Laurie's Notes

Overview of Section 9.2

Introduction

- ***FOCUS* on Major Work:** In this section, students will prove and use the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem.
- ***RIGOR* in the Section:** In the exploration, students develop **conceptual understanding** of relationships in special right triangles by applying their understanding of the Pythagorean Theorem and similar triangles. The lesson provides opportunities for **procedural fluency** with examples and Self-Assessment exercises on finding side lengths in special right triangles. **Application** examples and additional Self-Assessment exercises provide in-class practice with problem solving before homework.
- Two types of special right triangles are presented in this section, 45°-45°-90° and 30°-60°-90° triangles. Students will discover that the ratio of the length of the hypotenuse to the length of any leg of an isosceles right triangle is the same for all isosceles right triangles. They will also discover ratio relationships for the side lengths of 30°-60°-90° triangles. In the lesson, these relationships are stated as theorems.

Making Math Visible

- To appreciate and make sense of the relationship between the legs and the hypotenuse of an isosceles right triangle, I have found that it is important for students to complete some sort of investigation. A technological approach is presented in the exploration. Alternatively, the investigation can be done by folding paper.
- Have students begin with a 6-inch square piece of paper and fold the square on its diagonal to form two isosceles right triangles. Tell students to measure the lengths of a leg and the hypotenuse, and record the lengths in a table.
- Have students continue folding the paper into smaller isosceles right triangles and filling in the table.

Number of Folds	Leg	Hypotenuse	$\frac{\text{Hypotenuse}}{\text{Leg}}$
1			
2			
3			
4			
5			

- Students can use a calculator to find the ratios of hypotenuse length to leg length. Depending upon the accuracy of students' measurements, the ratios should be close to 1.41, an approximation of $\sqrt{2}$.
- Write $\frac{\text{hypotenuse}}{\text{leg}} = \sqrt{2}$ and hypotenuse $=$ leg $\cdot \sqrt{2}$.

Section Resources

PLAN

Chapter at a Glance
Everyday Connections Video Series
Lesson Plans
Pacing Guide
Skills Review Handbook

TEACH

Answer Presentation Tool
CalcChat®
CalcView®
Differentiating the Lesson
Dynamic Classroom
Interactive Tools
Resources by Chapter*
- Warm-Up
- Extra Practice
- Reteach
- Enrichment and Extension
- Puzzle Time

Skills Trainer
Tutorial Video Series

ASSESS

Dynamic Assessment System
- Self-Assessment
- Section Practice
- Section Review & Refresh
- Point-of-use Remediation
- Reports

Formative Check
Homework App
Practice Workbook and Test Prep*
- Extra Practice
- Review & Refresh
- Self-Assessment

Self-Assessment

**Available in print*

Common Core State Standards

HSG-SRT.B.4

Learning Target

Understand and use special right triangles.

Success Criteria

- Find side lengths in 45°-45°-90° triangles.
- Find side lengths in 30°-60°-90° triangles.
- Use special right triangles to solve real-life problems.

Warm-Up

Cumulative and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL SUPPORT

Review that a right triangle is a triangle that has one angle that measures 90°. Explain that its other angles may vary in measure, and the lengths of its sides may also vary. Explain that, in this section, students will learn how to find the lengths of the sides of special right triangles with angles measuring 45°, 45°, and 90°, or 30°, 60°, and 90°.

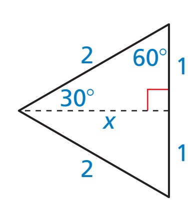
Laurie's Notes

Launch the Lesson

- Pose a baseball problem to students. If you are located near a Major League Baseball team, you could display the team's logo!
- ? "The distance between each pair of consecutive bases is 90 feet. What is the *exact* distance from home plate to second base?" Students should note the use of the word *exact*, and they should also realize that the problem involves finding the length of a diagonal of a square when the side lengths are known.
- This problem allows you to review simplifying expressions involving radicals, a skill that students will need in the lesson.
- The exact distance is $90\sqrt{2}$ feet.

EXPLORE IT!

- There are different techniques that students can use to construct special right triangles.
  - By folding paper, as described in *Making Math Visible* on page T-454, students can construct isosceles right triangles.
  - Using a coordinate plane and ordered pairs such as (0, 0), (4, 0), and (0, 4), students can construct isosceles right triangles.
  - Student can use dynamic geometry software or a compass and straightedge.
  - **Note:** One way my students have efficiently constructed a 30°-60°-90° triangle is by constructing an equilateral triangle and then constructing one of the angle bisectors.
- **MP8 Look for and Express Regularity in Repeated Reasoning:** Students should note that each construction and subsequent measurements are going to be repeated several times. Students might want to consider how to construct the triangles so that they are dynamic.
- Alternatively, you could have each pair of students construct one triangle and then collect their results in one table. Then each pair can write a conjecture based on the results of all the constructions.
- To find the *exact* ratios in part (a), students can use integer values for the legs and then use the Pythagorean Theorem to find the hypotenuse.
- Circulate as students construct the right triangles so that you see the various methods they use.
- To find the exact values in part (b), students can begin by constructing an equilateral triangle with side lengths of 2, as shown below. Then students can use the Pythagorean Theorem to find the length of the longer leg of the right triangle formed by the bisector of an angle of the equilateral triangle.



- **MP6 Attend to Precision:** Ask students to explain how they found the exact ratios.
- **DIG DEEPER** "Explain why your conjectures are true." The Pythagorean Theorem can be used to find the side lengths of each triangle. By the AA Similarity Theorem, all 45°-45°-90° triangles are similar and all 30°-60°-90° are similar. So, corresponding side lengths are proportional.

Where Are We In Our Learning?

"In the lesson, your conjectures will be stated as theorems. These relationships can be used to solve many real-life applications."

9.2 Special Right Triangles

Learning Target Understand and use special right triangles.

Success Criteria
- I can find side lengths in 45°-45°-90° triangles.
- I can find side lengths in 30°-60°-90° triangles.
- I can use special right triangles to solve real-life problems.

EXPLORE IT! Finding Side Ratios of Special Right Triangles

MP CHOOSE TOOLS **Work with a partner.**

a. One type of special right triangle is a 45°-45°-90° triangle.

i. Construct a right triangle with acute angle measures of 45°.

ii. Find the exact ratios of the side lengths.

$\frac{AB}{BC} = $ ▭

$\frac{AB}{AC} = $ ▭

$\frac{AC}{BC} = $ ▭

iii. Repeat parts (i) and (ii) for several other 45°-45°-90° triangles. Use your results to write a conjecture about the ratios of the side lengths of 45°-45°-90° triangles.

b. Another type of special right triangle is a 30°-60°-90° triangle.

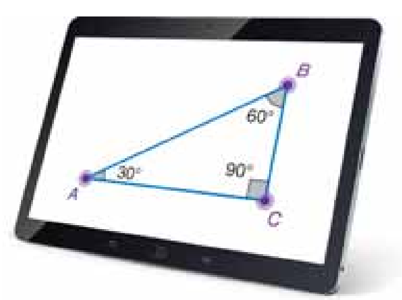
i. Construct a right triangle with acute angle measures of 30° and 60°.

ii. Find the exact ratios of the side lengths.

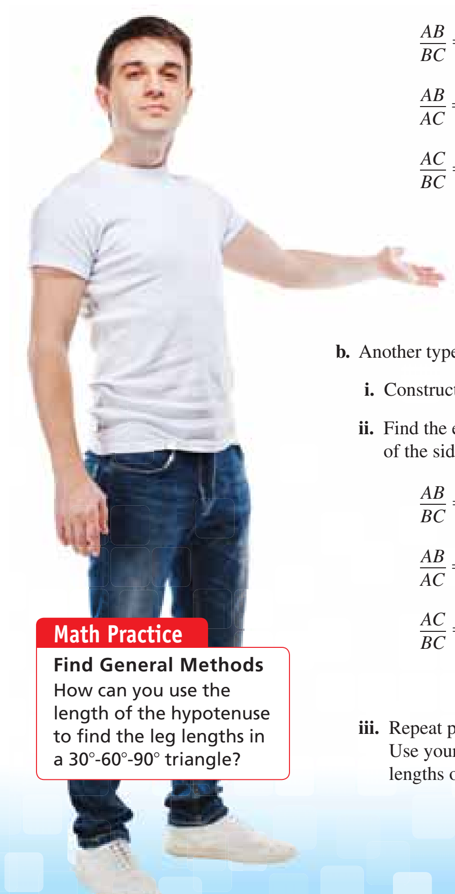
$\frac{AB}{BC} = $ ▭

$\frac{AB}{AC} = $ ▭

$\frac{AC}{BC} = $ ▭



iii. Repeat parts (i) and (ii) for several other 30°-60°-90° triangles. Use your results to write a conjecture about the ratios of the side lengths of 30°-60°-90° triangles.



Math Practice

Find General Methods
How can you use the length of the hypotenuse to find the leg lengths in a 30°-60°-90° triangle?

ANSWERS

a. i. Check students' work.

ii. $\sqrt{2}$; $\sqrt{2}$; 1

iii. Check students' work. The ratio of the length of the hypotenuse to the length of each leg is $\sqrt{2}$. The ratio of the length of one leg to the other is 1.

b. i. Check students' work.

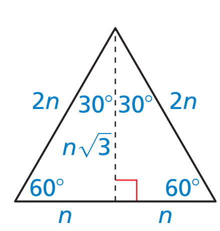
ii. 2; $\frac{2}{\sqrt{3}}$; $\sqrt{3}$

iii. Check students' work. The ratio of the length of the hypotenuse to the length of the longer leg is $\frac{2}{\sqrt{3}}$. The ratio of the length of the hypotenuse to the length of the shorter leg is 2. The ratio of the length of the longer leg to the length of the shorter leg is $\sqrt{3}$.

EVERYDAY CONNECTIONS
Learn more about how students can use paper folding to explore the 30°-60°-90° Triangle Theorem.

Laurie's Notes

Scaffolding Instruction

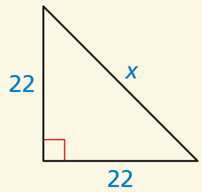


- ***EMERGING*** Several of the ratios students found in the exploration are stated as equations in the theorems. For each theorem, it is helpful to write the relationships in both forms. Demonstrate how the relationships in a 30°-60°-90° triangle can be found using an equilateral triangle. This diagram is a helpful model for students. Have students verify that the side lengths form a right triangle.
- ***PROFICIENT*** Students can prove and apply the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem. Verify that students are confident in simplifying radical expressions.

Extra Example 1

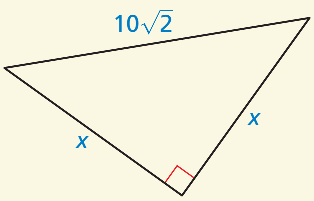
Find the value of x. Write your answer in simplest form.

a.



$x = 22\sqrt{2}$

b.

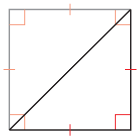


$x = 10$

Finding Side Lengths in Special Right Triangles

GO DIGITAL

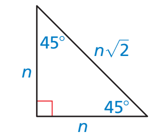
A 45°-45°-90° triangle is an *isosceles right triangle* that can be formed by cutting a square in half diagonally.



THEOREM

9.4 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



hypotenuse = leg • $\sqrt{2}$

Prove this Theorem Exercise 17, page 459

REMEMBER

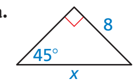
An expression involving a radical with index 2 is in simplest form when no radicands have perfect squares as factors other than 1, no radicands contain fractions, and no radicals appear in the denominator of a fraction.

EXAMPLE 1 Finding Side Lengths in 45°-45°-90° Triangles

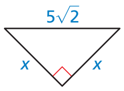
WATCH

Find the value of x. Write your answer in simplest form.

a.



b.



SOLUTION

a. By the Triangle Sum Theorem, the measure of the third angle must be 45°, so the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • $\sqrt{2}$	45°-45°-90° Triangle Theorem
$x = 8 \cdot \sqrt{2}$	Substitute.
$x = 8\sqrt{2}$	Simplify.

▶ The value of x is $8\sqrt{2}$.

b. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • $\sqrt{2}$	45°-45°-90° Triangle Theorem
$5\sqrt{2} = x \cdot \sqrt{2}$	Substitute.
$\frac{5\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$	Division Property of Equality
$5 = x$	Simplify.

▶ The value of x is 5.

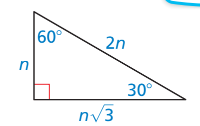
Laurie's Notes

- State the 45°-45°-90° Triangle Theorem and make connections to part (a) of the exploration.
- When solving problems like those in Example 1, students will often use the Pythagorean Theorem instead of applying the 45°-45°-90° Triangle Theorem.
- **MP2 Reason Abstractly and Quantitatively:** Mathematically proficient students understand that all isosceles right triangles possess the relationship stated in the 45°-45°-90° Triangle Theorem. They use the theorem to efficiently solve problems.

THEOREM

9.5 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



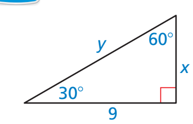
hypotenuse = shorter leg • 2
longer leg = shorter leg • $\sqrt{3}$

Prove this Theorem Exercise 19, page 460

EXAMPLE 2 Finding Side Lengths in a 30°-60°-90° Triangle

WATCH

Find the values of x and y. Write your answers in simplest form.



SOLUTION

Step 1 Find the value of x.

longer leg = shorter leg • $\sqrt{3}$	30°-60°-90° Triangle Theorem
$9 = x \cdot \sqrt{3}$	Substitute.
$\frac{9}{\sqrt{3}} = x$	Divide each side by $\sqrt{3}$.
$\frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$	Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.
$\frac{9\sqrt{3}}{3} = x$	Multiply fractions.
$3\sqrt{3} = x$	Simplify.

▶ The value of x is $3\sqrt{3}$.

Step 2 Find the value of y.

hypotenuse = shorter leg • 2	30°-60°-90° Triangle Theorem
$y = 3\sqrt{3} \cdot 2$	Substitute.
$y = 6\sqrt{3}$	Simplify.

▶ The value of y is $6\sqrt{3}$.

REMEMBER

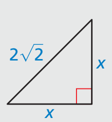
Because the angle opposite 9 is larger than the angle opposite x, the leg with length 9 is longer than the leg with length x by the Triangle Larger Angle Theorem.

SELF-ASSESSMENT

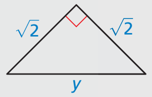
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the missing side length(s). Write your answer(s) in simplest form.

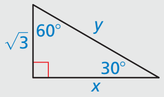
1.



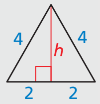
2.

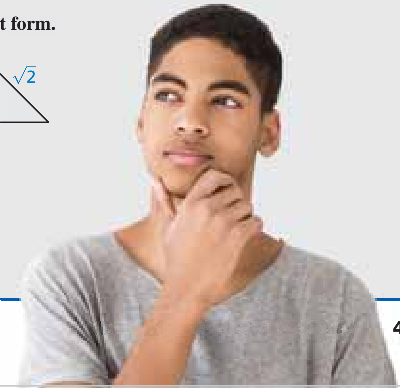


3.



4.





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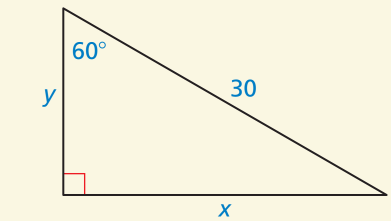
Extra Example 2

Find the values of x and y. Write your answers in simplest form.



$x = 15\sqrt{3}$, $y = 15$

ELL SUPPORT

After demonstrating Example 2, have students work in pairs for language practice to discuss and complete the Self-Assessment. Remind students to refer to the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem as needed. Expect students to perform according to their language levels.

Beginner: Use words and phrases to discuss.

Intermediate: Use simple sentences to discuss.

Advanced: Use detailed sentences to discuss.

ANSWERS

1. $x = 2$
2. $y = 2$
3. $x = 3$, $y = 2\sqrt{3}$
4. $h = 2\sqrt{3}$

Laurie's Notes

- State the 30°-60°-90° Triangle Theorem and make connections to part (b) of the exploration. Stress that there are two relationships stated in this theorem.

? FEEDBACK "How do you know which leg is the shorter leg? the longer leg?" The shorter leg is opposite the 30° angle; The longer leg is opposite the 60° angle.

- **THINK-PAIR-SHARE** Have students complete Self-Assessment Exercises 1–4, and then share and discuss as a class.
- **DIG DEEPER** "You have used two theorems to find side lengths in special right triangles. Tell your elbow partner why the hypotenuse of any 30°-60°-90° triangle is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg." Circulate and listen to students' conversations. Are they making connections to similar triangles?

Extra Example 3

The warning sticker is shaped like an equilateral triangle. Estimate the area of the sticker.

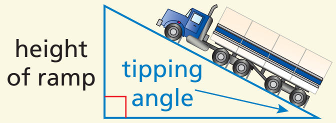
about 6.9 in.^{2}

ELL SUPPORT

Before demonstrating Example 3, you may want to discuss the meaning of a biohazard sign. Before demonstrating Example 4, you may want to discuss the context of a tipping platform. The diagrams may provide helpful support for students with limited language. After demonstrating Examples 3 and 4, allow students to work on the Self-Assessment in groups for extra support with language. To check understanding, have each group display their answers for your review.

Extra Example 4

A tipping platform is a ramp used to unload trucks. How high is the end of a 60-foot ramp when the tipping angle is 30°? 45°?



30 ft; about 42 ft 5 in.

ANSWERS

5–6. See Additional Answers.

Closure

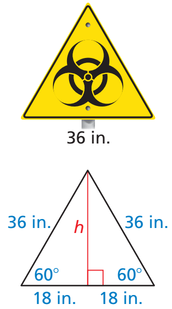
EXIT TICKET (a) The length of the hypotenuse of an isosceles right triangle is 12 units. Find the length of a leg. $6\sqrt{2}$ units (b) The length of the hypotenuse of a 30°-60°-90° triangle is 12 units. Find the lengths of the legs. 6 units, $6\sqrt{3}$ units

GO DIGITAL

Solving Real-Life Problems

EXAMPLE 3 **Modeling Real Life** WATCH INFO

The biohazard sign is shaped like an equilateral triangle. Estimate the area of the sign.



SOLUTION

First find the height h of the triangle by dividing it into two 30°-60°-90° triangles. The length of the longer leg of one of these triangles is h. The length of the shorter leg is 18 inches.

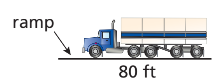
$h = 18 \cdot \sqrt{3} = 18\sqrt{3}$ 30°-60°-90° Triangle Theorem

Use $h = 18\sqrt{3}$ to find the area of the equilateral triangle.

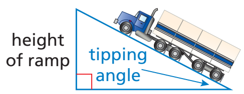
$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(36)(18\sqrt{3}) \approx 561.18$

▶ The area of the sign is about 561 square inches.

EXAMPLE 4 **Modeling Real Life** WATCH



A tipping platform is a ramp used to unload trucks. How high is the end of an 80-foot ramp when the tipping angle is 30°? 45°?



SOLUTION

When the tipping angle is 30°, the height h of the ramp is the length of the shorter leg of a 30°-60°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = 2h$ 30°-60°-90° Triangle Theorem

$40 = h$ Divide each side by 2.

When the tipping angle is 45°, the height is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 80 feet.

$80 = h \cdot \sqrt{2}$ 45°-45°-90° Triangle Theorem

$\frac{80}{\sqrt{2}} = h$ Divide each side by $\sqrt{2}$.

$56.6 \approx h$ Use technology.

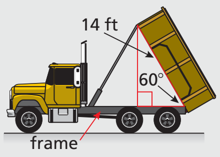
▶ When the tipping angle is 30°, the ramp height is 40 feet. When the tipping angle is 45°, the height is about 56 feet 7 inches.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5. The logo on a recycling bin resembles an equilateral triangle with side lengths of 6 centimeters. Approximate the area of the logo.

6. The body of a dump truck rests on a frame. The body is raised to empty a load of sand. How far from the frame is the front of the 14-foot-long body when it is tipped upward by a 60° angle?



Laurie's Notes

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Pose the problems in Examples 3 and 4, and allow time for students to work through each of them. Do not rush in to solve the problems for students. Trust that they can apply the theorems in this lesson to solve the problems.

? "In Example 3, why did you approximate the answer?" The problem asked for an estimate, and when using the answer for a contextual purpose, an approximation may be more helpful.

? "In Example 4, as the measure of the tipping angle increases, what happens to the height of the front of the truck?" It increases.

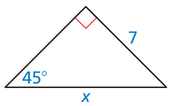
◎ Review the success criteria and have students assess where they are in their learning.

9.2 Practice WITH CalcChat® AND CalcView®

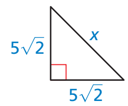
GO DIGITAL

In Exercises 1–4, find the value of x. Write your answer in simplest form. *Example 1*

1.



2.



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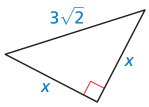
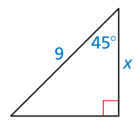
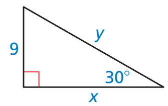


4.

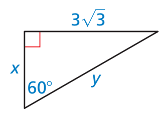


In Exercises 5–8, find the values of x and y. Write your answers in simplest form. *Example 2*

5.



6.



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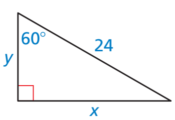

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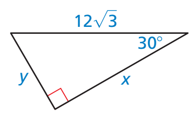
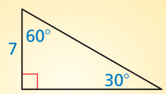


ERROR ANALYSIS **In Exercises 9 and 10, describe and correct the error in finding the length of the hypotenuse in the special right triangle.**

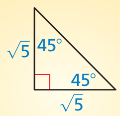
9.



hypotenuse = shorter leg • $\sqrt{3} = 7\sqrt{3}$

So, the length of the hypotenuse is $7\sqrt{3}$ units.

10.



hypotenuse = leg • 2 = $2\sqrt{5}$

So, the length of the hypotenuse is $2\sqrt{5}$ units.

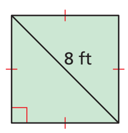
In Exercises 11 and 12, sketch the figure that is described. Find the indicated length. Write your answer in simplest form.

11. The perimeter of a square is 36 inches. Find the length of a diagonal.

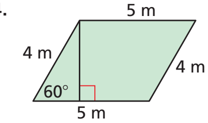
12. The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude.

In Exercises 13 and 14, find the area of the figure. Write your answer in simplest form. *Example 3*

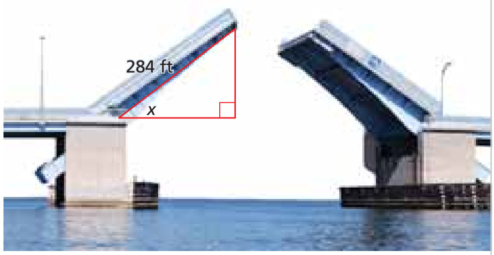
13.



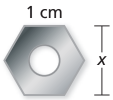
14.



15. MODELING REAL LIFE Each half of the drawbridge is about 284 feet long. How high does the drawbridge rise when x is 30°? 45°? 60°? *Example 4*



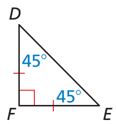
16. MODELING REAL LIFE A nut is shaped like a regular hexagon with side lengths of 1 centimeter. Find the value of x.



17. PROVING A THEOREM Write a paragraph proof of the 45°-45°-90° Triangle Theorem.

Given $\triangle DEF$ is a 45°-45°-90° triangle.

Prove The hypotenuse is $\sqrt{2}$ times as long as each leg.



Assignment Guide

Emerging: 1, 2, 3, 5, 7, 9, 11, 12, 13, 14, 15, 17, 18

Proficient: 1, 2, 4, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Advanced: 2, 4, 6, 8, 10, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23

- Use the red exercises to check understanding of the learning target.
- Assign Review & Refresh exercises as appropriate for continued spaced practice.

Item Leveling

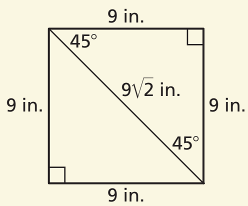
DOK	Exercises
1	1–8
2	9–18, 21
3	19, 20, 22, 23

Slow Reveal

For Exercise 15, draw a sketch or display a picture of a drawbridge. Ask students what measurements remain the same when the drawbridge rises or lowers. After a student says that the length of the bridge, half of which represents a hypotenuse, remains the same, ask students to describe the various right triangles that can be formed.

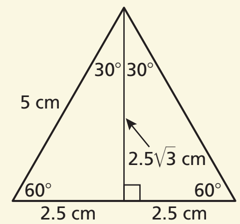
ANSWERS

1. $x = 7\sqrt{2}$
2. $x = 10$
3. $x = 3$
4. $x = \frac{9\sqrt{2}}{2}$
5. $x = 9\sqrt{3}$, $y = 18$
6. $x = 3$, $y = 6$
7. $x = 12\sqrt{3}$, $y = 12$
8. $x = 18$, $y = 6\sqrt{3}$
9. The hypotenuse of a 30°-60°-90° triangle is equal to the shorter leg times 2; hypotenuse = shorter leg • 2 = 7 • 2 = 14; So, the length of the hypotenuse is 14 units.
10. The hypotenuse of a 45°-45°-90° triangle is equal to a leg times $\sqrt{2}$; hypotenuse = leg • $\sqrt{2} = \sqrt{5} \cdot \sqrt{2} = \sqrt{10}$; So, the length of the hypotenuse is $\sqrt{10}$ units.
11.



$9\sqrt{2}$ in.

12.

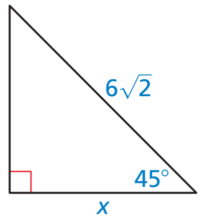


$\frac{5}{2}\sqrt{3}$ cm

13. 32 ft^2
14. $10\sqrt{3}$ m^2
15. 142 ft; about 200.82 ft; about 245.95 ft
16. $x = \sqrt{3}$ cm
17. See Additional Answers.

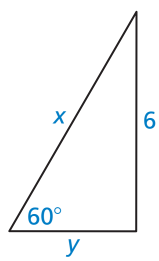
Mini-Assessment

1. Find the value of x. Write your answer in simplest form.



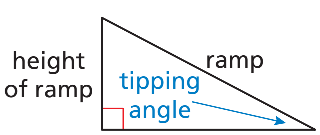
$x = 6$

2. Find the values of x and y. Write your answers in simplest form.



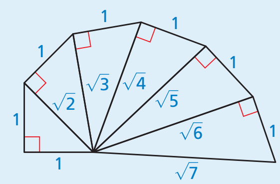
$x = 4\sqrt{3}, y = 2\sqrt{3}$

3. How high is the end of a 38-foot ramp when the tipping angle is 30°?



19 ft

18. HOW DO YOU SEE IT? The diagram shows part of the *Wheel of Theodorus*.



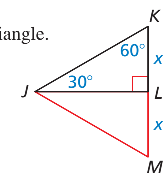
a. Which triangles, if any, are 45°-45°-90° triangles?

b. Which triangles, if any, are 30°-60°-90° triangles?

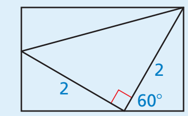
19. PROVING A THEOREM Write a paragraph proof of the 30°-60°-90° Triangle Theorem. (*Hint*: Construct $\triangle JML$ congruent to $\triangle JKL$.)

Given $\triangle JKL$ is a 30°-60°-90° triangle.

Prove The hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



20. THOUGHT PROVOKING The diagram below is called the *Ailles rectangle*. Each triangle in the diagram has rational angle measures, and each side length contains at most one square root. Label the sides and angles in the diagram. Describe the triangles.



GO DIGITAL

21. WRITING Describe two ways to show that all isosceles right triangles are similar to each other.

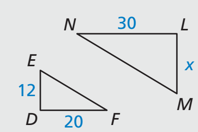
22. CRITICAL THINKING The area of an equilateral triangle is $3\sqrt{3}$ square units. Find the side length of the triangle. Justify your answer.

23. DIG DEEPER $\triangle TUV$ is a 30°-60°-90° triangle, where two vertices are $U(3, -1)$ and $V(-3, -1)$, $\overline{UV}$ is the hypotenuse, and point T is in Quadrant I. Find the coordinates of T.

REVIEW & REFRESH

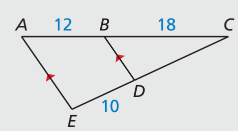
WATCH

24. In the diagram, $\triangle DEF \sim \triangle LMN$. Find the value of x.

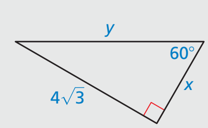


25. Determine whether segments with lengths of 2.6 feet, 4.8 feet, and 6.0 feet form a triangle. If so, is the triangle *acute*, *right*, or *obtuse*?

26. Find the length of $\overline{DC}$.



27. Find the values of x and y. Write your answers in simplest form.

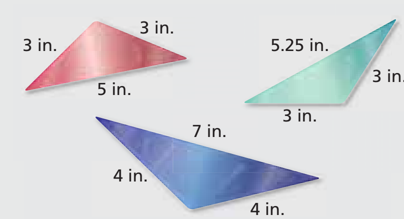


28. The endpoints of $\overline{CD}$ are $C(-2, 9)$ and $D(3, -1)$. Find the coordinates of the midpoint M.

29. Determine whether the polygons with the given vertices are congruent. Use transformations to explain your reasoning.

$D(3, 5), E(8, 0), F(4, -3)$ and
$M(-5, 3), N(0, 8), P(3, 4)$

30. MODELING REAL LIFE Which pieces of stained glass, if any, are similar? Explain.



31. Three vertices of ▱$JKLM$ are $J(0, 5)$, $K(4, 5)$, and $M(3, 0)$. Find the coordinates of vertex L.

32. Rewrite the definition as a biconditional statement.

Definition A *parallelogram* is a quadrilateral with both pairs of opposite sides parallel.

ANSWERS

18. a. the triangle farthest to the left with legs that are each 1 unit

b. the third triangle from the left with legs that are 1 unit and $\sqrt{3}$ units, and a hypotenuse that is $\sqrt{4} = 2$ units

19–20. See Additional Answers.

21. *Sample answer:* Because all isosceles right triangles are 45°-45°-90° triangles, they are similar by the AA Similarity Theorem. Because both legs of an isosceles right triangle are congruent, the legs will always be proportional. So, 45°-45°-90° triangles are similar by the SAS Similarity Theorem.

22. $2\sqrt{3}$ units; The height of the triangle is $\frac{\sqrt{3}}{2}x$, where x represents each side of the triangle. $A = \frac{1}{2}bh = \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$. Because $\frac{\sqrt{3}}{4}x^2 = 3\sqrt{3}$, $x = 2\sqrt{3}$.

23. $T(1.5, 1.5\sqrt{3} - 1)$

24. $x = 18$

25. yes; obtuse

26. 15

27. $x = 4, y = 8$

28. $\left(\frac{1}{2}, 4\right)$

29. yes; $\triangle DEF$ can be mapped to $\triangle MNP$ by a rotation 90° about the origin.

30. the pieces with side lengths of 5.25 inches and 7 inches

31. (7, 0)

32. A quadrilateral is a parallelogram if and only if both pairs of opposite sides are parallel.