b$ and $c < 0$, then $ac < bc$. If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.

If $a < b$ and $c < 0$, then $ac > bc$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

These properties are also true for $\le$ and $\ge$.

COMMON ERROR

A negative sign in an inequality does not necessarily mean you must reverse the inequality symbol, as shown in Example 1.

Only reverse the inequality symbol when you multiply or divide each side by a negative number.

EXAMPLE 2 Multiplying or Dividing by Negative Numbers

Solve each inequality. Graph each solution.

a. $2 < \frac{y}{-3}$ **b.** $-7y \le -35$

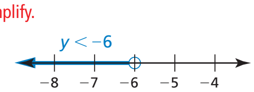
SOLUTION

a. $2 < \frac{y}{-3}$ — Write the inequality.

Multiplication Property of Inequality → $-3 \cdot 2 > -3 \cdot \frac{y}{-3}$ — Multiply each side by -3. Reverse the inequality symbol.

$-6 > y$ — Simplify.

▶ The solution is $y < -6$.

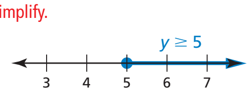


b. $-7y \le -35$ — Write the inequality.

Division Property of Inequality → $\frac{-7y}{-7} \ge \frac{-35}{-7}$ — Divide each side by -7. Reverse the inequality symbol.

$y \ge 5$ — Simplify.

▶ The solution is $y \ge 5$.



Monitoring Progress

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Solve the inequality. Graph the solution.

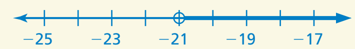
5. $\frac{p}{-4} < 7$ **6.** $\frac{x}{-5} \le -5$ **7.** $1 \ge -\frac{1}{10}z$

8. $-9m > 63$ **9.** $-2r \ge -22$ **10.** $-0.4y \ge -12$

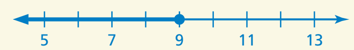
Extra Example 2

Solve each inequality. Graph each solution.

a. $3 > \frac{x}{-7}$ $x > -21$

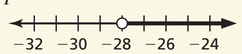


b. $-5m \ge -45$ $m \le 9$

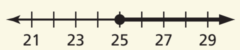


MONITORING PROGRESS ANSWERS

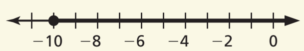
5. $p > -28$



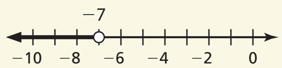
6. $x \ge 25$



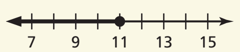
7. $z \ge -10$



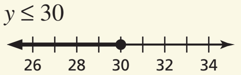
8. $m < -7$



9. $r \le 11$



10. $y \le 30$



Laurie's Notes — Teacher Actions

- Write the *Core Concept*. Discuss the difference between these properties and those at the beginning of the lesson.
- The short version of the properties: When you multiply or divide by a negative quantity, reverse the direction of the inequality symbol.
- After working each inequality in Example 2, check numbers that are in the solution set and numbers *not* in the solution set.

COMMON ERROR When students solve $2x < -4$, they sometimes reverse the inequality symbol because there is a negative number in the problem. The inequality symbol is reversed only when the coefficient is negative, *not* when the constant is negative.

- **Neighbor Check:** Have students work independently, and then have their neighbors check their work. Have students discuss any discrepancies.

Extra Example 3

You earn \$8.50 per hour at the sub shop. Write and solve an inequality that represents the numbers of hours you need to work to earn \$187 to buy a tablet computer. $8.5h \geq 187$; $h \geq 22$; at least 22 hours

MONITORING PROGRESS ANSWERS

11. $0.25c \leq 3.65$, $c \leq 14.6$

12. $\frac{165}{t} \leq 55$, $t \geq 3$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

You earn \$9.50 per hour at your summer job. Write and solve an inequality that represents the numbers of hours you need to work to buy a digital camera that costs \$247.

SOLUTION

1. **Understand the Problem** You know your hourly wage and the cost of the digital camera. You are asked to write and solve an inequality that represents the numbers of hours you need to work to buy the digital camera.
2. **Make a Plan** Use a verbal model to write an inequality. Then solve the inequality.
3. **Solve the Problem**

Words Hourly wage • Hours worked ≥ Cost of camera

Variable Let n be the number of hours worked.

Inequality 9.5 • n ≥ 247

$9.5n \geq 247$ Write the inequality.

Division Property of Inequality → $\frac{9.5n}{9.5} \geq \frac{247}{9.5}$ Divide each side by 9.5.

$n \geq 26$ Simplify.

▶ You need to work at least 26 hours for your gross pay to be at least \$247. If you have payroll deductions, such as Social Security taxes, you need to work more than 26 hours.

REMEMBER
Compatible numbers are numbers that are easy to compute mentally.

4. **Look Back** You can use estimation to check that your answer is reasonable.

\$247 ÷ \$9.50/h
↓ ↓
\$250 ÷ \$10/h = 25 h Use compatible numbers.

Your hourly wage is about \$10 per hour. So, to earn about \$250, you need to work about 25 hours.

Unit Analysis Each time you set up an equation or inequality to represent a real-life problem, be sure to check that the units balance.

$\frac{\$9.50}{\cancel{h}} \times 26\cancel{h} = \$247$

Monitoring Progress

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11. You have at most \$3.65 to make copies. Each copy costs \$0.25. Write and solve an inequality that represents the numbers of copies you can make.

12. The maximum speed limit for a school bus is 55 miles per hour. Write and solve an inequality that represents the numbers of hours it takes to travel 165 miles in a school bus.

Laurie's Notes Teacher Actions

- The problem posed is not difficult to solve with mental math. The point is not that you can solve the problem mentally. The practice you want to provide students is the opportunity to *read* a problem, *make sense* of the context, and *translate* words into symbols. This is a building block for future problems. I ask a volunteer to read Steps 1 and 2 in the problem-solving plan.
- **Use Appropriate Tools Strategically:** The color coding is a tool to help students connect words and symbols.
- **Think-Pair-Share:** Partners should work Questions 11 and 12 together.

Closure

- **I Used to Think … But Now I Know:** Take time for students to reflect on their current understanding of solving inequalities involving multiplication and division.
- **Exit Ticket:** Solve and graph.

$\frac{x}{-3} \leq -9$ $x \geq 27$

$-8 > 4x$ $x < -2$

2.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how solving $2x < -8$ is different from solving $-2x < 8$.

2. **OPEN-ENDED** Write an inequality that is solved using the Division Property of Inequality where the inequality symbol needs to be reversed.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the inequality. Graph the solution. *(See Example 1.)*

3. $4x < 8$
4. $3y \le -9$
5. $-20 \le 10n$
6. $35 < 7t$
7. $\frac{x}{2} > -2$
8. $\frac{a}{4} < 10.2$
9. $20 \ge \frac{4}{5}w$
10. $-16 \le \frac{8}{3}t$

In Exercises 11–18, solve the inequality. Graph the solution. *(See Example 2.)*

11. $-6t < 12$
12. $-9y > 9$
13. $-10 \ge -2z$
14. $-15 \le -3c$
15. $\frac{n}{-3} \ge 1$
16. $\frac{w}{-5} \le 16$
17. $-8 < -\frac{1}{4}m$
18. $-6 > -\frac{2}{3}y$

19. **MODELING WITH MATHEMATICS** You have \$12 to buy five goldfish for your new fish tank. Write and solve an inequality that represents the prices you can pay per fish. *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** A weather forecaster predicts that the temperature in Antarctica will decrease 8°F each hour for the next 6 hours. Write and solve an inequality to determine how many hours it will take for the temperature to drop at least 36°F.

USING TOOLS In Exercises 21–26, solve the inequality. Use a graphing calculator to verify your answer.

21. $36 < 3y$
22. $17v \ge 51$
23. $2 \le -\frac{2}{9}x$
24. $4 > \frac{n}{-4}$
25. $2x > \frac{3}{4}$
26. $1.1y < 4.4$

ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in solving the inequality.

27. ✗

$$-6 > \frac{2}{3}x$$
$$\frac{3}{2}\cdot(-6) < \frac{3}{2}\cdot\frac{2}{3}x$$
$$-\frac{18}{2} < x$$
$$-9 < x$$

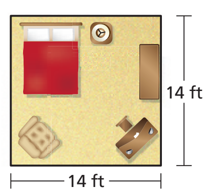
The solution is $x > -9$.

28. ✗

$$-4y \le -32$$
$$\frac{-4y}{-4} \le \frac{-32}{-4}$$
$$y \le 8$$

The solution is $y \le 8$.

29. **ATTENDING TO PRECISION** You have \$700 to buy new carpet for your bedroom. Write and solve an inequality that represents the costs per square foot that you can pay for the new carpet. Specify the units of measure in each step.



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–19 odd, 27, 30, 31, 33, 40–47

Average: 1, 2, 4–30 even, 31, 35, 40–47

Advanced: 1, 2, 6–28 even, 29–31, 32–38 even, 40–47

HOMEWORK CHECK

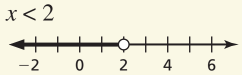
Basic: 3, 7, 11, 15, 19

Average: 4, 8, 12, 16, 20

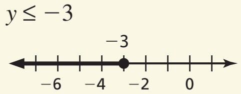
Advanced: 6, 8, 16, 18, 29

ANSWERS

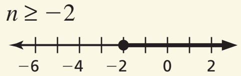
1. When solving $2x < -8$, the inequality symbol is not reversed when dividing each side by 2. When solving $-2x < 8$, the inequality is reversed when dividing each side by -2.
2. *Sample answer:* $-5x \ge 25$
3. $x < 2$



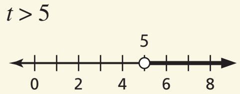
4. $y \le -3$



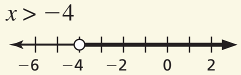
5. $n \ge -2$



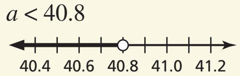
6. $t > 5$



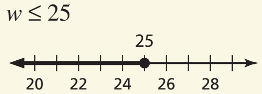
7. $x > -4$



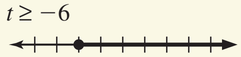
8. $a < 40.8$



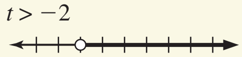
9. $w \le 25$



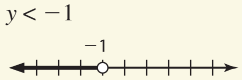
10. $t \ge -6$



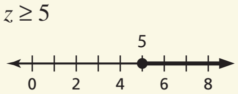
11. $t > -2$



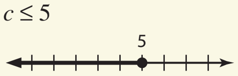
12. $y < -1$



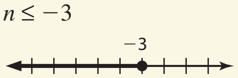
13. $z \ge 5$



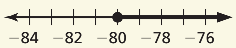
14. $c \le 5$



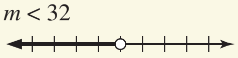
15. $n \le -3$



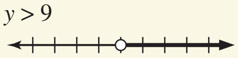
16. $w \ge -80$



17. $m < 32$



18. $y > 9$



19. $5p \le 12$, $p \le 2.4$
20. $-8t \le -36$, $t \ge 4.5$
21. $y > 12$
22. $v \ge 3$
23. $x \le -9$

24–29. See Additional Answers.

Dynamic Teaching Tools

Dynamic Assessment System
Dynamic Classroom

ANSWERS

30. **a.** C; Multiplying both sides by m gives $x < -m$.
b. A; Multiplying both sides by m gives $x > m$.
c. B; Multiplying both sides by m gives $x < m$.
d. D; Multiplying both sides by $-m$ and reversing the inequality symbol gives $x > -m$.

31. **a.** $d \le 6.3(2)$, $d \le 12.6$
b. yes; The distance traveled in 4 hours would be no more than 25.2 miles, which is less than the distance required for a marathon.

32. *Sample answer:* $x < 21$

33. more than 300 million pennies

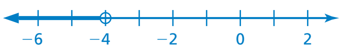
34. no; To get the second inequality, each side of the first is multiplied by -3. To be equivalent, the inequality symbol also needs to be reversed.

35–47. See Additional Answers.

Mini-Assessment

Solve the inequality. Graph the solution.

1. $6x < -24$ $\quad x < -4$



2. $\frac{s}{-4} \le -2$ $\quad s \ge 8$

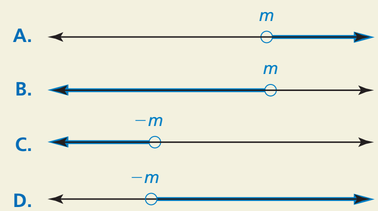
3. $18 < -\frac{2}{3}y$ $\quad y < -27$

4. You have \$12 to spend on ride tickets at the fair. Each ticket costs \$1.25. Write and solve an inequality that represents the number of tickets you can buy. $1.25x \le 12$; $x \le 9.6$; at most 9 tickets

30. HOW DO YOU SEE IT? Let $m > 0$. Match each inequality with its graph. Explain your reasoning.

a. $\frac{x}{m} < -1$ **b.** $\frac{x}{m} > 1$

c. $\frac{x}{m} < 1$ **d.** $-\frac{x}{m} < 1$



31. MAKING AN ARGUMENT You run for 2 hours at a speed no faster than 6.3 miles per hour.

a. Write and solve an inequality that represents the possible numbers of miles you run.

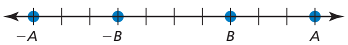
b. A marathon is approximately 26.2 miles. Your friend says that if you continue to run at this speed, you will not be able to complete a marathon in less than 4 hours. Is your friend correct? Explain.

32. THOUGHT PROVOKING The inequality $\frac{x}{4} \le 5$ has a solution of $x = p$. Write a second inequality that also has a solution of $x = p$.

33. PROBLEM SOLVING The U.S. Mint pays \$0.02 to produce every penny. How many pennies are produced when the U.S. Mint pays more than \$6 million in production costs?

34. REASONING Are $x \le \frac{2}{3}$ and $-3x \le -2$ equivalent? Explain your reasoning.

35. ANALYZING RELATIONSHIPS Consider the number line shown.



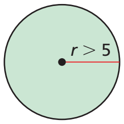
a. Write an inequality relating A and B.

b. Write an inequality relating $-A$ and $-B$.

c. Use the results from parts (a) and (b) to explain why the direction of the inequality symbol must be reversed when multiplying or dividing each side of an inequality by the same negative number.

36. REASONING Why might solving the inequality $\frac{4}{x} \ge 2$ by multiplying each side by x lead to an error? (*Hint:* Consider $x > 0$ and $x < 0$.)

37. MATHEMATICAL CONNECTIONS The radius of a circle is represented by the formula $r = \frac{C}{2\pi}$. Write and solve an inequality that represents the possible circumferences C of the circle.



38. CRITICAL THINKING A water-skiing instructor recommends that a boat pulling a beginning skier has a speed less than 18 miles per hour. Write and solve an inequality that represents the possible distances d (in miles) that a beginner can travel in 45 minutes of practice time.

39. CRITICAL THINKING A local zoo employs 36 people to take care of the animals each day. At most, 24 of the employees work full time. Write and solve an inequality that represents the fraction of employees who work part time. Graph the solution.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Section 1.2 and Section 1.3)*

40. $5x + 3 = 13$

41. $\frac{1}{2}y - 8 = -10$

42. $-3n + 2 = 2n - 3$

43. $\frac{1}{2}z + 4 = \frac{5}{2}z - 8$

Tell which number is greater. *(Skills Review Handbook)*

44. 0.8, 85%

45. $\frac{16}{30}$, 50%

46. 120%, 0.12

47. 60%, $\frac{2}{3}$

Section Resources

Surface Level	Deep Level
Resources by Chapter • Practice A and Practice B • Puzzle Time Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension • Cumulative Review Dynamic Assessment System • Section Practice

Laurie's Notes

Overview of Section 2.4

Introduction

- You solve multi-step inequalities the same way you solve multi-step equations. You only need to remember to change the direction of the inequality symbol when you multiply or divide by a negative quantity. Students solved multi-step equations in the first chapter, so they should be ready to apply that knowledge to inequalities.
- Just like linear equations, inequalities may have no solution or a solution of all real numbers. These special solutions are introduced after students have solved a number of multi-step inequalities.
- Students should be encouraged to check their solutions by substituting a value from the solution set to verify that it gives a true statement. They should also substitute a value that is *not* in the solution set to verify that it gives a false statement.

Formative Assessment Tips

- **No-Hands Questioning:** Typically when you ask a question there are hands that immediately go up, often the same hands each time. Some students need a longer time to process a question and think through their response. This technique instructs students not to put their hands in the air when the question is posed. *Wait Time** is exercised.
- This technique encourages all students to be active and engaged in the lesson. Students who need additional think time are provided that opportunity. Teachers can then use *Popsicle Sticks* to call on students for a response, or they can purposely call on those students whose voice they do not hear enough.
- To be effective, the question(s) posed during *No-Hands Questioning* have to require more than a simple response.

*See Section 2.5 for a description of *Wait Time*.

Another Way

- The exploration demonstrates that the solution of a multi-step inequality can be graphed using a graphing calculator. When the solution is a ray or half-line, the *trace* key and *zoom* feature can be used to get a good approximation of the endpoint even when it is not an integer value. The calculator is a tool to be used wisely.

Pacing Suggestion

- The inequalities students will work in the exploration are similar to Examples 1 and 2 in the formal lesson. If students have been successful, you may want to begin the formal lesson with Example 3.

Dynamic Teaching Tools

- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ALABAMA STANDARDS

A1.11 Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.

A1.13 Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.

Learning Target

Write and solve multi-step inequalities.

Success Criteria

- Use more than one property of inequality to generate equivalent inequalities.
- Solve multi-step inequalities using inverse operations.
- Apply multi-step inequalities to solve real-life problems.

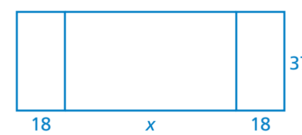
Laurie's Notes

Exploration

Motivate

? "Have you ever heard of or played Ultimate? Answers will vary.

- It is a sport played with a flying disc at colleges, high schools, and some middle schools. There are 10 simple rules, one of which is that there are no officials! Pretty cool.
- A regulation Ultimate field is 64 meters long and 37 meters wide, with two end zones of 18 meters each. Sometimes the length (64 meters) varies when using existing fields. Let the length be x. Draw a sketch and find the area of the field in terms of x.
  Area $= 37(x + 36) = (37x + 1332)\ \text{m}^2$



Discuss

- **Look For and Make Use of Structure:** Students have solved multi-step equations. Structurally, multi-step inequalities are solved in the same fashion. Mathematically proficient students recognize the similarity between solving $-4x - 5 = 41$ and $-4x - 5 < 41$. Students must recall that when the variable term has been isolated, if the coefficient is negative, then the inequality symbol is reversed.

Exploration 1

- This may seem like a stretch for students to solve multi-step inequalities of this type, yet the connection to multi-step equations and working with partners is generally enough for students to meet with success.
- Discuss the directions to ensure that students do not immediately use a calculator to determine the solution. The calculator is used to check their answers.
- In checking their solutions, students may find that their endpoint is correct but the shading is going in the opposite direction. Students should then be able to do a self-check to observe that they forgot to change the direction of the inequality symbol when they multiplied or divided by a negative number.
- Circulate and make note of common errors, questions, or procedures. When several groups are challenged by one of the inequalities, look for students who have solved it correctly. Ask those students to show the first step in their solution.
- **Construct Viable Arguments and Critique the Reasoning of Others:** Ask two volunteers who have their work displayed in a neat and organized fashion to share their solutions at the board for parts (e) and (f). You want to model good problem solving, as well as good mathematics. Hearing the reasoning process of classmates helps students improve their own problem-solving strategies.

? "Is it possible to have the same solutions but the steps in a different order? Explain." yes; There are different orders in which the steps might be done and still correctly solve the inequality.

Communicate Your Answer

- Ask two volunteers to share their work for Question 3.

Connecting to Next Step

- If students have been very successful with solving the inequalities in the exploration, be selective in examples you work in the formal lesson. It may be sufficient to try a few questions from *Monitoring Progress* and the real-life example.

2.4 Solving Multi-Step Inequalities

Essential Question
How can you solve a multi-step inequality?

ALABAMA STANDARDS
A1.11, A1.13

EXPLORATION 1 **Solving a Multi-Step Inequality**

Work with a partner.

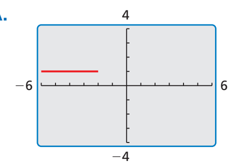
- Use what you already know about solving equations and inequalities to solve each multi-step inequality. Justify each step.
- Match each inequality with its graph. Use a graphing calculator to check your answer.

JUSTIFYING STEPS
To be proficient in math, you need to justify each step in a solution and communicate your justification to others.

a. $2x + 3 \le x + 5$ **b.** $-2x + 3 > x + 9$

c. $27 \ge 5x + 4x$ **d.** $-8x + 2x - 16 < -5x + 7x$

e. $3(x - 3) - 5x > -3x - 6$ **f.** $-5x - 6x \le 8 - 8x - x$

A.


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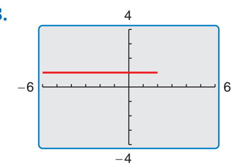

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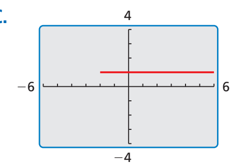

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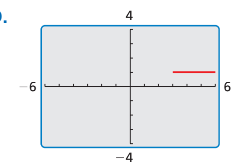
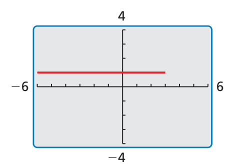
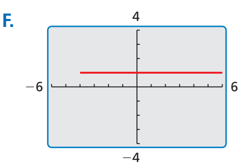


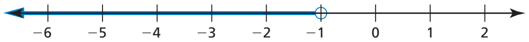
E.


F.


Communicate Your Answer

2. How can you solve a multi-step inequality?

3. Write two different multi-step inequalities whose solutions are represented by the graph.



Dynamic Teaching Tools
- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ANSWERS

1. a. $x \le 2$; Subtract x and 3 from each side; B

b. $x < -2$; Subtract x and 3 from each side; Divide each side by -3; A

c. $3 \ge x$; Divide each side by 9; E

d. $-2 < x$; Add $6x$ to each side; Divide each side by 8; C

e. $x > 3$; Add $3x$ and 9 to each side; D

f. $x \ge -4$; Add $9x$ to each side; Divide each side by -2; F

2. Simplify each side, if possible, then use inverse operations to isolate the variable. Reverse the inequality symbol if multiplying or dividing by a negative number.

3. *Sample answer:* $3x + 4 < 1$, $-2x - 10 > -8$

English Language Learners

Build on Past Knowledge

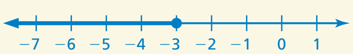
Explain to students that when they solve multi-step inequalities, they will use a process similar to the one they used to solve multi-step equations. They will be using inverse operations and performing one step at a time in order to isolate the variable.

Extra Example 1

Solve each inequality. Graph each solution.

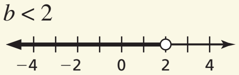
a. $\frac{x}{-2} + 4 > -1$ $x < 10$

b. $3c - 2 \leq -11$ $c \leq -3$

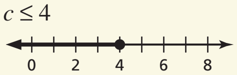


MONITORING PROGRESS ANSWERS

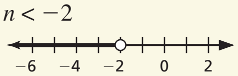
1. $b < 2$



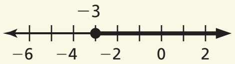
2. $c \leq 4$



3. $n < -2$



4. $v \geq -3$



2.4 Lesson

Learning Target: Write and solve multi-step inequalities.

Success Criteria:
- I can use more than one property of inequality to generate equivalent inequalities.
- I can solve multi-step inequalities using inverse operations.
- I can apply multi-step inequalities to solve real-life problems.

Solving Multi-Step Inequalities

To solve a multi-step inequality, simplify each side of the inequality, if necessary. Then use inverse operations to isolate the variable. Be sure to reverse the inequality symbol when multiplying or dividing by a negative number.

EXAMPLE 1 Solving Multi-Step Inequalities

Solve each inequality. Graph each solution.

a. $\frac{y}{-6} + 7 < 9$ **b.** $2v - 4 \geq 8$

SOLUTION

a. $\frac{y}{-6} + 7 < 9$ Write the inequality.

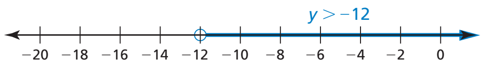
$-7 \quad -7$ Subtract 7 from each side.

$\frac{y}{-6} < 2$ Simplify.

$-6 \cdot \frac{y}{-6} > -6 \cdot 2$ Multiply each side by −6. Reverse the inequality symbol.

$y > -12$ Simplify.

▶ The solution is $y > -12$.



b. $2v - 4 \geq 8$ Write the inequality.

$+4 \quad +4$ Add 4 to each side.

$2v \geq 12$ Simplify.

$\frac{2v}{2} \geq \frac{12}{2}$ Divide each side by 2.

$v \geq 6$ Simplify.

▶ The solution is $v \geq 6$.

Monitoring Progress

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Solve the inequality. Graph the solution.

1. $4b - 1 < 7$
2. $8 - 9c \geq -28$
3. $\frac{n}{-2} + 11 > 12$
4. $6 \geq 5 - \frac{v}{3}$

Laurie's Notes Teacher Actions

- ? "What operations are being performed on the left side of the inequality?" division and addition
- ? "What is the first step in isolating the variable, meaning getting the *y*-term by itself?" Subtract 7 from each side of the inequality.
- ? "To solve for *y*, what is the last step?" Multiply each side by −6, and change the direction of the inequality symbol.
- Graph and check. Remember to use an open circle because the variable cannot equal −12.

COMMON ERROR In solving Question 2, students add 8 to each side. Suggest to students that they rewrite the inequality as $8 + (-9c) \geq -28$.

EXAMPLE 2 **Solving an Inequality with Variables on Both Sides**

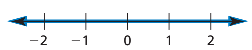
Solve $6x - 5 < 2x + 11$.

SOLUTION

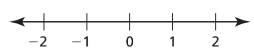
$6x - 5 < 2x + 11$	Write the inequality.
$+5 \qquad +5$	Add 5 to each side.
$6x < 2x + 16$	Simplify.
$-2x \qquad -2x$	Subtract $2x$ from each side.
$4x < 16$	Simplify.
$\frac{4x}{4} < \frac{16}{4}$	Divide each by 4.
$x < 4$	Simplify.

▶ The solution is $x < 4$.

When solving an inequality, if you obtain an equivalent inequality that is true, such as $-5 < 0$, the solutions of the inequality are *all real numbers*. If you obtain an equivalent inequality that is false, such as $3 \leq -2$, the inequality has *no solution*.



Graph of an inequality whose solutions are all real numbers



Graph of an inequality that has no solution

EXAMPLE 3 **Inequalities with Special Solutions**

Solve (a) $8b - 3 > 4(2b + 3)$ and (b) $2(5w - 1) \leq 7 + 10w$.

SOLUTION

a. $8b - 3 > 4(2b + 3)$	Write the inequality.
$8b - 3 > 8b + 12$	Distributive Property
$-8b \qquad -8b$	Subtract $8b$ from each side.
$-3 > 12$ ✗	Simplify.

▶ The inequality $-3 > 12$ is false. So, there is no solution.

b. $2(5w - 1) \leq 7 + 10w$	Write the inequality.
$10w - 2 \leq 7 + 10w$	Distributive Property
$-10w \qquad -10w$	Subtract $10w$ from each side.
$-2 \leq 7$	Simplify.

▶ The inequality $-2 \leq 7$ is true. So, all real numbers are solutions.

LOOKING FOR STRUCTURE

When the variable terms on each side of an inequality are the same, the constant terms will determine whether the inequality is true or false.

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Solve the inequality.

5. $5x - 12 \leq 3x - 4$

6. $2(k - 5) < 2k + 5$

7. $-4(3n - 1) > -12n + 5.2$

8. $3(2a - 1) \geq 10a - 11$

Extra Example 2

Solve $5x - 6 \geq 3x + 4$. $x \geq 5$

Extra Example 3

a. Solve $6t - 4 > 2(3t + 5)$. The inequality $-4 > 10$ is false. So, there is no solution.

b. Solve $2(2x - 3) \leq 9 + 4x$. The inequality $-6 \leq 9$ is true. So, all real numbers are solutions.

MONITORING PROGRESS ANSWERS

5. $x \leq 4$

6. all real numbers

7. no solution

8. $a \leq 2$

Laurie's Notes Teacher Actions

- Discuss with students the side on which they want to solve for the variable. In Example 2, solving for the variable on the left gives a coefficient of 4. Solving for the variable on the right gives a coefficient of –4. To avoid a negative coefficient, solve for the variable on the side with the coefficient with the greater value.

? **Turn and Talk:** Before introducing inequalities with special solutions, have students *Turn and Talk:* "What if Example 2 had been $6x - 5 < 6x + 11$? What would the solution have been?" Students will likely say it does not make sense; there is no x left in the inequality!

? **Assessing Question:** "Does every equation have a solution?" no; Students may have forgotten about an equation such as $5x - 2 = 5x + 3$.

- **Look For and Make Use of Structure:** Work through both parts of Example 3.
- **No-Hands Questioning:** "How will you know that an inequality will have a special solution? Explain." When like terms have been combined and the variable terms are the same on both sides.
- **Make Sense of Problems and Persevere in Solving Them:** In Example 3(a), have students substitute different values for b. This does not prove that there is no solution, but it helps students make sense of the problem. In Example 3(b), have students substitute different values of w. They will find that any value works!

Extra Example 4

Your grades on 4 math tests are 79, 85, 86, and 88. You want your mean grade to be at least 85. What grade on your fifth test will give you a mean score of at least 85? a grade of at least 87

MONITORING PROGRESS ANSWER

9. a score of at least 73

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

You need a mean score of at least 90 points to advance to the next round of the touch-screen trivia game. What scores in the fifth game will allow you to advance?

SOLUTION

REMEMBER

The mean in Example 4 is equal to the sum of the game scores divided by the number of games.

1. **Understand the Problem** You know the scores of your first four games. You are asked to find the scores in the fifth game that will allow you to advance.
2. **Make a Plan** Use the definition of the mean of a set of numbers to write an inequality. Then solve the inequality and answer the question.
3. **Solve the Problem** Let x be your score in the fifth game.

$$\frac{95 + 91 + 77 + 89 + x}{5} \geq 90 \quad \text{Write an inequality.}$$

$$\frac{352 + x}{5} \geq 90 \quad \text{Simplify.}$$

$$5 \cdot \frac{352 + x}{5} \geq 5 \cdot 90 \quad \text{Multiply each side by 5.}$$

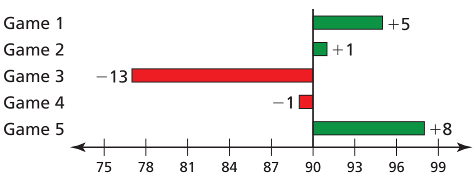
$$352 + x \geq 450 \quad \text{Simplify.}$$

$$-352 \qquad -352 \quad \text{Subtract 352 from each side.}$$

$$x \geq 98 \quad \text{Simplify.}$$

▶ A score of at least 98 points will allow you to advance.

4. **Look Back** You can draw a diagram to check that your answer is reasonable. The horizontal bar graph shows the differences between the game scores and the desired mean of 90.



To have a mean of 90, the sum of the differences must be zero.

$$5 + 1 - 13 - 1 + 8 = 0 \; ✓$$

Monitoring Progress

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9. **WHAT IF?** You need a mean score of at least 85 points to advance to the next round. What scores in the fifth game will allow you to advance?

Laurie's Notes Teacher Actions

- This is a classic problem. Students always want to know what they have to score on a test in order to have an average (mean) of __________. This is the same type of problem.
- Set up the inequality to compute the mean. Because you want your score to be a minimum of 90, you need to set the mean greater than or equal to 90.
- **Use Appropriate Tools Strategically:** The horizontal bar graph is a helpful tool for students to judge the reasonableness of their solutions.

Closure

- **Writing:** How are these alike? How are they different?
  $3n - 4 = -25$ $\quad 3n - 4 > -25$ $\quad 3n - 4 \leq -25$
  Sample answer: They are alike because they each use the expressions $(3n - 4)$ and (-25). They are different because of the way the expressions are related: equal to, greater than, and less than or equal to.

2.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

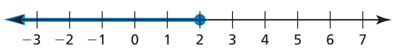
1. **WRITING** Compare solving multi-step inequalities and solving multi-step equations.
2. **WRITING** Without solving, how can you tell that the inequality $4x + 8 \leq 4x - 3$ has no solution?

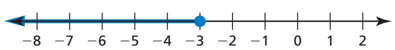
Monitoring Progress and Modeling with Mathematics

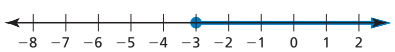
In Exercises 3–6, match the inequality with its graph.

3. $7b - 4 \leq 10$
4. $4p + 4 \geq 12$
5. $-6g + 2 \geq 20$
6. $3(2 - f) \leq 15$

A. 

B. 

C. 

D. 

In Exercises 7–16, solve the inequality. Graph the solution. *(See Example 1.)*

7. $2x - 3 > 7$
8. $5y + 9 \leq 4$
9. $-9 \leq 7 - 8v$
10. $2 > -3t - 10$
11. $\frac{w}{2} + 4 > 5$
12. $1 + \frac{m}{3} \leq 6$
13. $\frac{p}{-8} + 9 > 13$
14. $3 + \frac{r}{-4} \leq 6$
15. $6 \geq -6(a + 2)$
16. $18 \leq 3(b - 4)$

In Exercises 17–28, solve the inequality. *(See Examples 2 and 3.)*

17. $4 - 2m > 7 - 3m$
18. $8n + 2 \leq 8n - 9$
19. $-2d - 2 < 3d + 8$
20. $8 + 10f > 14 - 2f$
21. $8g - 5g - 4 \leq -3 + 3g$
22. $3w - 5 > 2w + w - 7$
23. $6(\ell + 3) < 3(2\ell + 6)$
24. $2(5c - 7) \geq 10(c - 3)$
25. $4\left(\frac{1}{2}t - 2\right) > 2(t - 3)$
26. $15\left(\frac{1}{3}b + 3\right) \leq 6(b + 9)$
27. $9j - 6 + 6j \geq 3(5j - 2)$
28. $6h - 6 + 2h < 2(4h - 3)$

ERROR ANALYSIS **In Exercises 29 and 30, describe and correct the error in solving the inequality.**

29. ✗

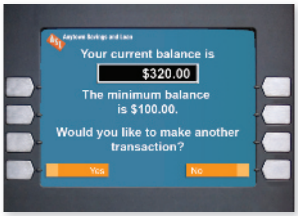
$$\frac{x}{4} + 6 \geq 3$$
$$x + 6 \geq 12$$
$$x \geq 6$$

30. ✗

$$-2(1 - x) \leq 2x - 7$$
$$-2 + 2x \leq 2x - 7$$
$$-2 \leq -7$$

All real numbers are solutions.

31. **MODELING WITH MATHEMATICS** Write and solve an inequality that represents how many \$20 bills you can withdraw from the account without going below the minimum balance. *(See Example 4.)*



Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–23 odd, 29, 31, 34, 36, 41–43

Average: 1, 2, 4–30 even, 31–36, 41–43

Advanced: 1, 2, 13–16, 22–28 even, 30–43

HOMEWORK CHECK

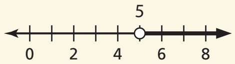
Basic: 7, 13, 17, 19, 31

Average: 8, 14, 18, 20, 32

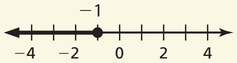
Advanced: 14, 16, 22, 28, 32

ANSWERS

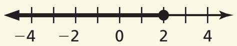
1. *Sample answer:* The same steps can be applied when solving multi-step inequalities and multi-step equations, except that when each side of an inequality is divided by a negative number, the inequality must be reversed.
2. *Sample answer:* Because the terms with the variable are the same, they will cancel, and $8 \leq -3$ is not true.
3. B
4. A
5. C
6. D
7. $x > 5$



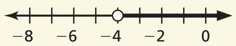
8. $y \leq -1$



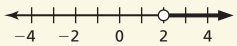
9. $v \leq 2$



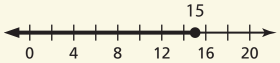
10. $t > -4$



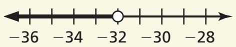
11. $w > 2$



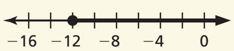
12. $m \leq 15$



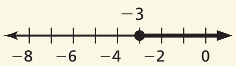
13. $p < -32$



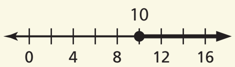
14. $r \geq -12$



15. $a \geq -3$



16. $b \geq 10$



17. $m > 3$
18. no solution
19. $d > -2$
20. $f > 0.5$
21. all real numbers
22. all real numbers
23. no solution
24. all real numbers
25. no solution
26. $b \geq -9$
27. all real numbers
28. no solution

29–31. See Additional Answers.

Dynamic Teaching Tools

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ANSWERS

32. $25h + 125 \le 500,\ h \le 15$

33. $12(2x - 3) > 60;\ x > 4$

34. no; For Woodland to be cheaper, its total cost needs to be less than Forest Park's total cost. Solving the inequality $20 + 55n \le 100 + 35n$ indicates that Woodland is cheaper for no more than 4 nights, not more than 4 nights.

35. 7 stories; Using the Pythagorean Theorem, the 74-foot ladder can reach at most 70 feet. Solving the inequality $10n - 8 \le 70$ gives $n \le 7.8$, so the ladder cannot quite reach the 8th story.

36. **a.** $40

b. $3.55; $8

c. $3.55x + 8 \le 40$

d. no more than 9 gallons

37. $r \ge 3$

38–43. See Additional Answers.

Mini-Assessment

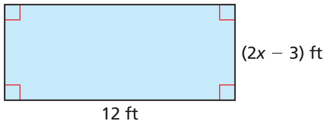
1. Solve the inequality $-2x + 6 > 16$. Graph the solution. $x < -5$

Solve the inequality.

2. $6n + 5 > 2n - 3$ $n > -2$
3. $10x - 3 < 2(5x + 1)$ The inequality $-3 < 2$ is true. So, all real numbers are solutions.
4. $3(4s - 3) \ge 5 + 12s$ The inequality $-9 \ge 5$ is false. So, there is no solution.
5. During the first 3 weeks of training, a runner runs 20 miles, 22 miles, and 21 miles. How many miles must she run during the fourth week to have a mean of at least 22 miles per week? at least 25 miles

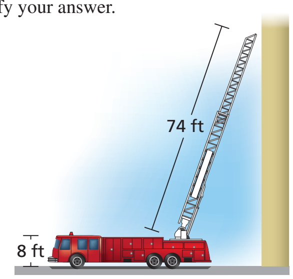
32. MODELING WITH MATHEMATICS A woodworker wants to earn at least \$25 an hour making and selling cabinets. He pays \$125 for materials. Write and solve an inequality that represents how many hours the woodworker can spend building the cabinet.

33. MATHEMATICAL CONNECTIONS The area of the rectangle is greater than 60 square feet. Write and solve an inequality to find the possible values of x.

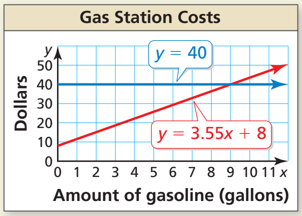


34. MAKING AN ARGUMENT Forest Park Campgrounds charges a \$100 membership fee plus \$35 per night. Woodland Campgrounds charges a \$20 membership fee plus \$55 per night. Your friend says that if you plan to camp for four or more nights, then you should choose Woodland Campgrounds. Is your friend correct? Explain.

35. PROBLEM SOLVING The height of one story of a building is about 10 feet. The bottom of the ladder on the fire truck must be at least 24 feet away from the building. How many stories can the ladder reach? Justify your answer.



36. HOW DO YOU SEE IT? The graph shows your budget and the total cost of x gallons of gasoline and a car wash. You want to determine the possible amounts (in gallons) of gasoline you can buy within your budget.



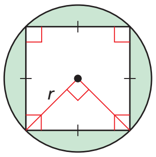
a. What is your budget?

b. How much does a gallon of gasoline cost? How much does a car wash cost?

c. Write an inequality that represents the possible amounts of gasoline you can buy.

d. Use the graph to estimate the solution of your inequality in part (c).

37. PROBLEM SOLVING For what values of r will the area of the shaded region be greater than or equal to $9(\pi - 2)$?



38. THOUGHT PROVOKING A runner's times (in minutes) in the four races he has completed are 25.5, 24.3, 24.8, and 23.5. The runner plans to run at least one more race and wants to have an average time less than 24 minutes. Write and solve an inequality to show how the runner can achieve his goal.

REASONING In Exercises 39 and 40, find the value of *a* for which the solution of the inequality is all real numbers.

39. $a(x + 3) < 5x + 15 - x$

40. $3x + 8 + 2ax \ge 3ax - 4a$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. *(Section 2.1)*

41. Six times a number y is less than or equal to 10.

42. A number p plus 7 is greater than 24.

43. The quotient of a number r and 7 is no more than 18.

Section Resources

Surface Level	Deep Level	Transfer Level
Resources by Chapter • Practice A and Practice B • Puzzle Time Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension • Cumulative Review Dynamic Assessment System • Section Practice	Dynamic Assessment System • Mid-Chapter Quiz Assessment Book • Mid-Chapter Quiz

2.1–2.4 What Did You Learn?

Core Vocabulary

inequality, *p. 54*
solution of an inequality, *p. 55*
solution set, *p. 55*
graph of an inequality, *p. 56*
equivalent inequalities, *p. 62*

Core Concepts

Section 2.1

Representing Linear Inequalities, *p. 57*

Section 2.2

Addition Property of Inequality, *p. 62*
Subtraction Property of Inequality, *p. 63*

Section 2.3

Multiplication and Division Properties of Inequality ($c > 0$), *p. 68*
Multiplication and Division Properties of Inequality ($c < 0$), *p. 69*

Section 2.4

Solving Multi-Step Inequalities, *p. 74*
Special Solutions of Linear Inequalities, *p. 75*

Mathematical Practices

1. Explain the meaning of the inequality symbol in your answer to Exercise 47 on page 59. How did you know which symbol to use?
2. In Exercise 30 on page 66, why is it important to check the reasonableness of your answer in part (a) before answering part (b)?
3. Explain how considering the units involved in Exercise 29 on page 71 helped you answer the question.

Study Skills

Analyzing Your Errors

Application Errors

What Happens: You can do numerical problems, but you struggle with problems that have context.

How to Avoid This Error: Do not just mimic the steps of solving an application problem. Explain out loud what the question is asking and why you are doing each step. After solving the problem, ask yourself, "Does my solution make sense?"

Dynamic Teaching Tools

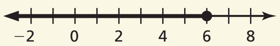
Dynamic Assessment System
Dynamic Classroom

ANSWERS

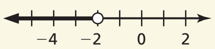
1. The inequality symbol means that all the other bridges are shorter; Because the Xianren Bridge is the longest, all other bridges must have a shorter length.
2. The value from part (a) is used to answer part (b), so checking its reasonableness is important to be sure that the correct result is used.
3. Showing the units in each step helps to compare the units in the final answer with the expected units.

ANSWERS

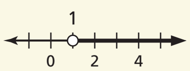
1. $z - 6 \geq 11$
2. $12 \leq -1.5w + 4$
3. $x < 0$
4. $x \geq 8$
5. $q \leq 6$



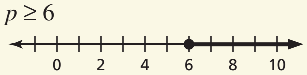
6. $z < -2$



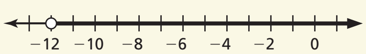
7. $y > 1$



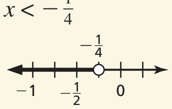
8. $p \geq 6$



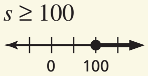
9. $w > -12$



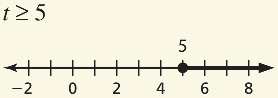
10. $x < -\frac{1}{4}$



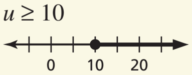
11. $y \geq 8$
12. all real numbers
13. no solution
14. a. $s \geq 100$



$t \geq 5$



$u \geq 10$



b. no; One of the requirements is to swim 100 yards, which is equal to 300 feet. If you only swim 250 feet you do not meet this requirement.

15. $V + 100 \leq 700$; $V \leq 600$
16. a. $15w \geq 120$; $w \geq 8$

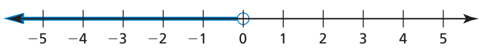
b. The number of weeks is reduced; $65 + 15w \geq 120$; $w \geq 3\frac{2}{3}$

2.1–2.4 Quiz

Write the sentence as an inequality. *(Section 2.1)*

1. A number z minus 6 is greater than or equal to 11.
2. Twelve is no more than the sum of -1.5 times a number w and 4.

Write an inequality that represents the graph. *(Section 2.1)*

3. 

4. 

Solve the inequality. Graph the solution. *(Section 2.2 and Section 2.3)*

5. $9 + q \leq 15$
6. $z - (-7) < 5$
7. $-3 < y - 4$
8. $3p \geq 18$
9. $6 > \frac{w}{-2}$
10. $-20x > 5$

Solve the inequality. *(Section 2.4)*

11. $3y - 7 \geq 17$
12. $8(3g - 2) \leq 12(2g + 1)$
13. $6(2x - 1) \geq 3(4x + 1)$

14. Three requirements for a lifeguard training course are shown. *(Section 2.1)*

a. Write and graph three inequalities that represent the requirements.

b. You can swim 250 feet, tread water for 6 minutes, and swim 35 feet underwater without taking a breath. Do you satisfy the requirements of the course? Explain.

15. The maximum volume of an American white pelican's bill is about 700 cubic inches. A pelican scoops up 100 cubic inches of water. Write and solve an inequality that represents the additional volumes the pelican's bill can contain. *(Section 2.2)*

16. You save \$15 per week to purchase one of the bikes shown. *(Section 2.3 and Section 2.4)*

a. Write and solve an inequality to find the numbers of weeks you need to save to purchase a bike.

b. Your parents give you \$65 to help you buy the new bike. How does this affect you answer in part (a)? Use an inequality to justify your answer.

Laurie's Notes

Overview of Section 2.5

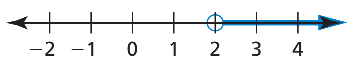
Introduction

- Compound inequalities are introduced in this lesson. The skills involved with solving and graphing the individual inequalities are familiar to students. What is new is using the conjunction "and" or "or" to combine two inequalities. It is helpful to first review what it means to form the intersection (and) or the union (or) of two sets.
- There are two ways in which an "and" compound inequality can be written, and hence, there are two strategies for solving as shown in Example 2. When reading the solutions of a compound inequality, students may need to be reminded to use the correct conjunction.

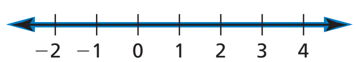
Common Misconceptions

- Students often believe that the graph of an "and" compound inequality is a segment (with open or closed endpoints) and the graph of an "or" compound inequality is two rays in opposite directions (with open or closed endpoints). There are other possibilities.

$x > 2$ *and* $x > -1$



$x > -1$ *or* $x < 3$



Formative Assessment Tips

- **Wait Time:** Wait time is the interval between a question being posed and a student (or the teacher) response. Silence can be uncomfortable in a classroom, but research has shown that increasing wait time increases class participation and answers are more detailed. For complex, higher-order thinking questions, increased wait time is necessary.
- **Reason Abstractly and Quantitatively** and **Construct Viable Arguments and Critique the Reasoning of Others:** If we want students to reason abstractly and quantitatively, and to construct viable arguments, we need to first pose complex questions. Students then need additional *Wait Time* to allow for thinking and formulation of responses. With increased participation, teachers learn more about students' progress and their learning.

Another Way

- **Teaching Tip:** To introduce graphing compound inequalities, use two transparencies or use the *copy* command on an interactive whiteboard. You want students to see that a compound inequality is composed of two parts. When joined with the word "and," you are graphing the intersection of the two parts. When joined with the word "or," you are graphing the union of the two parts. You can overlay the two parts using the transparencies or *copy* command.

Pacing Suggestion

- The explorations are similar to Example 1 in the formal lesson. If you use the *Motivate* and both explorations, you may want to omit Example 1 in the formal lesson.

Dynamic Teaching Tools

- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ALABAMA STANDARDS

A1.11 Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.

A1.13 Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.

Learning Target

Write and solve compound inequalities.

Success Criteria

- Write word sentences as compound inequalities.
- Solve compound inequalities.
- Graph solutions of compound inequalities.

Laurie's Notes

Exploration

Motivate

- **FYI:** Every state has a law on the books that says something along the lines of: "A person shall not drive a motor vehicle at such a slow speed so as to impede or block the normal and reasonable forward movement of traffic."
- Some states specify a minimum speed limit for highway driving. In Florida, that minimum speed is 40 miles per hour on highways with at least four lanes. The maximum speed on a four-lane divided highway is 65 miles per hour.

? "Can you graph the legal speeds on a four-lane highway in Florida?"

- Explain to students that this is the graph of a compound inequality, the type they will look at in this lesson.

Discuss

- Have students discuss the difference between your boss saying, "You can take a 15-minute break *or* leave 15 minutes early today," versus "You can take a 15-minute break *and* leave 15 minutes early today."
- The words "and" and "or" have meaning in mathematics that students will see today.

Exploration 1

- Superimposed above each graph is a description of the interval. A definition is not necessary because students can make sense of the words.
- It is natural for students to think (and say) that the shaded part is the numbers between ____ and ____. Remind students of the directions: use two inequalities to describe the interval.
- **Whiteboarding:** Have students write their answers and share with the class.
- **Attend to Precision:** Students should check that they have used the correct inequality symbols.
- **Construct Viable Arguments and Critique the Reasoning of Others:** Ask several students to share their reasoning for part (e).

Exploration 2

- The second exploration is very similar to the first. Now that the first exploration has been discussed, it should not take long for students to write two inequalities for the intervals shown.
- **Whiteboarding:** Have students write their answers and share with the class.

Communicate Your Answer

- **Turn and Talk:** Give students time to answer Question 3 with their partners.

Connecting to Next Step

- If you feel students are comfortable using two inequalities to describe an interval, assign Exercises 3–6 on page 85. Explain that two inequalities joined by "and" or "or" are called compound inequalities. You could also have students copy the *Core Vocabulary* into their notebooks.

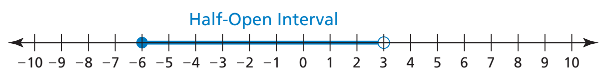
2.5 Solving Compound Inequalities

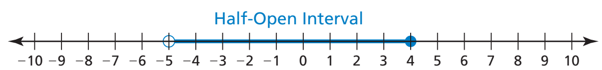
Essential Question How can you use inequalities to describe intervals on the real number line?

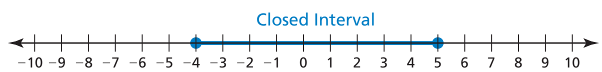
ALABAMA STANDARDS
A1.11, A1.13

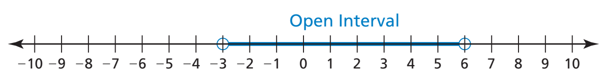
EXPLORATION 1 Describing Intervals on the Real Number Line

Work with a partner. In parts (a)–(d), use two inequalities to describe the interval.

a.


b.


c.


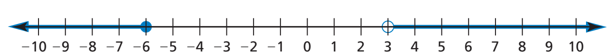
d.


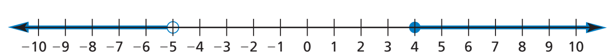
e. Do you use “and” or “or” to connect the two inequalities in parts (a)–(d)? Explain.

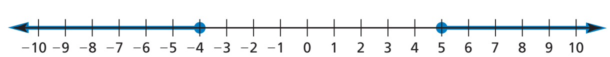
REASONING ABSTRACTLY
To be proficient in math, you need to create a clear representation of the problem at hand.

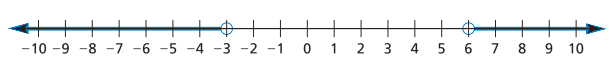
EXPLORATION 2 Describing Two Infinite Intervals

Work with a partner. In parts (a)–(d), use two inequalities to describe the interval.

a.


b.


c.


d.


e. Do you use “and” or “or” to connect the two inequalities in parts (a)–(d)? Explain.

Communicate Your Answer

3. How can you use inequalities to describe intervals on the real number line?

Dynamic Teaching Tools
- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ANSWERS

1. **a.** $x \geq -6$ and $x < 3$
b. $x > -5$ and $x \leq 4$
c. $x \geq -4$ and $x \leq 5$
d. $x > -3$ and $x < 6$
e. and; Both inequalities need to be true for values that are in the interval.

2. **a.** $x \leq -6$ or $x > 3$
b. $x < -5$ or $x \geq 4$
c. $x \leq -4$ or $x \geq 5$
d. $x < -3$ or $x > 6$
e. or; Either inequality needs to be true for values that are in the interval.

3. Write 2 inequalities joined by “and” or “or.”

Differentiated Instruction

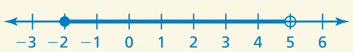
Kinesthetic/Visual

Have pairs of students work together to graph compound inequalities. Have one student use a red pencil to graph the first part of the inequality. The other student uses a different color to graph the second part on the same number line. For a compound inequality with "and," the graph is the interval where both colors overlap.

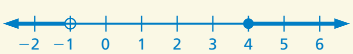
Extra Example 1

Write each sentence as an inequality. Graph each inequality.

a. A number c is less than 5 and greater than or equal to -2. $-2 \le c < 5$

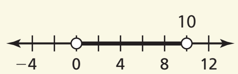


b. A number t is less than -1 or greater than or equal to 4. $t < -1$ *or* $t \ge 4$

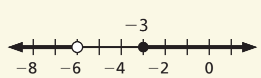


MONITORING PROGRESS ANSWERS

1. $0 < d < 10$



2. $a < -6$ *or* $a \ge -3$



2.5 Lesson

Learning Target: Write and solve compound inequalities.

Success Criteria:
- I can write word sentences as compound inequalities..
- I can solve compound inequalities.
- I can graph solutions of compound inequalities.

Core Vocabulary

compound inequality, *p. 82*

Writing and Graphing Compound Inequalities

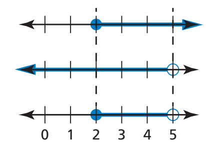
A **compound inequality** is an inequality formed by joining two inequalities with the word "and" or the word "or."

The graph of a compound inequality with "and" is the *intersection* of the graphs of the inequalities. The graph shows numbers that are solutions of *both* inequalities.

$x \ge 2$

$x < 5$

$2 \le x$ *and* $x < 5$
$2 \le x < 5$

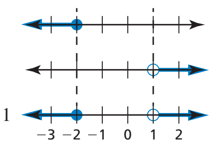


The graph of a compound inequality with "or" is the *union* of the graphs of the inequalities. The graph shows numbers that are solutions of *either* inequality.

$y \le -2$

$y > 1$

$y \le -2$ *or* $y > 1$



EXAMPLE 1 **Writing and Graphing Compound Inequalities**

Write each sentence as an inequality. Graph each inequality.

a. A number x is greater than -8 and less than or equal to 4.

b. A number y is at most 0 or at least 2.

REMEMBER

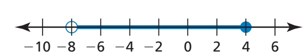
A compound inequality with "and" can be written as a single inequality. For example, you can write $x > -8$ and $x \le 4$ as $-8 < x \le 4$.

SOLUTION

a. A number x is greater than -8 and less than or equal to 4.

$x > -8$ *and* $x \le 4$

▶ An inequality is $-8 < x \le 4$.

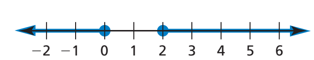


Graph the intersection of the graphs of $x > -8$ and $x \le 4$.

b. A number y is at most 0 or at least 2.

$y \le 0$ *or* $y \ge 2$

▶ An inequality is $y \le 0$ *or* $y \ge 2$.



Graph the union of the graphs of $y \le 0$ and $y \ge 2$.

Monitoring Progress

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Write the sentence as an inequality. Graph the inequality.

1. A number d is more than 0 and less than 10.
2. A number a is fewer than -6 or no less than -3.

Laurie's Notes Teacher Actions

- Define *compound inequality* and connect to the explorations or *Motivate*.
- Review what intersection and union mean in the context of a set of numbers.
- Discuss the *Remember* note. Be sure students understand that when they encounter a compound inequality written in the form $a < x < b$, the conjunction is always "and."
- As you graph say, "We want to shade the values of x that are greater than -8 and less than or equal to 4."

? "What does the phrase 'at most' mean?" As an example, if you have at most 3 cookies, then you could have 3, 2, 1, or no cookies.

Solving Compound Inequalities

You can solve a compound inequality by solving two inequalities separately. When a compound inequality with "and" is written as a single inequality, you can solve the inequality by performing the same operation on each expression.

LOOKING FOR STRUCTURE

To be proficient in math, you need to see complicated things as single objects or as being composed of several objects.

EXAMPLE 2 Solving Compound Inequalities with "And"

Solve each inequality. Graph each solution.

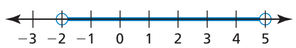
a. $-4 < x - 2 < 3$ **b.** $-3 < -2x + 1 \le 9$

SOLUTION

a. Separate the compound inequality into two inequalities, then solve.

$-4 < x - 2$	*and*	$x - 2 < 3$	Write two inequalities.
$\underline{+2 \quad\; +2}$		$\underline{+2 \quad +2}$	Add 2 to each side.
$-2 < x$	*and*	$x < 5$	Simplify.

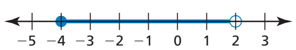
▶ The solution is $-2 < x < 5$.



b.

$-3 < -2x + 1 \le 9$	Write the inequality.
$\underline{-1} \quad \underline{-1} \quad \underline{-1}$	Subtract 1 from each expression.
$-4 < -2x \le 8$	Simplify.
$\dfrac{-4}{-2} > \dfrac{-2x}{-2} \ge \dfrac{8}{-2}$	Divide each expression by −2. Reverse each inequality symbol.
$2 > x \ge -4$	Simplify.

▶ The solution is $-4 \le x < 2$.



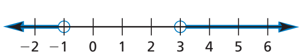
EXAMPLE 3 Solving a Compound Inequality with "Or"

Solve $3y - 5 < -8$ *or* $2y - 1 > 5$. Graph the solution.

SOLUTION

$3y - 5 < -8$	*or*	$2y - 1 > 5$	Write the inequality.
$\underline{+5 \quad\; +5}$		$\underline{+1 \quad +1}$	Addition Property of Inequality
$3y < -3$		$2y > 6$	Simplify.
$\dfrac{3y}{3} < \dfrac{-3}{3}$		$\dfrac{2y}{2} > \dfrac{6}{2}$	Division Property of Inequality
$y < -1$	*or*	$y > 3$	Simplify.

▶ The solution is $y < -1$ *or* $y > 3$.



Monitoring Progress

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Solve the inequality. Graph the solution.

3. $5 \le m + 4 < 10$

4. $-3 < 2k - 5 < 7$

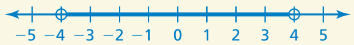
5. $4c + 3 \le -5$ *or* $c - 8 > -1$

6. $2p + 1 < -7$ *or* $3 - 2p \le -1$

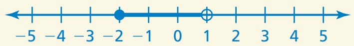
Extra Example 2

Solve each inequality. Graph each solution.

a. $-3 < x + 1 < 5$ $-4 < x < 4$

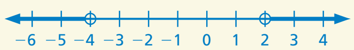


b. $-7 < -3w - 4 \le 2$ $-2 \le w < 1$



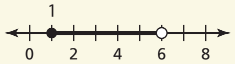
Extra Example 3

Solve $2x + 6 < -2$ *or* $4x - 5 > 3$. Graph the solution. $x < -4$ *or* $x > 2$

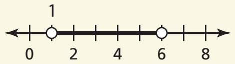


MONITORING PROGRESS ANSWERS

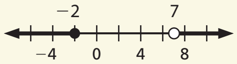
3. $1 \le m < 6$



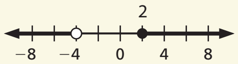
4. $1 < k < 6$



5. $c \le -2$ *or* $c > 7$



6. $p < -4$ *or* $p \ge 2$



Laurie's Notes Teacher Actions

- Discuss the general process for solving compound inequalities. Explain that each inequality can be solved separately. When the compound inequality with "and" is written as a single inequality statement, perform the same operation on each of the three parts (left, middle, and right).
- Part (a) is solved by separating the inequalities, while part (b) models performing operations on all three parts of the inequality.

COMMON ERROR When dividing each of the three expressions by −2, students may forget to reverse the direction of the inequality symbols.

- Discuss why the solution $2 > x \ge -4$ is rewritten. By convention we read inequalities from the least value (−4) to the greatest value (2), just as we would read the graph of the solution from left to right.
- In both examples, be sure to substitute values not in the solution set to show that they do not make the inequality true.

Extra Example 4

A recommended storage temperature for chocolate is from 15°C to 20°C.

a. Write and solve a compound inequality that represents the possible storage temperatures (in degrees Fahrenheit) for chocolate. $15 \le \frac{5}{9}(F - 32) \le 20$; $59 \le F \le 68$; The storage temperature is 59°F to 68°F.

b. Describe one situation in which the surrounding temperature could be below the storage temperature range and one in which it could be above. *Sample answer:* The surrounding temperature could be below this range outdoors on a cold day. The surrounding temperature could be above this range in a car parked in the sun on a hot day.

MONITORING PROGRESS ANSWER

7. $-40 \le \frac{5}{9}(F - 32) \le 15$, $-40 \le F \le 59$

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

Operating temperature: 0°C to 35°C

Electrical devices should operate effectively within a specified temperature range. Outside the operating temperature range, the device may fail.

a. Write and solve a compound inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone.

b. Describe one situation in which the surrounding temperature could be below the operating range and one in which it could be above.

SOLUTION

1. **Understand the Problem** You know the operating temperature range in degrees Celsius. You are asked to write and solve a compound inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone. Then you are asked to describe situations outside this range.

2. **Make a Plan** Write a compound inequality in degrees Celsius. Use the formula $C = \frac{5}{9}(F - 32)$ to rewrite the inequality in degrees Fahrenheit. Then solve the inequality and describe the situations.

3. **Solve the Problem** Let C be the temperature in degrees Celsius, and let F be the temperature in degrees Fahrenheit.

$0 \le C \le 35$	Write the inequality using C.
$0 \le \frac{5}{9}(F - 32) \le 35$	Substitute $\frac{5}{9}(F - 32)$ for C.
$9 \cdot 0 \le 9 \cdot \frac{5}{9}(F - 32) \le 9 \cdot 35$	Multiply each expression by 9.
$0 \le 5(F - 32) \le 315$	Simplify.
$0 \le 5F - 160 \le 315$	Distributive Property
$+160 \quad +160 \quad +160$	Add 160 to each expression.
$160 \le 5F \le 475$	Simplify.
$\frac{160}{5} \le \frac{5F}{5} \le \frac{475}{5}$	Divide each expression by 5.
$32 \le F \le 95$	Simplify.

STUDY TIP

You can also solve the inequality by first multiplying each expression by $\frac{9}{5}$.

▶ The solution is $32 \le F \le 95$. So, the operating temperature range of the smartphone is 32°F to 95°F. One situation when the surrounding temperature could be below this range is winter in Alaska. One situation when the surrounding temperature could be above this range is daytime in the Mojave Desert of the American Southwest.

4. **Look Back** You can use the formula $C = \frac{5}{9}(F - 32)$ to check that your answer is correct. Substitute 32 and 95 for F in the formula to verify that 0°C and 35°C are the minimum and maximum operating temperatures in degrees Celsius.

Monitoring Progress

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−40°C to 15°C

7. Write and solve a compound inequality that represents the temperature rating (in degrees Fahrenheit) of the winter boots.

Laurie's Notes Teacher Actions

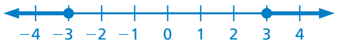
- ? Hold up a cell phone and ask, "Have any of you ever had difficulty getting your phone to work properly because the outside temperature was too hot or too cold?" Answers will vary.
- Share the range of operating temperatures for a generic cell phone.
- ? **Wait Time:** Pose and give *Wait Time*, and even alert students that this is a *No-Hands Question*. "How can you determine the range of operating temperatures in degrees Fahrenheit?" Wait 8–10 seconds and solicit a response from a student whose voice is not often heard. Students may offer strategies that do not involve writing the compound inequality first and then solving for the interval of operating temperatures.

Closure

- **Exit Ticket:** Solve and graph.

a. $-1 \le 2x + 3 \le 7$ $\quad -2 \le x \le 2$

b. $4x + 1 \le -11$ *or* $3x - 4 \ge 5$ $\quad x \le -3$ *or* $x \ge 3$



2.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

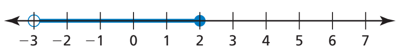
1. **WRITING** Compare the graph of $-6 \le x \le -4$ with the graph of $x \le -6$ *or* $x \ge -4$.

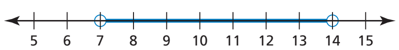
2. **WHICH ONE DOESN'T BELONG?** Which compound inequality does *not* belong with the other three? Explain your reasoning.

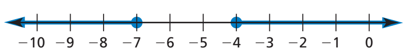
$a > 4$ *or* $a < -3$	$a < -2$ *or* $a > 8$	$a > 7$ *or* $a < -5$	$a < 6$ *or* $a > -9$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, write a compound inequality that is represented by the graph.

3. 

4. 

5. 

6. 

In Exercises 7–10, write the sentence as an inequality. Graph the inequality. *(See Example 1.)*

7. A number p is less than 6 and greater than 2.

8. A number n is less than or equal to -7 or greater than 12.

9. A number m is more than $-7\frac{2}{3}$ or at most -10.

10. A number r is no less than -1.5 and fewer than 9.5.

11. **MODELING WITH MATHEMATICS** Slitsnails are large mollusks that live in deep waters. They have been found in the range of elevations shown. Write and graph a compound inequality that represents this range.

12. **MODELING WITH MATHEMATICS** The life zones on Mount Rainier, a mountain in Washington, can be approximately classified by elevation, as follows.

Low-elevation forest: above 1700 feet to 2500 feet
Mid-elevation forest: above 2500 feet to 4000 feet
Subalpine: above 4000 feet to 6500 feet
Alpine: above 6500 feet to the summit

Elevation of Mount Rainier: 14,410 ft

Write a compound inequality that represents the elevation range for each type of plant life.

a. trees in the low-elevation forest zone
b. flowers in the subalpine and alpine zones

In Exercises 13–20, solve the inequality. Graph the solution. *(See Examples 2 and 3.)*

13. $6 < x + 5 \le 11$

14. $24 > -3r \ge -9$

15. $v + 8 < 3$ *or* $-8v < -40$

16. $-14 > w + 3$ *or* $3w \ge -27$

17. $2r + 3 < 7$ *or* $-r + 9 \le 2$

18. $-6 < 3n + 9 < 21$

19. $-12 < \frac{1}{2}(4x + 16) < 18$

20. $35 < 7(2 - b)$ *or* $\frac{1}{3}(15b - 12) \ge 21$

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–11 odd, 12, 13–17 odd, 21, 25, 33–40

Average: 1, 2, 4–18 even, 22, 23, 24–28 even, 33–40

Advanced: 1, 2, 6, 9–12, 18–20, 22–24, 28–40

HOMEWORK CHECK

Basic: 5, 7, 13, 15, 21

Average: 8, 10, 14, 16, 23

Advanced: 9, 10, 19, 20, 23

ANSWERS

1. The graph of $-6 \le x \le -4$ shows a single segment between -6 and -4. The graph of $x \le -6$ *or* $x \ge -4$ shows two opposite rays with endpoints at -6 and -4.

2. $a < 6$ *or* $a > -9$; The graph of this inequality is the only one with overlapping rays that represent all real numbers.

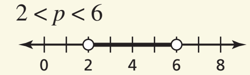
3. $-3 < x \le 2$

4. $7 < x < 14$

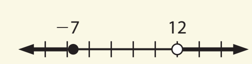
5. $x \le -7$ *or* $x \ge -4$

6. $x \le 4$ *or* $x > 6$

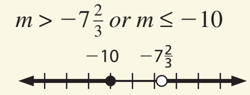
7. $2 < p < 6$



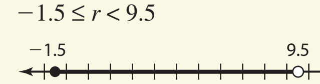
8. $n \le -7$ *or* $n > 12$



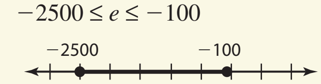
9. $m > -7\frac{2}{3}$ *or* $m \le -10$



10. $-1.5 \le r < 9.5$

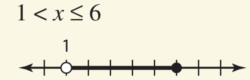


11. $-2500 \le e \le -100$

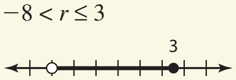


12. a. $1700 < h \le 2500$
b. $4000 < h \le 14{,}410$

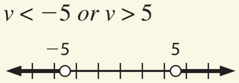
13. $1 < x \le 6$



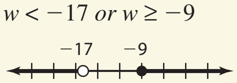
14. $-8 < r \le 3$



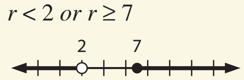
15. $v < -5$ *or* $v > 5$



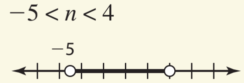
16. $w < -17$ *or* $w \ge -9$



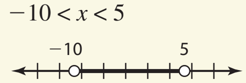
17. $r < 2$ *or* $r \ge 7$



18. $-5 < n < 4$



19. $-10 < x < 5$



20. See Additional Answers.

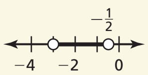
Dynamic Teaching Tools

Dynamic Assessment System

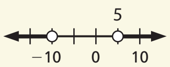
Dynamic Classroom

ANSWERS

21. In the second step, 3 should have been subtracted from 4 on the left side; $1 < -2x < 6$; $-\frac{1}{2} > x > -3$



22. The graph should have 2 opposite rays, not 1 segment.



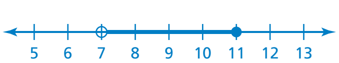
23. $-20 \le \frac{5}{9}(F - 32) \le -15$, $-4 \le F \le 5$

24–40. See Additional Answers.

Mini-Assessment

1. Write the sentence as an inequality. Graph the inequality. A number p is greater than −3 and less than 4.
   $-3 < p < 4$

2. Solve $22 < 4x - 6 \le 38$. Graph the solution. $7 < x \le 11$

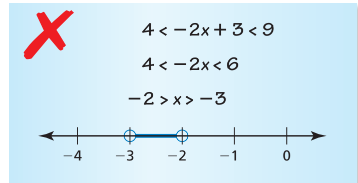


3. Solve $2c + 5 \le -11$ *or* $3c - 1 \ge -7$. Graph the solution.
   $c \le -8$ *or* $c \ge -2$

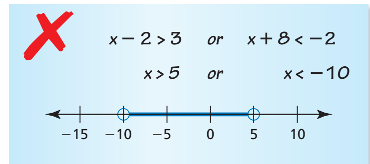
4. In a tropical rain forest, the temperature ranges from 22°C to 30°C. Write and solve a compound inequality that represents the possible temperatures (in degrees Fahrenheit) of a tropical rain forest.
   $22 \le \frac{5}{9}(F - 32) \le 30$; $71.6 \le F \le 86$; 71.6°F to 86°F

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in solving the inequality or graphing the solution.

21.



22.



23. **MODELING WITH MATHEMATICS** Write and solve a compound inequality that represents the possible temperatures (in degrees Fahrenheit) of the interior of the iceberg. *(See Example 4.)*

24. **PROBLEM SOLVING** A ski shop sells skis with lengths ranging from 150 centimeters to 220 centimeters. The shop says the length of the skis should be about 1.16 times a skier's height (in centimeters). Write and solve a compound inequality that represents the heights of skiers the shop does *not* provide skis for.

In Exercises 25–30, solve the inequality. Graph the solution, if possible.

25. $22 < -3c + 4 < 14$

26. $2m - 1 \ge 5$ *or* $5m > -25$

27. $-y + 3 \le 8$ *and* $y + 2 > 9$

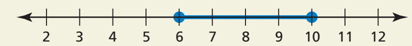
28. $x - 8 \le 4$ *or* $2x + 3 > 9$

29. $2n + 19 \le 10 + n$ *or* $-3n + 3 < -2n + 33$

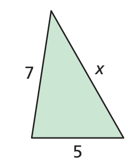
30. $3x - 18 < 4x - 23$ *and* $x - 16 < -22$

31. **REASONING** Fill in the compound inequality $4(x - 6)$ ▭ $2(x - 10)$ *and* $5(x + 2) \ge 2(x + 8)$ with <, ≤, >, or ≥ so that the solution is only one value.

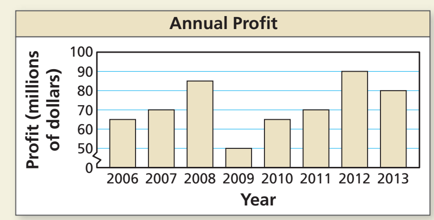
32. **THOUGHT PROVOKING** Write a real-life story that can be modeled by the graph.



33. **MAKING AN ARGUMENT** The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Use the triangle shown to write and solve three inequalities. Your friend claims the value of x can be 1. Is your friend correct? Explain.



34. **HOW DO YOU SEE IT?** The graph shows the annual profits of a company from 2006 to 2013.



a. Write a compound inequality that represents the annual profits from 2006 to 2013.

b. You can use the formula $P = R - C$ to find the profit P, where R is the revenue and C is the cost. From 2006 to 2013, the company's annual cost was about \$125 million. Is it possible the company had an annual revenue of \$160 million from 2006 to 2013? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Graph the solutions, if possible. *(Section 1.4)*

35. $\left|\frac{d}{9}\right| = 6$ **36.** $7|5p - 7| = -21$ **37.** $|r + 2| = |3r - 4|$ **38.** $\left|\frac{1}{2}w - 6\right| = |w + 7|$

Find and interpret the mean absolute deviation of the data. *(Skills Review Handbook)*

39. 1, 1, 2, 5, 6, 8, 10, 12, 12, 13

40. 24, 26, 28, 28, 30, 30, 32, 32, 34, 36

Section Resources

Surface Level	Deep Level
Resources by Chapter • Practice A and Practice B • Puzzle Time Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension • Cumulative Review Dynamic Assessment System • Section Practice

Laurie's Notes

Overview of Section 2.6

Introduction

- In Section 1.4, students learned to solve absolute value equations. The technique used for solving an equation such as $|x - 4| = 6$ was to consider when the expression inside the absolute value symbols was equal to 6 or -6.
- Now consider the inequality $|x - 4| \leq 6$. This question is asking when the expression inside the absolute value symbols is between -6 and 6, inclusive, or when is $-6 \leq x - 4 \leq 6$? This is an "and" compound inequality.
- For the inequality $|x - 4| \geq 6$, the question is asking when the expression inside the absolute value symbols is greater than or equal to 6 or less than or equal to -6. This translates into the "or" compound inequality $x - 4 \geq 6$ or $x - 4 \leq -6$.
- This lesson connects the lessons on solving absolute value equations (Section 1.4) and solving compound inequalities (Section 2.5).

Resources

- I have a long number line (8 feet) above one of my whiteboards. The number line is a helpful visual reference for students in this lesson.

Teaching Strategy

- **Human Graph:** Creating a human graph is an engaging visual way for students to display (plot) data. You need to have room to form a number line or an entire *xy*-coordinate grid in your classroom or adjacent space. If there are square tiles on the floor, the tiles can be used to help increment the axis (axes). You can also use masking tape and a dark pen to increment.
- In this lesson, a human graph is made to display the solution to an absolute value inequality. Students viewing the volunteers quickly see that the solution is either a segment (and) or two rays (or). A student volunteer who moves incorrectly sees the immediate feedback!

Another Way

- You can use the words "between" and "beyond" to describe absolute value inequalities. For example, $|x| < 2$ means that x is between -2 and 2; $|x| > 2$ means that x is beyond -2 or beyond 2.

Pacing Suggestion

- Students will work the same inequality three different ways in the explorations. Begin with the *Motivate* and discussion, followed by the explorations. When students have finished the explorations, begin the *Core Concept* in the formal lesson.

Dynamic Teaching Tools

- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ALABAMA STANDARDS

A1.11 Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.

A1.13 Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions.

Learning Target

Write and solve inequalities involving absolute value.

Success Criteria

- Write a compound inequality related to a given absolute value inequality.
- Solve absolute value inequalities.
- Use absolute value inequalities to solve real-life problems.

Laurie's Notes

Exploration

Motivate

- **Human Graph:** Have nine volunteers, each with a number from –4 to 4, form a number line at the front of the room facing their classmates.
- When you show an inequality, the volunteers holding a solution are to take a step forward. The others should remain in place. *Examples:*

  $|x| \leq 2$ The following step forward: $-2, -1, 0, 1, 2$

  $|x| > 1$ The following step forward: $-4, -3, -2, 2, 3, 4$

? "The less than inequalities remind you of what type of compound inequality?" and

? "The greater than inequalities remind you of what type of compound inequality?" or

Discuss

- Students solved absolute value equations in Section 1.4. Now they are combining that skill with their understanding of compound inequalities to solve absolute value inequalities.

? "What does $|x| = 2$ mean geometrically?" all values that are 2 units from 0

- Graph the solutions for $|x| = 2$.

? "What does $|x| < 2$ mean geometrically?" all values that are less than 2 units from 0

- Graph the solutions for $|x| < 2$.

? "What does $|x| > 2$ mean geometrically?" all values that are more than 2 units from 0

- Graph the solutions for $|x| > 2$.
- Connect these two types of absolute value inequalities to student understanding of "and" and "or" compound inequalities.

Exploration 1

- Students could use trial and error to see what values work. Connecting their knowledge of solving absolute value equations and compound inequalities, students should realize that they are looking for an interval bounded by $x = 1$ above and a negative number below.
- Students may be uncertain of how to describe the values of $x + 2$ that make the inequality true. Say, "The quantity inside the absolute value symbols is less than or equal to 3 or greater than or equal to -3." That leads to the inequalities $x + 2 \leq 3$ *and* $x + 2 \geq -3$.
- The solution is an "and" compound inequality.

Exploration 2

- **Connection:** When solving $|x| = 3$, you are finding the two values that are 3 units from 0, thus making 0 the midpoint. When solving $|x + 2| = 3$, you are still finding two values that are 3 units from a midpoint, but the midpoint has been translated 2 units to the left, where $x + 2 = 0$. Making the connection to the translation ($|x| = 3 \rightarrow |x + 2| = 3$) is a *Big Idea* that students have already seen and will continue to see in their study of mathematics.

? "Using the number line, what solutions did you have?" -5 and 1

- The solution to the absolute value inequality is all of the x-values between -5 and 1, inclusive.

Exploration 3

- Students can use the *table* feature on a graphing calculator. The solution, of course, will be the same as the two previous explorations.

Communicate Your Answer

- **Turn and Talk:** Have students share their thoughts about the three approaches with their partners.

Connecting to Next Step

- If you feel students are comfortable with solving simple absolute value inequalities, assign some of Exercises 3–8 on page 91.

2.6 Solving Absolute Value Inequalities

Essential Question

How can you solve an absolute value inequality?

ALABAMA STANDARDS
A1.11, A1.13

EXPLORATION 1 Solving an Absolute Value Inequality Algebraically

Work with a partner. Consider the absolute value inequality

$|x + 2| \le 3.$

a. Describe the values of $x + 2$ that make the inequality true. Use your description to write two linear inequalities that represent the solutions of the absolute value inequality.

b. Use the linear inequalities you wrote in part (a) to find the solutions of the absolute value inequality.

c. How can you use linear inequalities to solve an absolute value inequality?

MAKING SENSE OF PROBLEMS

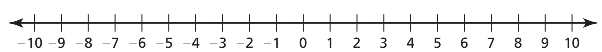
To be proficient in math, you need to explain to yourself the meaning of a problem and look for entry points to its solution.

EXPLORATION 2 Solving an Absolute Value Inequality Graphically

Work with a partner. Consider the absolute value inequality

$|x + 2| \le 3.$

a. On a real number line, locate the point for which $x + 2 = 0$.



b. Locate the points that are within 3 units from the point you found in part (a). What do you notice about these points?

c. How can you use a number line to solve an absolute value inequality?

EXPLORATION 3 Solving an Absolute Value Inequality Numerically

Work with a partner. Consider the absolute value inequality

$|x + 2| \le 3.$

a. Use a spreadsheet, as shown, to solve the absolute value inequality.

b. Compare the solutions you found using the spreadsheet with those you found in Explorations 1 and 2. What do you notice?

c. How can you use a spreadsheet to solve an absolute value inequality?

	A	B
1	*x*	|x + 2|
2	-6	4
3	-5	
4	-4	
5	-3	
6	-2	
7	-1	
8	0	
9	1	
10	2	

abs(A2 + 2)

Communicate Your Answer

4. How can you solve an absolute value inequality?

5. What do you like or dislike about the algebraic, graphical, and numerical methods for solving an absolute value inequality? Give reasons for your answers.

Dynamic Teaching Tools

- Dynamic Assessment System
- Lesson Plans
- Dynamic Classroom

ANSWERS

1. a. The inequality is true if the distance between $x + 2$ and 0 is 3 or less; $x + 2 \le 3$; $x + 2 \ge -3$

b. $-5 \le x \le 1$

c. Write a compound inequality representing the distance between the absolute value expression and 0.

2. See Additional Answers.

3. a. $-5 \le x \le 1$

b. They are the same.

c. Have the spreadsheet calculate the value of the absolute value expression for many values of x, and find the ones that give the expected solution.

4. solving algebraic equations, graphically on a number line, or by trial and error with a spreadsheet

5. *Sample answers:* The algebraic method is nice to use because it is the quickest method. The graphical method is nice to use because it helps to visualize absolute value. The numerical method is not favorable because setting up the spreadsheet is time-consuming.

Differentiated Instruction

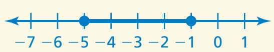
Inclusion

When rewriting $|x + 7| \leq 2$ as a compound inequality, students may have difficulty understanding why the direction of the inequality sign and the sign of the number 2 are changed to get $x + 7 \geq -2$. Remind them that the value of the expression inside the absolute value sign can be positive or negative. Write $-(x + 7) \leq 2$ on the board and then multiply each side by -1 to get $x + 7 \geq -2$.

Extra Example 1

Solve each inequality. Graph each solution, if possible.

a. $|x + 3| \leq 2$ $\quad -5 \leq x \leq -1$



b. $|3c - 5| < -2$ The expression $|3c - 5|$ cannot be negative. So, the inequality has no solution.

2.6 Lesson

Learning Target: Write and solve inequalities involving absolute value.

Success Criteria:
- I can write a compound inequality related to a given absolute value inequality.
- I can solve absolute value inequalities.
- I can use absolute value inequalities to solve real-life problems.

Core Vocabulary

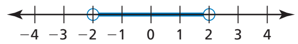
absolute value inequality, *p. 88*
absolute deviation, *p. 90*

Previous
compound inequality
mean

Solving Absolute Value Inequalities

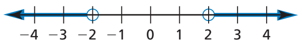
An **absolute value inequality** is an inequality that contains an absolute value expression. For example, $|x| < 2$ and $|x| > 2$ are absolute value inequalities. Recall that $|x| = 2$ means the distance between x and 0 is 2.

The inequality $|x| < 2$ means the distance between x and 0 is *less than* 2.



The graph of $|x| < 2$ is the graph of $x > -2$ *and* $x < 2$.

The inequality $|x| > 2$ means the distance between x and 0 is *greater than* 2.



The graph of $|x| > 2$ is the graph of $x < -2$ *or* $x > 2$.

You can solve these types of inequalities by solving a compound inequality.

Core Concept

Solving Absolute Value Inequalities

To solve $|ax + b| < c$ for $c > 0$, solve the compound inequality

$$ax + b > -c \quad \textit{and} \quad ax + b < c.$$

To solve $|ax + b| > c$ for $c > 0$, solve the compound inequality

$$ax + b < -c \quad \textit{or} \quad ax + b > c.$$

In the inequalities above, you can replace $<$ with $\leq$ and $>$ with $\geq$.

EXAMPLE 1 Solving Absolute Value Inequalities

Solve each inequality. Graph each solution, if possible.

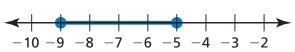
a. $|x + 7| \leq 2$ **b.** $|8x - 11| < 0$

SOLUTION

a. Use $|x + 7| \leq 2$ to write a compound inequality. Then solve.

$x + 7 \geq -2$	*and*	$x + 7 \leq 2$	Write a compound inequality.
$\underline{-7} \quad \underline{-7}$		$\underline{-7} \quad \underline{-7}$	Subtract 7 from each side.
$x \geq -9$	*and*	$x \leq -5$	Simplify.

▶ The solution is $-9 \leq x \leq -5$.



REMEMBER

A compound inequality with "and" can be written as a single inequality.

b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|8x - 11|$ cannot be less than 0.

▶ So, the inequality has no solution.

Laurie's Notes Teacher Actions

- Write the *Core Concept* on the board.
- **Teaching Tip:** When solving an absolute value inequality such as $|2x + 1| < 4$, I place my fingers over the expression inside the absolute value bars and say, "We are looking for a quantity whose absolute value is less than 4 units from 0. What is under my fingers is between -4 *and* 4." That leads directly into writing the compound inequality $-4 < 2x + 1 < 4$. By covering up the expression, I find it helps focus student attention on the process, without getting overwhelmed by symbols. If the inequality had been $|2x + 1| > 4$, then I would have said, "We are looking for a quantity whose absolute value is more than 4 units from 0. What is under my fingers is less than -4 *or* greater than 4."
- Work through part (a) as shown.
- ? Write the inequality in part (b) and ask, "What value(s) of x will make the expression on the left side of the inequality less than 0? Explain." none; Any value substituted for x and then multiplied by 8 and decreased by 11 will not be negative once you find the absolute value of the result.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

Solve the inequality. Graph the solution, if possible.

1. $|x| \leq 3.5$ **2.** $|k - 3| < -1$ **3.** $|2w - 1| < 11$

EXAMPLE 2 Solving Absolute Value Inequalities

Solve each inequality. Graph each solution.

a. $|c - 1| \geq 5$ **b.** $|10 - m| \geq -2$ **c.** $4|2x - 5| + 1 > 21$

SOLUTION

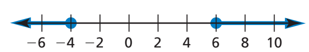
a. Use $|c - 1| \geq 5$ to write a compound inequality. Then solve.

$c - 1 \leq -5$ *or* $c - 1 \geq 5$ Write a compound inequality.

$+1 \quad +1 \qquad +1 \quad +1$ Add 1 to each side.

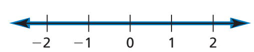
$c \leq -4$ *or* $c \geq 6$ Simplify.

▶ The solution is $c \leq -4$ *or* $c \geq 6$.



b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|10 - m|$ will always be greater than -2.

▶ So, all real numbers are solutions.



c. First isolate the absolute value expression on one side of the inequality.

$4|2x - 5| + 1 > 21$ Write the inequality.

$-1 \quad -1$ Subtract 1 from each side.

$4|2x - 5| > 20$ Simplify.

$\dfrac{4|2x - 5|}{4} > \dfrac{20}{4}$ Divide each side by 4.

$|2x - 5| > 5$ Simplify.

Then use $|2x - 5| > 5$ to write a compound inequality. Then solve.

$2x - 5 < -5$ *or* $2x - 5 > 5$ Write a compound inequality.

$+5 \quad +5 \qquad +5 \quad +5$ Add 5 to each side.

$2x < 0 \qquad 2x > 10$ Simplify.

$\dfrac{2x}{2} < \dfrac{0}{2} \qquad \dfrac{2x}{2} > \dfrac{10}{2}$ Divide each side by 2.

$x < 0$ *or* $x > 5$ Simplify.

▶ The solution is $x < 0$ *or* $x > 5$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

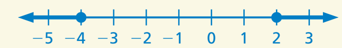
Solve the inequality. Graph the solution.

4. $|x + 3| > 8$ **5.** $|n + 2| - 3 \geq -6$ **6.** $3|d + 1| - 7 \geq -1$

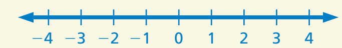
Extra Example 2

Solve each inequality. Graph each solution.

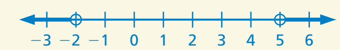
a. $|x + 1| \geq 3$ $x \leq -4$ *or* $x \geq 2$



b. $|12 + w| \geq -5$ All real numbers are solutions.

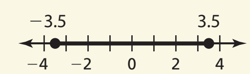


c. $3|2t - 3| - 5 > 16$ $t < -2$ *or* $t > 5$



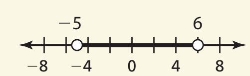
MONITORING PROGRESS ANSWERS

1. $-3.5 \leq x \leq 3.5$

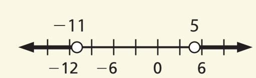


2. no solution

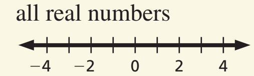
3. $-5 < w < 6$



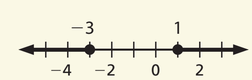
4. $x < -11$ *or* $x > 5$



5. all real numbers



6. $d \leq -3$ *or* $d \geq 1$



Laurie's Notes Teacher Actions

- **Think-Pair-Share:** Have partners work part (a) of Example 2.
- **Wait Time:** "In part (b), is it possible for the absolute value of an expression, like $10 - m$, to be greater than or equal to -2?" Before soliciting an answer, ask students to give a *Thumbs Up* response. This can be a challenging inequality for students to make sense of. One approach might be to change the inequality to $|10 - m| < -2$. There are no values of m that make the inequality true. Now return to the original inequality with the $\geq$ symbol. yes; All values of m must be solutions.
- **Look For and Make Use of Structure:** When solving part (c), students need to think of $|2x - 5|$ as a term that needs to be isolated first. Students should think solving $4|2x - 5| + 1 > 21$ is similar to solving $4x + 1 > 21$. Additionally, solving $4x + 1 > 21$ is similar to solving $4x + 1 = 21$. To isolate the variable (or the absolute value expression), subtract 1 and then divide by 4.
- **Make Sense of Problems and Persevere in Solving Them:** By connecting the process back to prior skills, students generally can follow and make sense of the problems. Performing the process on their own is still a skill that needs to be practiced.

Extra Example 3

Use the table in Example 3. You are willing to pay the mean price with an absolute deviation of at most $125. How many of the computers are priced to meet your condition? $539 \le x \le 789$; Six prices meet your condition: $750, $650, $660, $670, $650, and $725.

MONITORING PROGRESS ANSWER

7. 5

Solving Real-Life Problems

The **absolute deviation** of a number x from a given value is the absolute value of the difference of x and the given value.

absolute deviation $= |x - \text{given value}|$

EXAMPLE 3 Modeling with Mathematics

Computer prices	
$890	$750
$650	$370
$660	$670
$450	$650
$725	$825

You are buying a new computer. The table shows the prices of computers in a store advertisement. You are willing to pay the mean price with an absolute deviation of at most $100. How many of the computer prices meet your condition?

SOLUTION

1. **Understand the Problem** You know the prices of 10 computers. You are asked to find how many computers are at most $100 from the mean price.

2. **Make a Plan** Calculate the mean price by dividing the sum of the prices by the number of prices, 10. Use the absolute deviation and the mean price to write an absolute value inequality. Then solve the inequality and use it to answer the question.

3. **Solve the Problem**

The mean price is $\frac{6640}{10} = \$664$. Let x represent a price you are willing to pay.

$|x - 664| \le 100$ Write the absolute value inequality.

$-100 \le x - 664 \le 100$ Write a compound inequality.

$564 \le x \le 764$ Add 664 to each expression and simplify.

▶ The prices you will consider must be at least $564 and at most $764. Six prices meet your condition: $750, $650, $660, $670, $650, and $725.

4. **Look Back** You can check that your answer is correct by graphing the computer prices and the mean on a number line. Any point within 100 of 664 represents a price that you will consider.

STUDY TIP
The absolute deviation of at most $100 from the mean, $664, is given by the inequality $|x - 664| \le 100$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. **WHAT IF?** You are willing to pay the mean price with an absolute deviation of at most $75. How many of the computer prices meet your condition?

Concept Summary

Solving Inequalities

One-Step and Multi-Step Inequalities

- Follow the steps for solving an equation. Reverse the inequality symbol when multiplying or dividing by a negative number.

Compound Inequalities

- If necessary, write the inequality as two separate inequalities. Then solve each inequality separately. Include *and* or *or* in the solution.

Absolute Value Inequalities

- If necessary, isolate the absolute value expression on one side of the inequality. Write the absolute value inequality as a compound inequality. Then solve the compound inequality.

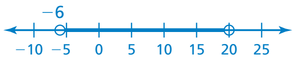
Laurie's Notes Teacher Actions

- **Connection:** In middle school, students studied the *mean absolute deviation* of a data set. Define the *absolute deviation* of a number from a given value.
- **Teaching Tip:** Use the context of the speed of a car on the highway. There is a given value (speed limit) posted. If you drive too fast or too slow, you risk getting a ticket. The mean absolute deviation might be 10 miles per hour.
- **Make Sense of Problems and Persevere in Solving Them:** Discuss the context of the situation and why someone would be willing to pay the mean price with an absolute deviation of at most $100. Students must first understand and make sense of the problem before they can make a plan to solve the problem.
- Work through the example as shown.
- **Whiteboarding:** Discuss each of the types of inequalities in the *Concept Summary* box. Have students solve 1 or 2 problems of each type with their partners.

Closure

- **Exit Ticket:** Solve and graph.

$|x - 7| < 13$ $-6 < x < 20$



$|x + 4| \ge 8$ $x \ge 4$ or $x \le -12$

2.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** Can you determine the solution of $|4x - 2| \geq -6$ without solving? Explain.
2. **WRITING** Describe how solving $|w - 9| \leq 2$ is different from solving $|w - 9| \geq 2$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–18, solve the inequality. Graph the solution, if possible. *(See Examples 1 and 2.)*

3. $|x| < 3$
4. $|y| \geq 4.5$
5. $|d + 9| > 3$
6. $|h - 5| \leq 10$
7. $|2s - 7| \geq -1$
8. $|4c + 5| > 7$
9. $|5p + 2| < -4$
10. $|9 - 4n| < 5$
11. $|6t - 7| - 8 \geq 3$
12. $|3j - 1| + 6 > 0$
13. $3|14 - m| > 18$
14. $-4|6b - 8| \leq 12$
15. $2|3w + 8| - 13 \leq -5$
16. $-3|2 - 4u| + 5 < -13$
17. $6|-f + 3| + 7 > 7$
18. $\frac{2}{3}|4v + 6| - 2 \leq 10$

19. **MODELING WITH MATHEMATICS** The rules for an essay contest say that entries can have 500 words with an absolute deviation of at most 30 words. Write and solve an absolute value inequality that represents the acceptable numbers of words. *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** The normal body temperature of a camel is 37°C. This temperature varies by up to 3°C throughout the day. Write and solve an absolute value inequality that represents the range of normal body temperatures (in degrees Celsius) of a camel throughout the day.

ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in solving the absolute value inequality.**

21. ✗
$|x - 5| < 20$
$x - 5 < 20$
$x < 25$

22. ✗
$|x + 4| > 13$
$x + 4 > -13$ and $x + 4 < 13$
$x > -17$ and $x < 9$
$-17 < x < 9$

In Exercises 23–26, write the sentence as an absolute value inequality. Then solve the inequality.

23. A number is less than 6 units from 0.
24. A number is more than 9 units from 3.
25. Half of a number is at most 5 units from 14.
26. Twice a number is no less than 10 units from −1.

27. **PROBLEM SOLVING** An auto parts manufacturer throws out gaskets with weights that are not within 0.06 pound of the mean weight of the batch. The weights (in pounds) of the gaskets in a batch are 0.58, 0.63, 0.65, 0.53, and 0.61. Which gasket(s) should be thrown out?

28. **PROBLEM SOLVING** Six students measure the acceleration (in meters per second per second) of an object in free fall. The measured values are shown. The students want to state that the absolute deviation of each measured value x from the mean is at most d. Find the value of d.

10.56, 9.52, 9.73, 9.80, 9.78, 10.91

Assignment Guide and Homework Check

ASSIGNMENT

Basic: 1, 2, 3–15 odd, 19, 21, 27, 35, 38, 41–46

Average: 1, 2, 4–16 even, 20–28 even, 30, 35, 38, 41–46

Advanced: 1, 2, 10–34 even, 35–46

HOMEWORK CHECK

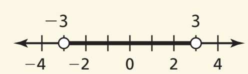
Basic: 5, 9, 11, 19

Average: 6, 10, 12, 20

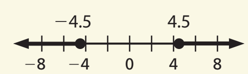
Advanced: 10, 14, 16, 28

ANSWERS

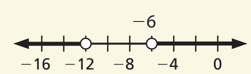
1. yes; Because the absolute value is never negative, all values will be greater than −6.
2. Solving $|w - 9| \leq 2$ requires a compound inequality joined by "and." Solving $|w - 9| \geq 2$ requires a compound inequality joined by "or."
3. $-3 < x < 3$



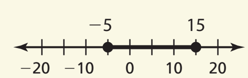
4. $y \leq -4.5$ *or* $y \geq 4.5$



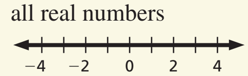
5. $d < -12$ *or* $d > -6$



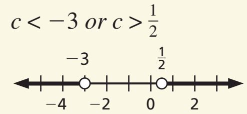
6. $-5 \leq h \leq 15$



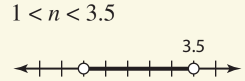
7. all real numbers



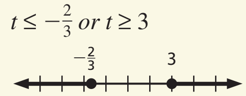
8. $c < -3$ *or* $c > \frac{1}{2}$



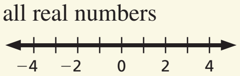
9. no solution
10. $1 < n < 3.5$



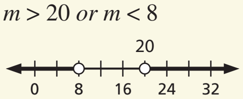
11. $t \leq -\frac{2}{3}$ *or* $t \geq 3$



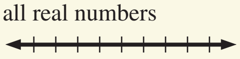
12. all real numbers



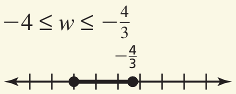
13. $m > 20$ *or* $m < 8$



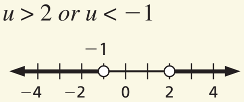
14. all real numbers



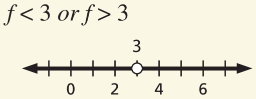
15. $-4 \leq w \leq -\frac{4}{3}$



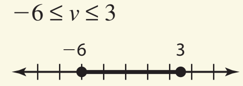
16. $u > 2$ *or* $u < -1$



17. $f < 3$ *or* $f > 3$



18. $-6 \leq v \leq 3$



19–28. See Additional Answers.

Dynamic Teaching Tools

Dynamic Assessment System

Dynamic Classroom

ANSWERS

29. $\left|\frac{1}{2} \cdot 4(x+6) - 2 \cdot 6\right| < 2$; $-1 < x < 1$

30. $|2(x+1) + 2 \cdot 3 - 4x| \leq 3$; $\frac{5}{2} \leq x \leq \frac{11}{2}$

31. true

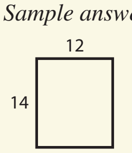
32. false; It could also be a solution of $x + 3 < -8$.

33. false; It has to be a solution of $x + 3 \leq -8$ *or* $x + 3 \geq 8$.

34. true

35. no; If n is 0, the statement is false.

36. *Sample answer:*

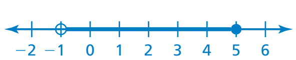


37–46. See Additional Answers.

Mini-Assessment

Solve the inequality. Graph the solution, if possible.

1. $|s - 2| < 3$ $-1 < s < 5$



2. $|6x - 5| < 0$ The inequality has no solution.

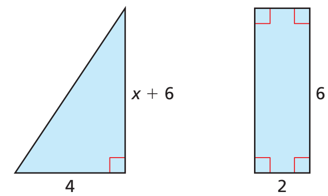
3. $|n + 4| \geq 1$ $n \leq -5$ *or* $n \geq -3$

4. $2|2y - 3| - 4 \geq 6$ $y \leq -1$ *or* $y \geq 4$

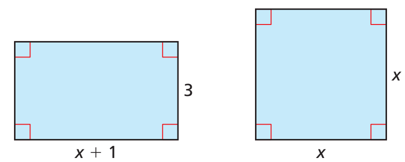
5. You are looking at ads for a new bicycle. The price you want to pay is \$200 with an absolute deviation of at most \$25. Write and solve an absolute value inequality that represents the price you want to pay. $|x - 200| \leq 25$; $175 \leq x \leq 225$; at least \$175 and at most \$225

MATHEMATICAL CONNECTIONS **In Exercises 29 and 30, write an absolute value inequality that represents the situation. Then solve the inequality.**

29. The difference between the areas of the figures is less than 2.



30. The difference between the perimeters of the figures is less than or equal to 3.



REASONING **In Exercises 31–34, tell whether the statement is true or false. If it is false, explain why.**

31. If a is a solution of $|x + 3| \leq 8$, then a is also a solution of $x + 3 \geq -8$.

32. If a is a solution of $|x + 3| > 8$, then a is also a solution of $x + 3 > 8$.

33. If a is a solution of $|x + 3| \geq 8$, then a is also a solution of $x + 3 \geq -8$.

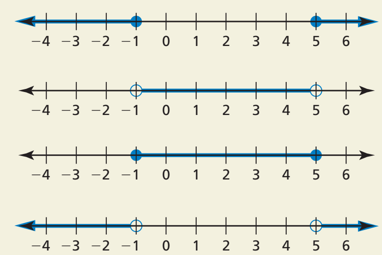
34. If a is a solution of $x + 3 \leq -8$, then a is also a solution of $|x + 3| \geq 8$.

35. **MAKING AN ARGUMENT** One of your classmates claims that the solution of $|n| > 0$ is all real numbers. Is your classmate correct? Explain your reasoning.

36. **THOUGHT PROVOKING** Draw and label a geometric figure so that the perimeter P of the figure is a solution of the inequality $|P - 60| \leq 12$.

37. **REASONING** What is the solution of the inequality $|ax + b| < c$, where $c < 0$? What is the solution of the inequality $|ax + b| > c$, where $c < 0$? Explain.

38. **HOW DO YOU SEE IT?** Write an absolute value inequality for each graph.



How did you decide which inequality symbol to use for each inequality?

39. **WRITING** Explain why the solution set of the inequality $|x| < 5$ is the *intersection* of two sets, while the solution set of the inequality $|x| > 5$ is the *union* of two sets.

40. **PROBLEM SOLVING** Solve the compound inequality below. Describe your steps.

$$|x - 3| < 4 \text{ and } |x + 2| > 8$$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Plot the ordered pair in a coordinate plane. Describe the location of the point. *(Skills Review Handbook)*

41. $A(1, 3)$ **42.** $B(0, -3)$ **43.** $C(-4, -2)$ **44.** $D(-1, 2)$

Copy and complete the table. *(Skills Review Handbook)*

45.

x	0	1	2	3	4
$5x + 1$					

46.

x	−2	−1	0	1	2
$-2x - 3$					

Section Resources

Surface Level	Deep Level	Transfer Level
Resources by Chapter • Practice A and Practice B • Puzzle Time Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer	Resources by Chapter • Enrichment and Extension • Cumulative Review Dynamic Assessment System • Section Practice	Dynamic Assessment System • End-of-Chapter Quiz

2.5–2.6 What Did You Learn?

Core Vocabulary

compound inequality, *p. 82*
absolute value inequality, *p. 88*
absolute deviation, *p. 90*

Core Concepts

Section 2.5

Writing and Graphing Compound Inequalities, *p. 82*
Solving Compound Inequalities, *p. 83*

Section 2.6

Solving Absolute Value Inequalities, *p. 88*

Mathematical Practices

1. How can you use a diagram to help you solve Exercise 12 on page 85?
2. In Exercises 13 and 14 on page 85, how can you use structure to break down the compound inequality into two inequalities?
3. Describe the given information and the overall goal of Exercise 27 on page 91.
4. For false statements in Exercises 31–34 on page 92, use examples to show the statements are false.

Performance Task

Grading Calculations

You are not doing as well as you had hoped in one of your classes. So, you want to figure out the minimum grade you need on the final exam to receive the semester grade that you want. Is it still possible to get an A? How would you explain your calculations to a classmate?

To explore the answers to this question and more, go to ***BigIdeasMath.com***.

Dynamic Teaching Tools

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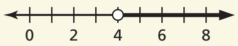
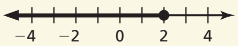
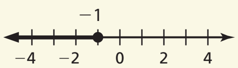
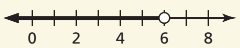
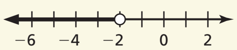
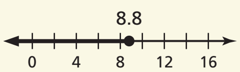
ANSWERS

1. A diagram can be used to show the different life zones, and the elevation ranges for each.
2. Take the left expression, first inequality symbol, and middle expression to form the first inequality. Take the middle expression, second inequality symbol, and last expression to form the second inequality.
3. The given information represents the weight in pounds of the gaskets in a batch, and the goal is to determine which gaskets should be thrown out because they are not within the tolerance range.
4. **32.** *Sample answer:* $a = -12$; $|-12 + 3| > 8$ is true, but $-12 + 3 > 8$ is false.

   33. *Sample answer:* $a = -14$; $|-14 + 3| \geq 8$ is true, but $-14 + 3 \geq -8$ is false.

ANSWERS

1. $d - 2 < -1$
2. $5h \le 10$
3. 
4. 
5. 
6. $p < 6$

7. $r < -2$

8. $m \le 8.8$


2 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

2.1 Writing and Graphing Inequalities *(pp. 53–60)*

a. A number x plus 36 is no more than 40. Write this sentence as an inequality.

A number x plus 36 | is no more than | 40.

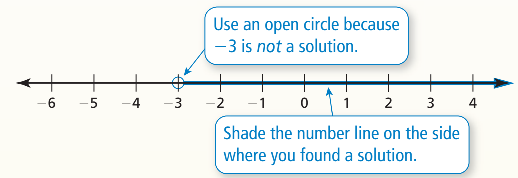
$x + 36$ | $\le$ | 40

▶ An inequality is $x + 36 \le 40$.

b. Graph $w > -3$.

Test a number to the left of -3. — $w = -4$ is *not* a solution.

Test a number to the right of -3. — $w = 0$ is a solution.



Write the sentence as an inequality.

1. A number d minus 2 is less than -1.
2. Ten is at least the product of a number h and 5.

Graph the inequality.

3. $x > 4$
4. $y \le 2$
5. $-1 \ge z$

2.2 Solving Inequalities Using Addition or Subtraction *(pp. 61–66)*

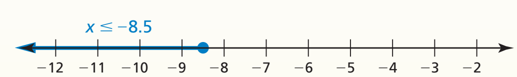
Solve $x + 2.5 \le -6$. Graph the solution.

$x + 2.5 \le -6$ — Write the inequality.

Subtraction Property of Inequality → $-\ 2.5 \quad -\ 2.5$ — Subtract 2.5 from each side.

$x \le -8.5$ — Simplify.

▶ The solution is $x \le -8.5$.



Solve the inequality. Graph the solution.

6. $p + 4 < 10$
7. $r - 4 < -6$
8. $2.1 \ge m - 6.7$

2.3 Solving Inequalities Using Multiplication or Division (pp. 67–72)

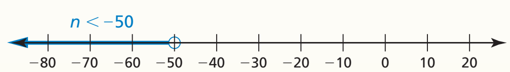
Solve $\frac{n}{-10} > 5$. Graph the solution.

$\frac{n}{-10} > 5$ Write the inequality.

Multiplication Property of Inequality → $-10 \cdot \frac{n}{-10} < -10 \cdot 5$ Multiply each side by −10. Reverse the inequality symbol.

$n < -50$ Simplify.

▶ The solution is $n < -50$.



Solve the inequality. Graph the solution.

9. $3x > -21$ **10.** $-4 \leq \frac{g}{5}$ **11.** $-\frac{3}{4}n \leq 3$

12. $\frac{s}{-8} \geq 11$ **13.** $36 < 2q$ **14.** $-1.2k > 6$

2.4 Solving Multi-Step Inequalities (pp. 73–78)

Solve $22 + 3y \geq 4$. Graph the solution.

$22 + 3y \geq 4$ Write the inequality.

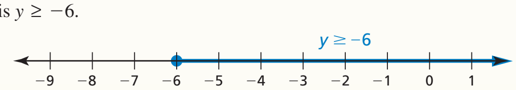
$-22 \quad -22$ Subtract 22 from each side.

$3y \geq -18$ Simplify.

$\frac{3y}{3} \geq \frac{-18}{3}$ Divide each side by 3.

$y \geq -6$ Simplify.

▶ The solution is $y \geq -6$.



Solve the inequality. Graph the solution, if possible.

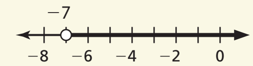
15. $3x - 4 > 11$ **16.** $-4 < \frac{b}{2} + 9$

17. $7 - 3n \leq n + 3$ **18.** $2(-4s + 2) \geq -5s - 10$

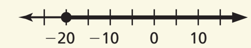
19. $6(2t + 9) \leq 12t - 1$ **20.** $3r - 8 > 3(r - 6)$

ANSWERS

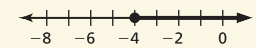
9. $x > -7$



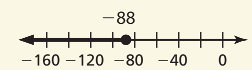
10. $g \geq -20$



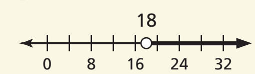
11. $n \geq -4$



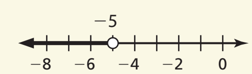
12. $s \leq -88$



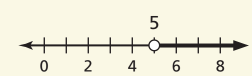
13. $q > 18$



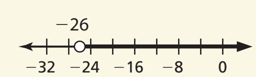
14. $k < -5$



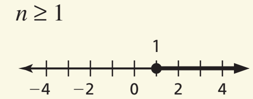
15. $x > 5$



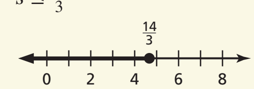
16. $b > -26$



17. $n \geq 1$

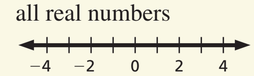


18. $s \leq \frac{14}{3}$



19. no solution

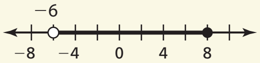
20. all real numbers



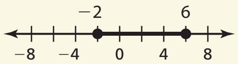
ALABAMA EDITION Chapter 2 95

ANSWERS

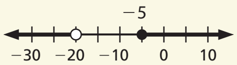
21. $-6 < x \le 8$



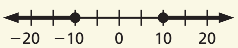
22. $-2 \le z \le 6$



23. $r < -20$ *or* $r \ge -5$

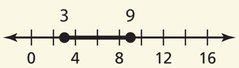


24. $m \ge 10$ *or* $m \le -10$

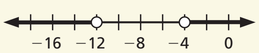


25. no solution

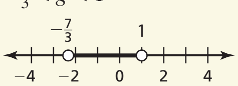
26. $3 \le f \le 9$



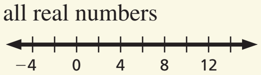
27. $b < -12$ *or* $b > -4$



28. $-\frac{7}{3} < g < 1$



29. all real numbers



30. $|h - 106| \le 7$; 99 cm to 113 cm

2.5 Solving Compound Inequalities (pp. 81–86)

Solve $-1 \le -2d + 7 \le 9$. Graph the solution.

$$-1 \le -2d + 7 \le 9$$ Write the inequality.

$$\underline{-7} \quad \underline{-7} \quad \underline{-7}$$ Subtract 7 from each expression.

$$-8 \le -2d \le 2$$ Simplify.

$$\frac{-8}{-2} \ge \frac{-2d}{-2} \ge \frac{2}{-2}$$ Divide each expression by −2. Reverse each inequality symbol.

$$4 \ge d \ge -1$$ Simplify.

▶ The solution is $-1 \le d \le 4$.

21. A number x is more than −6 and at most 8. Write this sentence as an inequality. Graph the inequality.

Solve the inequality. Graph the solution.

22. $19 \ge 3z + 1 \ge -5$

23. $\frac{r}{4} < -5$ *or* $-2r - 7 \le 3$

2.6 Solving Absolute Value Inequalities (pp. 87–92)

Solve $|2x + 11| + 3 > 8$. Graph the solution.

$$|2x + 11| + 3 > 8$$ Write the inequality.

$$\underline{-3} \quad \underline{-3}$$ Subtract 3 from each side.

$$|2x + 11| > 5$$ Simplify.

$$2x + 11 < -5 \quad \textit{or} \quad 2x + 11 > 5$$ Write a compound inequality.

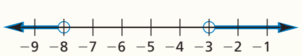
$$\underline{-11} \quad \underline{-11} \qquad \underline{-11} \quad \underline{-11}$$ Subtract 11 from each side.

$$2x < -16 \qquad 2x > -6$$ Simplify.

$$\frac{2x}{2} < \frac{-16}{2} \qquad \frac{2x}{2} > \frac{-6}{2}$$ Divide each side by 2.

$$x < -8 \quad \textit{or} \quad x > -3$$ Simplify.

▶ The solution is $x < -8$ *or* $x > -3$.



Solve the inequality. Graph the solution, if possible.

24. $|m| \ge 10$

25. $|k - 9| < -4$

26. $4|f - 6| \le 12$

27. $5|b + 8| - 7 > 13$

28. $|-3g - 2| + 1 < 6$

29. $|9 - 2j| + 10 \ge 2$

30. A safety regulation states that the height of a guardrail should be 106 centimeters with an absolute deviation of no more than 7 centimeters. Write and solve an absolute value inequality that represents the acceptable heights of a guardrail.

2 Chapter Test

Write the sentence as an inequality.

1. The sum of a number y and 9 is at least -1.
2. A number r is more than 0 or less than or equal to -8.
3. A number k is less than 3 units from 10.

Solve the inequality. Graph the solution, if possible.

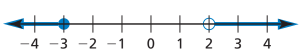
4. $\frac{x}{2} - 5 \geq -9$
5. $-4s < 6s + 1$
6. $4p + 3 \geq 2(2p + 1)$
7. $-7 < 2c - 1 < 10$
8. $-2 \leq 4 - 3a \leq 13$
9. $-5 < 2 - h$ *or* $6h + 5 > 71$
10. $|2q + 8| > 4$
11. $-2|y - 3| - 5 \geq -4$
12. $4|-3b + 5| - 9 < 7$

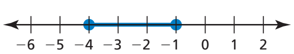
13. You start a small baking business, and you want to earn a profit of at least \$250 in the first month. The expenses in the first month are \$155. What are the possible revenues that you need to earn to meet the profit goal?

14. A manufacturer of bicycle parts requires that a bicycle chain have a width of 0.3 inch with an absolute deviation of at most 0.0003 inch. Write and solve an absolute value inequality that represents the acceptable widths.

15. Let a, b, c, and d be constants. Describe the possible solution sets of the inequality $ax + b < cx + d$.

Write and graph a compound inequality that represents the numbers that are *not* solutions of the inequality represented by the graph shown. Explain your reasoning.

16. 

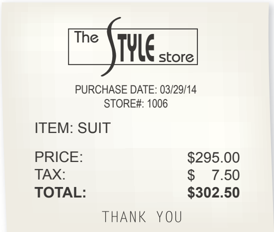
17. 

18. A state imposes a sales tax on items of clothing that cost more than \$175. The tax applies only to the difference of the price of the item and \$175.

    a. Use the receipt shown to find the tax rate (as a percent).

    b. A shopper has \$430 to spend on a winter coat. Write and solve an inequality to find the prices p of coats that the shopper can afford. Assume that $p \geq 175$.

    c. Another state imposes a 5% sales tax on the entire price of an item of clothing. For which prices would paying the 5% tax be cheaper than paying the tax described above? Write and solve an inequality to find your answer and list three prices that are solutions.

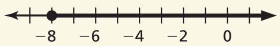


Chapter Resources

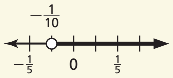
Surface Level	Deep Level	Transfer Level
Resources by Chapter • Practice A and Practice B • Puzzle Time Differentiating the Lesson Tutorial Videos Skills Review Handbook Skills Trainer Game Library	Resources by Chapter • Enrichment and Extension • Cumulative Review Game Library	STEM Video STEM Video Performance Task Dynamic Assessment System • Chapter Test Assessment Book • Chapter Tests A and B • Alternative Assessment • Performance Task

ANSWERS

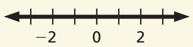
1. $y + 9 \geq -1$
2. $r > 0$ *or* $r \leq -8$
3. $|k - 10| < 3$
4. $x \geq -8$



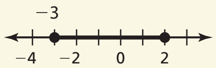
5. $s > -\frac{1}{10}$



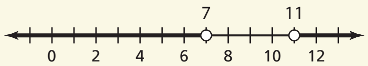
6. all real numbers



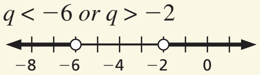
7. See Additional Answers.
8. $-3 \leq a \leq 2$



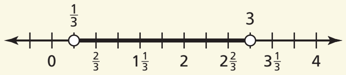
9. $h < 7$ *or* $h > 11$



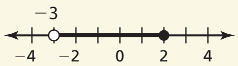
10. $q < -6$ *or* $q > -2$



11. no solution
12. $\frac{1}{3} < b < 3$

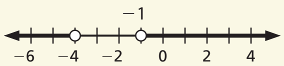


13. at least \$405
14. $|w - 0.3| \leq 0.0003$; between 0.2997 in. and 0.3003 in.
15. all real numbers; no solution; $x < \frac{d - b}{a - c}$; $x > \frac{d - b}{a - c}$
16. $-3 < x \leq 2$



The values between -3 and 2, including 2, are not solutions.

17. $x < -4$ *or* $x > -1$



The values greater than -1 or less than -4 are not solutions.

18. a. 6.25%
    b. $175 + 1.0625(p - 175) \leq 430$; $p \leq 415$
    c. $1.05p < 175 + 1.0625(p - 175)$; $p > 875$; *Sample answers:* \$900, \$950, \$1000

ANSWERS

1. A
2. **a.** -2
   b. 3; *Sample answer:* 10
   c. 3; *Sample answer:* 0
3. At least one integer solution: $5x - 6 + x \geq 2x - 8$, $9x - 3 < 12$ *or* $6x + 2 > -10$, $5(x - 1) \leq 5x - 3$;
   No integer solutions: $2(3x + 8) > 3(2x + 6)$, $17 < 4x + 5 < 21$, $x - 8 + 4x \leq 3(x - 3) + 2x$
4. **a.** $180 < 25x$
   b. 0, 1, 2, 3, 4, 5, 6, 7

2 Cumulative Assessment

1. The expected attendance at a school event is 65 people. The actual attendance can vary by up to 30 people. Which equation can you use to find the minimum and maximum attendances?

Ⓐ $|x - 65| = 30$ Ⓑ $|x + 65| = 30$
Ⓒ $|x - 30| = 65$ Ⓓ $|x + 30| = 65$

2. Fill in values for a and b so that each statement is true for the inequality $ax + 4 \leq 3x + b$.

a. When $a = 5$ and $b =$ _____, $x \leq -3$.

b. When $a =$ _____ and $b =$ _____, the solution of the inequality is all real numbers.

c. When $a =$ _____ and $b =$ _____, the inequality has no solution.

3. Place each inequality into one of the two categories.

At least one integer solution	No integer solutions

$5x - 6 + x \geq 2x - 8$

$2(3x + 8) > 3(2x + 6)$

$17 < 4x + 5 < 21$

$x - 8 + 4x \leq 3(x - 3) + 2x$

$9x - 3 < 12$ *or* $6x + 2 > -10$

$5(x - 1) \leq 5x - 3$

4. Admission to a play costs \$25. A season pass costs \$180.

a. Write an inequality that represents the numbers x of plays you must attend for the season pass to be a better deal.

b. Select the numbers of plays for which the season pass is *not* a better deal.

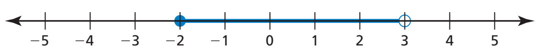
0	1	2	3	4
5	6	7	8	9
10	11	12	13	14

5. Select the values of a that make the solution of the equation $3(2x - 4) = 4(ax - 2)$ positive.

−2	−1	0	1	2	3	4	5

6. Fill in the compound inequality with $<$, $\leq$, $=$, $\geq$, or $>$ so the solution is shown in the graph.

$$4x - 18 \;\square\; -x - 3 \;\; and \;\; -3x - 9 \;\square\; -3$$



7. You have a $250 gift card to use at a sporting goods store.

a. Write an inequality that represents the possible numbers x of pairs of socks you can buy when you buy 2 pairs of sneakers. Can you buy 8 pairs of socks? Explain.

b. Describe what the inequality $60 + 80x \leq 250$ represents in this context.

8. Consider the equation shown, where a, b, c, and d are integers.

$$ax + b = cx + d$$

Student A claims the equation will always have one solution. Student B claims the equation will always have no solution. Use the numbers shown to answer parts (a)–(c).

−1	0	1	2	3	4	5	6

a. Select values for a, b, c, and d to create an equation that supports Student A's claim.

b. Select values for a, b, c, and d to create an equation that supports Student B's claim.

c. Select values for a, b, c, and d to create an equation that shows both Student A and Student B are incorrect.

ANSWERS

5. −2, −1, 0, 1

6. $<$; $\leq$

7. **a.** $160 + 12x \leq 250$; no; 8 is not a solution of the inequality.

b. the possible number of pairs of sneakers you can buy when you buy 5 pairs of socks

8. **a.** *Sample answer:* 1; 2; 3; 4

b. *Sample answer:* 2; 3; 2; 4

c. *Sample answer:* 6; 5; 6; 5