Suggested Pacing

The recommended pacing for this chapter is 14 days. Here is one way to pace out the chapter.

Chapter Opener	1 Day
Section 1	2 Days
Section 2	2 Days
Section 3	1 Day
Section 4	1 Day
Section 5	1 Day
Section 6	2 Days
Section 7	2 Days
Chapter Review	1 Day
Chapter Test/ Performance Task	1 Day
Total Chapter 1	14 Days
Year-to-Date	14 Days

ANSWERS

- *Sample answer:* Take shorter showers or turn the water off while brushing your teeth.
- about 10 gallons; about 20 gallons; about 30 gallons
- 100,000 gallons

1 Solving Linear Equations

GO DIGITAL

NATIONAL GEOGRAPHIC EXPLORER

Dalal Hanna

WATCH INFO

Ecologist Dalal Hanna specializes in how to sustain Earth's environment. She focuses on different ways rivers contribute to human well-being, and has published research on mercury contamination in African freshwater fish. Hanna has also developed the podcast "Science Faction" which explores discoveries in all fields of science.

- What are ways that you can conserve water?
- How much water is used to take a 5-minute shower? a 10-minute shower? a 15-minute shower?
- How much water is conserved if 10,000 people each use 10 fewer gallons of water every month?

Water Conservation

STEM

Water conservation is critical for human health and development. In the Performance Task, you will make a plan to conserve water in your own daily life.

Laurie's Notes

Math Connections: Encourage students to research online. Possible search terms are *mercury contamination* and *water conservation*. For the third bullet, students may say 100,000 gallons a month or 1,200,000 gallons a year. Encourage students to develop the problem-solving strategy of writing an equation. This strategy will be especially helpful when solving more difficult problems throughout the chapter.

About the Performance Task

Every Drop Counts on page 57 asks students to apply their understanding of rates, unit analysis, and solving equations to make a plan to reduce the amount of water they use each week.

Laurie's Notes

Chapter 1 Overview

Welcome to a new school year, and for many students, a new school. There is always great excitement, and students are anxious to start anew. Capitalize on this opportunity by establishing norms and routines for student discourse and classroom climate.

In this course, students are expected to work together on explorations, make conjectures, construct viable arguments, and critique the reasoning of others. Take time in this first chapter to clarify what productive classroom dialogue sounds like. Listen for students explaining their thinking, not just their processes.

***FOCUS* on Major Work:** Chapter 1 presents the foundational skills related to solving linear equations and the connected skills of solving absolute value equations and rewriting equations and formulas.

Most students will have prior experience with the properties of equality and techniques presented in the first two sections and perhaps the fifth section. It will sound familiar that whatever operation is performed on one side of the equation must be performed on the other side of the equation to keep equality, or balance.

In middle school, students worked with quantities, numbers with units that relate to measurement. The quantities were often attributes, such as height, length, perimeter, and area. The third and fourth sections build upon this work, as well as previous work with rates and ratios from middle school. Although they may have encountered derived units in middle school, students will now become more specific in their use of derived units and their abbreviations (e.g., m/sec).

The last two sections look at solving absolute value equations and rewriting equations and formulas. In each case, students are applying an understanding of equation solving to a context that seems quite different. An absolute value equation is an equation that can be rewritten as two linear equations. Rewriting equations and formulas requires students to see the structure of equations and perform operations on variable terms (e.g., $4x$) as they would perform operations on constants (e.g., 4).

Essential to success in this chapter is accuracy in computation, as well as an understanding of equation solving. Feedback to students should distinguish between errors in computation and process errors.

Chapter Learning Target

Understand solving linear equations.

Chapter Success Criteria

- ◆ Solve simple and multi-step equations.
- ◆ Describe how to solve equations.
- ■ Analyze the measurements used to solve a problem and judge the level of accuracy appropriate for the solution.
- ■ Apply equation-solving techniques to solve real-life problems.

◆ Surface ■ Deep

EVERYDAY CONNECTIONS
Learn more about Learning Targets and Success Criteria.

ELL SUPPORT

Write the chapter title on the board. Underline the word *linear*. Ask students what word they recognize within *linear*. Circle *line*. Using a straightedge, draw a line as a visual. Point to the line and say, "line." Explain that *linear* describes something that is related to a line. Tell students that there will be equations in this chapter that represent lines. Explain that when they do not know a word, one strategy for guessing its meaning is to look for a word they do know that is contained within it. Have students look at the other two words in the title and name words they know that could be related. Be sure to discuss the words *solve* and *equal*.

Chapter 1: Explore Chapter Resources

This chart highlights three of the many program resources available at *BigIdeasMath.com*.
You will learn more about program resources in each chapter.

PLAN	TEACH	ASSESS
Chapter at a Glance Complete Materials List **Everyday Connections Video Series** Graphic Organizers Lesson Plans Math Tool Paper Pacing Guide Skills Review Handbook	Answer Presentation Tool CalcChat® CalcView® Differentiating the Lesson **Dynamic Classroom** Everyday Explorations Video Series Game Library Interactive Tools Resources by Chapter* • Family Letter • Warm-Up • Extra Practice • Reteach • Enrichment and Extension • Puzzle Time Skills Trainer STEM Video and Performance Task Tutorial Video Series	Assessment Book* • Quizzes • Chapter Tests • Alternative Assessments • Performance Tasks • Course Benchmark Tests **Dynamic Assessment System** **• Practice** **• Assessments** • Point-of-use Remediation • Reports Formative Check Homework App Practice Workbook and Test Prep* • Extra Practice • Review & Refresh • Self-Assessments • Test Prep • Post-Course Test Self-Assessment

**Available in print*

PLAN with the Everyday Connections Video Series

Quick and easy **Everyday Connections Videos** help you:

- Develop a deeper understanding of the math content
- Extend concepts within the examples and exercises
- Integrate technology in the classroom

EVERYDAY CONNECTIONS
Learn more about determining the accuracy of a thermometer.

to Help Your Instructional Pathway

TEACH with the Dynamic Classroom

Use the **Dynamic Classroom** to:

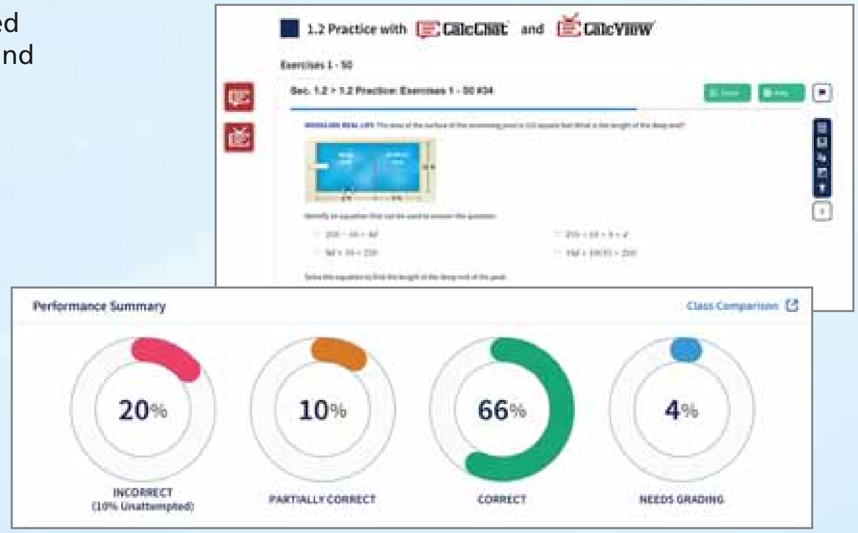
- Present interactive content from the Dynamic Student Edition such as Explore Its! and Performance Tasks
- Collect and act on real-time student formative and self-assessment data
- Get students on task with the Flip-To feature

ASSESS with the Dynamic Assessment System

The **Dynamic Assessment System** is a complete digital assessment and reporting system.

With digital **Practice** and **Assessments**, you can:

- Create pre-made or customized homework and assessments, and receive immediate feedback through robust reporting
- Give students exposure to a wide variety of technology-enhanced items
- Encourage students to build on understanding with point-of-use support



Chapter 1 Progressions

COHERENCE **Through the Grades**		
Prior Learning	**Current Learning**	**Future Learning**
Middle School • **6.NS.C.7c, 6.NS.C.7d** Find the absolute values of numbers and use absolute value to compare numbers in real-life situations. • **7.NS.A.3** Solve real-life problems involving operations with rational numbers. • **7.EE.B.4** Use variables and create simple equations to solve real-life problems. • **7.RP.A.3** Use proportionality to solve multi-step ratio problems. • **7.RP.A.2b** Find unit rates associated with ratios of fractions, areas, and other quantities in like or different units. • **7.RP.A.2c** Represent proportional relationships with equations. • **8.EE.C.7b** Solve linear equations with rational-number coefficients, including equations whose solutions require expanding expressions using the Distributive Property and collecting like terms. • **8.EE.B.4** Choose units of appropriate size for measurements of very large or very small quantities.	**Chapter 1** • **HSA-REI.A.1, HSA-REI.B.3** Solve multi-step linear equations with a variable on one side or both sides, and identify equations with no solution or infinitely many solutions. • **HSA-CED.A.1** Create multi-step linear equations and use them to solve real-life problems. • **HSN-Q.A.1, HSN-Q.A.2** Use unit analysis to model real-life problems. • **HSN-Q.A.3** Choose an appropriate level of accuracy for measurements when solving real-life problems. • **HSA-REI.A.1, HSA-REI.B.3** Solve absolute value equations involving one or two absolute values, and identify equations with extraneous solutions. • **HSA-CED.A.4** Rewrite and use literal equations and common formulas.	**Algebra 1** • **HSA-CED.A.1** Create linear and absolute value inequalities and use them to solve real-life problems. • **HSA-REI.B.3** Solve multi-step, compound, and absolute value inequalities, and identify inequalities with no solution, one solution, or infinitely many solutions. • **HSF-LE.A.2** Create linear functions, including arithmetic sequences. • **HSA-REI.C.5, HSA-REI.C.6** Solve systems of linear equations by graphing, substitution, and elimination. • **HSA-REI.D.11** Solve linear and absolute value equations by graphing. • **HSA-CED.A.1** Create exponential equations and use them to solve real-life problems. • **HSF-IF.B.4, HSA-REI.B.4a, HSA-REI.B.4b** Solve quadratic equations by graphing, using square roots, completing the square, and the Quadratic Formula. • **HSA-REI.C.7** Solve nonlinear systems of equations graphically and algebraically. **Algebra 2** • **HSN-CN.C.7** Solve quadratic equations with real and imaginary solutions. • **HSA-REI.A.2, HSF-LE.A.4** Solve radical, exponential, logarithmic, and rational equations, including equations with extraneous solutions.

COHERENCE Through the Chapter

Standard	1.1	1.2	1.3	1.4	1.5	1.6	1.7
HSN-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.			●				
HSN-Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.			★				
HSN-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.				★			
HSA-CED.A.1 Create equations and inequalities in one variable and use them to solve problems.	●	●			●	●	
HSA-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.							●
HSA-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	●	●			●	●	
HSA-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	●	●			●	●	

Key

▲ = preparing ● = learning ★ = complete

Chapter 1 Learning Targets and Success Criteria

	Learning Target	Success Criteria
Chapter 1 Solving Linear Equations	Understand solving linear equations.	• Solve simple and multi-step equations. • Describe how to solve equations. • Analyze the measurements used to solve a problem and judge the level of accuracy appropriate for the solution. • Apply equation-solving techniques to solve real-life problems.
1.1 Solving Simple Equations	Write and solve one-step linear equations.	• Apply properties of equality to produce equivalent equations. • Solve linear equations using addition, subtraction, multiplication, or division. • Write linear equations that model real-life situations.
1.2 Solving Multi-Step Equations	Write and solve multi-step linear equations.	• Apply more than one property of equality to produce equivalent equations. • Solve multi-step linear equations using inverse operations. • Write multi-step linear equations that model real-life situations.
1.3 Modeling Quantities	Use proportional reasoning and analyze units when solving problems.	• Use ratios to solve real-life problems. • Use rates to solve real-life problems. • Convert units and rates.
1.4 Accuracy with Measurements	Choose an appropriate level of accuracy when calculating with measurements.	• Choose an appropriate level of accuracy when measuring to solve real-life problems. • Determine where to round numbers when finding estimates.
1.5 Solving Equations with Variables on Both Sides	Write and solve equations with variables on both sides.	• Apply properties of equality using variable terms. • Solve equations with variables on both sides. • Recognize when an equation has zero, one, or infinitely many solutions.
1.6 Solving Absolute Value Equations	Write and solve equations involving absolute value.	• Write the two linear equations related to a given absolute value equation. • Solve equations involving one or two absolute values. • Identify special solutions of absolute value equations.
1.7 Rewriting Equations and Formulas	Solve literal equations for given variables.	• Identify a literal equation. • Use properties of equality to rewrite literal equations. • Use rewritten formulas to solve problems.

Preparing for Chapter 1

GO DIGITAL

Chapter Learning Target Understand solving linear equations.

Chapter Success Criteria
- ◆ I can solve simple and multi-step equations.
- ◆ I can describe how to solve equations.
- ■ I can analyze the measurements used to solve a problem and judge the level of accuracy appropriate for the solution.
- ■ I can apply equation-solving techniques to solve real-life problems.

◆ Surface
■ Deep

Chapter Vocabulary

AZ VOCAB

Work with a partner. Discuss each of the vocabulary terms.

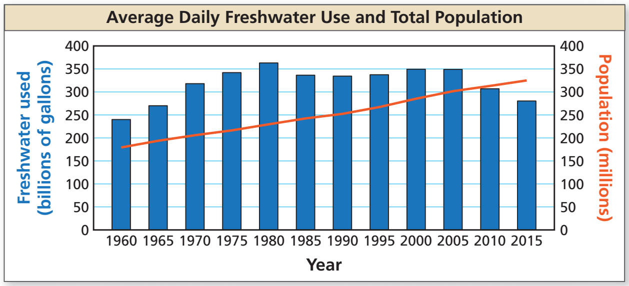
solution	ratio	rate
equivalent equations	proportion	accuracy

Mathematical Practices

Construct Viable Arguments and Critique the Reasoning of Others

Mathematically proficient students justify conclusions, communicate them to others, and respond to the arguments of others.

Work with a partner. The chart shows the population and the average amount of freshwater used daily in the United States from 1960 to 2015.



1. Make several observations about the data in the chart.
2. What conclusions can you make about water conservation efforts in the United States? Explain your reasoning to another pair.

1

Laurie's Notes

Mathematical Practices

The *Mathematical Practices* focus attention on how mathematics is learned—process versus content. The practice of constructing viable arguments and critiquing the reasoning of others (MP3) is particularly important in this chapter about solving linear equations.

ANSWERS

1. The population from 1960 to 2015 steadily increased from about 180 million to about 325 million. The average amount of freshwater used daily increased from 1960 until 1980, when it peaked. Then it remained constant until around 2005, before decreasing in 2010 and 2015.
2. *Sample answer:* As the population continued to increase, water usage leveled out and then decreased. So, water conservation efforts were successful, especially in the past few years.

Mathematical Practices

The content standards are enacted and framed by the specific expertise you help students develop to support their understanding and application of mathematics. Opportunities are embedded throughout this chapter for students to develop varieties of expertise in the mathematical practices as they learn. Here are some examples.

1. **Make Sense of Problems and Persevere in Solving Them**
   1.2 Exercise 50, page 18
2. **Reason Abstractly and Quantitatively**
   1.3 Exercise 23, page 24
3. **Construct Viable Arguments and Critique the Reasoning of Others**
   1.6 Exercise 43, page 43
4. **Model with Mathematics**
   1.4 Exercise 6, page 29
5. **Use Appropriate Tools Strategically**
   1.1 Exercise 35, page 8
6. **Attend to Precision**
   1.3 Math Practice mote, page 19
7. **Look for and Make Use of Structure**
   1.2 Math Practice note, page 13
8. **Look for and Express Regularity in Repeated Reasoning**
   1.4 Math Practice note, page 25

ANSWERS

1. -7
2. -13
3. 8
4. 4.9
5. -2
6. 1.6
7. $\frac{11}{10}$
8. $\frac{7}{3}$
9. 6
10. -24
11. 63
12. -28
13. $\frac{1}{24}$
14. -0.8
15. -0.4
16. $\frac{3}{8}$
17. 3
18. 14.875
19. a. Both numbers are positive or the number with the greater absolute value is positive; *Sample answer:* 2 and 5, or 6 and -2
    b. Both numbers have the same sign; *Sample answer:* 3 and 5, or -2 and -7
    c. Both numbers have the same sign; *Sample answer:* 10 and 2, or -8 and -4

GO DIGITAL

1 Prepare WITH CalcChat®

Adding and Subtracting Rational Numbers

WATCH **Example 1** **Evaluate 4.5 + (−12.5).**

$$4.5 + (-12.5) = -8$$

$|-12.5| > |4.5|$. So, subtract $|4.5|$ from $|-12.5|$.

Use the sign of -12.5.

WATCH **Example 2** **Evaluate $-7 - \left(-\frac{5}{2}\right)$.**

$$-7 - \left(-\frac{5}{2}\right) = -7 + \frac{5}{2} \quad \text{Add the opposite of } -\tfrac{5}{2}.$$
$$= -\frac{9}{2} \quad \text{Add.}$$

Add or subtract.

1. $-5 + (-2)$
2. $0 + (-13)$
3. $-6 + 14$
4. $1.9 - (-3)$
5. $-\frac{1}{2} - \frac{3}{2}$
6. $-5.6 - (-7.2)$
7. $\frac{4}{5} + \frac{3}{10}$
8. $\frac{8}{3} + \left(-\frac{1}{3}\right)$
9. $11.5 - 5\frac{1}{2}$

Multiplying and Dividing Rational Numbers

WATCH **Example 3** **Evaluate −3.5 • (−5).**

The numbers have the same sign.

$$-3.5 \cdot (-5) = 17.5$$

The product is positive.

WATCH **Example 4** **Evaluate $\frac{1}{5} \div (-3)$.**

The numbers have different signs.

$$\frac{1}{5} \div (-3) = -\frac{1}{15}$$

The quotient is negative.

Multiply or divide.

10. $-3(8)$
11. $-7 \cdot (-9)$
12. $4 \cdot (-7)$
13. $-\frac{1}{4} \div (-6)$
14. $-1.6 \div 2$
15. $1.2 \div (-3)$
16. $\frac{3}{4} \cdot \frac{1}{2}$
17. $\frac{4}{3} \div \frac{4}{9}$
18. $-3.5(-4.25)$
19. **MP LOGIC** Describe the signs of two rational numbers when (a) their sum is positive, (b) their product is positive, and (c) their quotient is positive. Give examples to support your answers.

Laurie's Notes

Adding and Subtracting Rational Numbers

Students may ignore the signs and just add or subtract the absolute values. Remind students of the rules for adding and subtracting integers.

Multiplying and Dividing Rational Numbers

When both factors are negative, students may write the product as negative. Remind students that the product of two numbers with the same sign is positive.

Laurie's Notes

Overview of Section 1.1

Introduction

- As you begin the school year, your students' recall and understanding of basic computational skills will become apparent. As you progress through the first few sections, it will be important to assess your students' understanding of equation solving and accuracy of computation. If an answer is incorrect, is the process correct and the computation flawed, or is the process flawed and the computation correct? Feedback to students should distinguish between these two possibilities.

- ***FOCUS* on Major Work:** Students will have prior experience solving simple equations and will recall the basic concept that you must perform the same operation on each side of an equation when solving for the variable. This section refreshes and extends the concept of equation solving by asking students to justify each step and explain that each step creates an equivalent equation.

- ***RIGOR* in the Section:** In the exploration, students develop **conceptual understanding** as they examine a solution that involves creating and solving a simple equation. The lesson provides opportunities for **procedural fluency** with examples and Self-Assessment exercises on solving one-step equations using properties of equality. An **application** example related to the Distance Formula and additional Self-Assessment exercises provide in-class practice with problem solving before homework.

- Students often say that if they use mental math and simple reasoning, they can solve the equations in this section. For instance, to solve $x - 3 = -5$, they think, "What number do you subtract 3 from to get -5?" Students need to understand that solving an equation is a process of reasoning, which they will need to solve more challenging equations.

- You want students to reason about the contexts used in the real-life applications. Reasoning about the context will help students think about how to solve the problem. For instance, in Example 3, students are told that Usain Bolt won the 200-meter dash with a time of 19.78 seconds. They are asked to find his average speed. Students are familiar with rates such as miles per hour and feet per second. So, they may reason that dividing the distance (200 meters) by the time (19.78 seconds) will yield his average speed. The reasoning is correct, but students may not make a connection to the equation $d = rt$, which involves dividing d by t to solve for r. Be sure to make this connection for students.

Formative Assessment Tip

THUMBS UP

- This technique asks students to indicate the extent to which they understand a concept, procedure, or even the directions for an activity.

I get it. I don't get it. I'm not sure.

- **FEEDBACK** This technique is a quick way to receive feedback from students. They can assess themselves and give a thumb signal to show where they are in their learning.

Section Resources

PLAN
Chapter at a Glance
Everyday Connections Video Series
Lesson Plans
Pacing Guide
Skills Review Handbook
TEACH
Answer Presentation Tool
CalcChat®
CalcView®
Differentiating the Lesson
Dynamic Classroom
Interactive Tools
Resources by Chapter*
• Warm-Up
• Extra Practice
• Reteach
• Enrichment and Extension
• Puzzle Time
Skills Trainer
Tutorial Video Series
ASSESS
Dynamic Assessment System
• Self-Assessment
• Section Practice
• Section Review & Refresh
• Point-of-use Remediation
• Reports
Formative Check
Homework App
Practice Workbook and Test Prep*
• Extra Practice
• Review & Refresh
• Self-Assessment
Self-Assessment

**Available in print*

Common Core State Standards

HSA-CED.A.1, HSA-REI.A.1, HSA-REI.B.3

Learning Target

Write and solve one-step linear equations.

Success Criteria

- Apply properties of equality to produce equivalent equations.
- Solve linear equations using addition, subtraction, multiplication, or division.
- Write linear equations that model real-life situations.

Warm-Up

Cumulative and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL SUPPORT

Students may know the word *solution* in the context of working out a resolution to a problem. In the context of mathematics, when you solve a problem, such as a simple equation, the answer is known as its solution.

Laurie's Notes

Launch the Lesson

- Begin each class with a question or situation that gets students thinking, engaging, and talking with one another. You want to motivate the learning for the day.
- ? "How much water do you use during a five-minute shower?" Answers will vary.
- ? "How did you calculate how much water you use?" Listen for students using water flow rates (e.g., 1.5 gallons per minute).
- Share the following information with students.
  - Prior to 1994, the average flow rate of showerheads was about 5.5 gallons per minute.
  - To conserve water and fuel for water heaters, the Department of Energy now limits the flow rate of showerheads to 2.5 gallons per minute.
- Explain to students that the exploration is about water flow.

EXPLORE IT!

- ? "Do you know where the Okavango Delta is located?" Students may know it is in Africa, or more specifically in the Republic of Botswana. Probe to see what students know about southern Africa or the Republic of Botswana. Share information about the Republic of Botswana, such as population and geography.
- **MP6 Attend to Precision:** As students discuss part (a), listen for titles, use of units, and references to the context. Saying, "The graph goes up and down," is not informative nor accurate. If necessary, model a better description by saying, "The water flow rate increases and decreases several times."
- The graph shows a relationship between the month and the flow rate of water in the Okavango Delta. The concepts of relations and functions are taught in Chapter 3.
- ? In part (b), you may need to scaffold the question.
  - "Where is the peak?" at 800 m^3 per sec
  - "What does a flow rate of 800 m^3 per sec mean?" The water flows at a rate of 800 cubic meters per second.
  - "If the peak is 800 m^3 per sec, how much water flows in 2 seconds? 3 seconds? 10 seconds?" 1600 m^3; 2400 m^3; 8000 m^3
- ***COHERENCE*** In part (c), ask students to explain what the work with units is about. You want students to understand that when solving real-life problems involving units, the units are part of the computation. Typically, you do not write the units in the equation-solving steps, but the answer must make sense in terms of the units that result from the unit analysis.

Where Are We In Our Learning?

"In the exploration, you looked at a problem that you could solve by reasoning. You understood which operation was necessary to solve the problem. An equation was written to represent that reasoning. The lesson looks at how you can use properties of equality to solve equations."

1.1 Solving Simple Equations

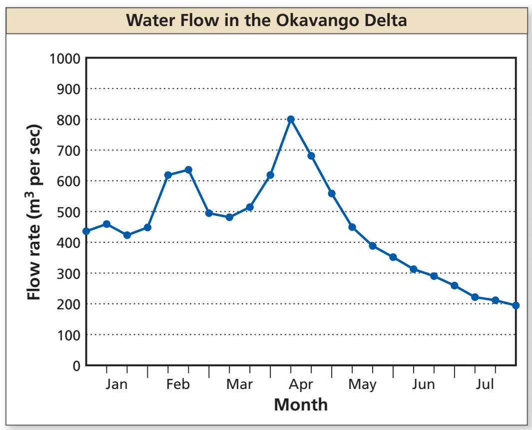
GO DIGITAL

Learning Target Write and solve one-step linear equations.

Success Criteria
- I can apply properties of equality to produce equivalent equations.
- I can solve linear equations using addition, subtraction, multiplication, or division.
- I can write linear equations that model real-life situations.

EXPLORE IT! Modeling a Real-Life Problem

Work with a partner. The Okavango Delta is the largest freshwater wetland in southern Africa and is the main source of water for one million people.



Math Practice

Use a Graph
How can you use the graph to determine the quantities involved and the relationship between the quantities?

a. What does the graph show? Make several observations from the graph.

b. When the water flow was at its peak, about how long did it take 100,000 cubic meters of water to flow past a point in the Okavango Delta? Explain your reasoning.

c. Your friend uses an equation to answer part (b) as shown. Is your friend's reasoning valid? Explain.

$$f = 800t$$
$$100{,}000 = 800t$$
$$\frac{100{,}000}{800} = t$$
$$\frac{1000}{8} = t$$
$$125 \text{ sec} = t \qquad m^3 \div \frac{m^3}{sec} = m^3 \times \frac{sec}{m^3} = sec$$

ANSWERS

a. The graph shows the flow rate from January through July in the Okavango Delta; *Sample answer:* The highest flow rate occurred in April then steadily decreased through July.

b. 125 sec; Divide the total volume of water by the flow rate to yield the time. Because the peak is at 800, divide 100,000 cubic meters by 800 cubic meters per second.

c. yes; *Sample answer:* The water flow can be found by multiplying the flow rate by the time. Using the Division Property of Equality, solve the equation to find the time. During division, the cubic-meter units divide out and the second units remain.

Laurie's Notes

Scaffolding Instruction

- ***EMERGING*** In the lesson, remind students that variables are taking the place of the unknowns. Variables are generally letters and may be upper or lower case. Before students solve an equation, encourage them to explain which operation is represented in each expression.
- ***PROFICIENT*** Students may recall how to solve one-step equations, but they may make computational errors. If an error is made, clarify whether it is an error in computation or the process.

Extra Example 1

Solve each equation. Justify each step. Check your solution.

a. $d - 4 = -2$

$d - 4 = -2$	Write the equation.
$\underline{+4} \quad \underline{+4}$	Addition Property of Equality
$d = 2$	Simplify.

b. $m + 4.8 = 9.2$

$m + 4.8 = 9.2$	Write the equation.
$\underline{-4.8} \quad \underline{-4.8}$	Subtraction Property of Equality
$m = 4.4$	Simplify.

ELL SUPPORT

Demonstrate Example 1. Then have students work in groups to discuss and complete the Self-Assessment. Have them first determine which property of equality is needed to solve each equation. Expect students to perform according to their language levels.

Beginner: Use simple phrases to discuss and write out each solution.

Intermediate: Use simple sentences to discuss the process for solving.

Advanced: Use detailed sentences and help guide discussion.

ANSWERS

1. $n = -10$; Subtract 3 from each side.
2. $g = -\frac{1}{3}$; Add $\frac{1}{3}$ to each side.
3. $p = -10.4$; Subtract 3.9 from each side.
4. yes; Subtract 4 from each side of the second equation to yield the first equation.

Solving Equations Using Addition or Subtraction

GO DIGITAL

Vocabulary (AZ VOCAB)

equation, *p. 4*
linear equation in one variable, *p. 4*
solution, *p. 4*
equivalent equations, *p. 4*

An **equation** is a statement that two expressions are equal. A **linear equation in one variable** is an equation that can be written in the form $ax + b = 0$, where a and b are constants and $a \neq 0$. When you *solve an equation*, you use properties of real numbers to find a **solution**, which is a value that makes the equation true.

Equivalent equations are equations that have the same solution(s). When you perform the same operation on each side of an equation, you produce an equivalent equation.

KEY IDEA

Addition, Subtraction, and Substitution Properties of Equality

Adding or subtracting the same number on each side of an equation produces an equivalent equation.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Substitution Property of Equality	If $a = b$, then a can be substituted for b (or b for a) in any equation or expression.

REMEMBER

Two operations that undo each other, such as addition and subtraction, are *inverse operations*. Use inverse operations to isolate the variable.

EXAMPLE 1 Solving Equations Using Addition or Subtraction

Solve each equation. Justify each step. Check your solution.

WATCH

a. $x - 3 = -5$ **b.** $0.9 = y + 2.8$

SOLUTION

a. $x - 3 = -5$ Write the equation.

Undo the subtraction. → $\underline{+3} \quad \underline{+3}$ Addition Property of Equality

$x = -2$ Simplify.

▶ The solution is $x = -2$.

Check

$x - 3 = -5$

$-2 - 3 \stackrel{?}{=} -5$

$-5 = -5$ ✓

b. $0.9 = y + 2.8$ Write the equation.

Undo the addition. → $\underline{-2.8} \quad \underline{-2.8}$ Subtraction Property of Equality

$-1.9 = y$ Simplify.

▶ The solution is $y = -1.9$.

Check

$0.9 = y + 2.8$

$0.9 \stackrel{?}{=} -1.9 + 2.8$

$0.9 = 0.9$ ✓

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Justify each step. Check your solution.

1. $n + 3 = -7$
2. $g - \frac{1}{3} = -\frac{2}{3}$
3. $-6.5 = p + 3.9$
4. **VOCABULARY** Are the equations $6x = -5$ and $-1 = 6x + 4$ equivalent? Explain your reasoning.

4 **Chapter 1** Solving Linear Equations

Laurie's Notes

- ? **DIG DEEPER** "In Example 1(a), can you use the Subtraction Property of Equality to solve the equation? Explain." Yes, subtract –3 from each side of the equation.
- **FEEDBACK** *Actively listen* as you probe student understanding of the vocabulary and *Key Idea*.
- **MP8 Look for and Express Regularity in Repeated Reasoning:** "In each example, you used a property of equality to produce an equivalent equation. Explain to a partner what this means." Listen for understanding that because an equation is an equality of two expressions, performing the same operation on each side preserves equality and produces an equivalent equation.

GO DIGITAL

Solving Equations Using Multiplication or Division

REMEMBER
Multiplication and division are inverse operations.

KEY IDEA

Multiplication and Division Properties of Equality

Multiplying or dividing each side of an equation by the same nonzero number produces an equivalent equation.

Multiplication Property of Equality If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

Division Property of Equality If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.

EXAMPLE 2 Solving Equations Using Multiplication or Division

WATCH

Solve each equation. Justify each step. Check your solution.

a. $-\frac{n}{5} = -3$ **b.** $\pi x = -2\pi$ **c.** $1.3z = 5.2$

SOLUTION

a. $-\frac{n}{5} = -3$ Write the equation.

Undo the division. → $-5 \cdot \left(-\frac{n}{5}\right) = -5 \cdot (-3)$ Multiplication Property of Equality

$n = 15$ Simplify.

▶ The solution is $n = 15$.

Check
$-\frac{n}{5} = -3$
$-\frac{15}{3} \stackrel{?}{=} -3$
$-3 = -3$ ✓

b. $\pi x = -2\pi$ Write the equation.

Undo the multiplication. → $\frac{\pi x}{\pi} = \frac{-2\pi}{\pi}$ Division Property of Equality

$x = -2$ Simplify.

▶ The solution is $x = -2$.

Check
$\pi x = -2\pi$
$\pi(-2) \stackrel{?}{=} -2\pi$
$-2\pi = -2\pi$ ✓

c. $1.3z = 5.2$ Write the equation.

Undo the multiplication. → $\frac{1.3z}{1.3} = \frac{5.2}{1.3}$ Division Property of Equality

$z = 4$ Simplify.

▶ The solution is $z = 4$.

Check
$1.3z = 5.2$
$1.3(4) \stackrel{?}{=} 5.2$
$5.2 = 5.2$ ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Justify each step. Check your solution.

5. $\frac{y}{3} = -6$ **6.** $z \div 25 = -4.5$ **7.** $9\pi = \pi x$ **8.** $0.05w = 1.4$

9. WHICH ONE DOESN'T BELONG? Which equation does not belong with the other three? Explain your reasoning.

$8 = \frac{x}{2}$ $3 = x \div 4$ $x - 6 = 5$ $\frac{x}{3} = 9$

Extra Example 2

Solve each equation. Justify each step. Check your solution.

a. $\frac{m}{5} = -4$

$\frac{m}{5} = -4$ Write the equation.

$5 \cdot \left(\frac{m}{5}\right) = 5 \cdot (-4)$ Multiplication Property of Equality

$m = -20$ Simplify.

b. $3\pi p = -12\pi$

$3\pi p = -12\pi$ Write the equation.

$\frac{3\pi p}{3\pi} = \frac{-12\pi}{3\pi}$ Division Property of Equality

$p = -4$ Simplify.

c. $2.5r = 75$

$2.5r = 75$ Write the equation.

$\frac{2.5r}{2.5} = \frac{75}{2.5}$ Division Property of Equality

$r = 30$ Simplify.

ANSWERS

5. $y = -18$; Multiply each side by 3.
6. $z = -112.5$; Multiply each side by 25.
7. $x = 9$; Divide each side by π.
8. $w = 28$; Divide each side by 0.05.
9. $x - 6 = 5$; It is the only one that involves addition and subtraction.

Laurie's Notes

- ? "Why is $c \neq 0$ in the Multiplication and Division Properties of Equality?" *Sample answer:* When multiplying, if $c = 0$, then $a \cdot c = b \cdot c$ is true when $a \neq b$. Division by 0 is undefined.
- **Math Misconception:** Students often view π as a variable and not a constant. Be sure to model a problem with π as the coefficient.
- ? Write $-\frac{n}{5} = \frac{-n}{5} = \frac{n}{-5}$ on the board and ask, "Are all three expressions equivalent? Explain." yes; *Sample answer:* The expressions are equivalent to $-1\left(\frac{n}{5}\right)$.
- ◎ Connect the examples to the first two success criteria. Be explicit. Say, "In Example 2(a), when you multiply each side of the equation by –5, you produce an equivalent equation. The equations $-\frac{n}{5} = -3$ and $n = 15$ have the same solution."

Extra Example 3

You clean a community park for 6.5 hours. You earn $43.88. Find how much you earn per hour.

about $6.75 per hour

ANSWERS

10. 3391.2 mi

11. 2012; about 0.46 sec

Solving Real-Life Problems

GO DIGITAL

KEY IDEA

Problem-Solving Plan

1. **Understand the Problem** What is the unknown? What information is given? What is being asked?
2. **Make a Plan** Decide how you will solve the problem. Your plan might involve one or more of the problem-solving strategies shown on the next page.
3. **Solve and Check** Carry out your plan. Examine your solution. Then check that your solution makes sense in the original statement of the problem.

WATCH

EXAMPLE 3 Modeling Real Life

In the 2016 Olympics, Usain Bolt won the 200-meter dash with a time of 19.78 seconds. Find his average speed to the nearest hundredth of a meter per second.

SOLUTION

1. **Understand the Problem** You know the winning time and the distance of the race. You are asked to find his average speed.
2. **Make a Plan** Use the Distance Formula to write an equation that represents the problem. Then solve the equation.
3. **Solve and Check**

$d = r \cdot t$	Write the Distance Formula.
$200 = r \cdot 19.78$	Substitute 200 for *d* and 19.78 for *t*.
$\dfrac{200}{19.78} = \dfrac{19.78r}{19.78}$	Division Property of Equality
$10.11 \approx r$	Simplify.

▶ Bolt's average speed was about 10.11 meters per second.

REMEMBER
The formula that relates distance *d*, rate or speed *r*, and time *t* is

$d = rt.$

REMEMBER
The symbol ≈ means "approximately equal to."

Check Reasonableness Round Bolt's average speed to 10 meters per second. At this speed, it would take

$$200 \text{ m} \div \frac{10 \text{ m}}{1 \text{ sec}} = 200 \cancel{\text{m}} \times \frac{1 \text{ sec}}{10 \cancel{\text{m}}} = 20 \text{ sec}$$

to run 200 meters. Because 20 is close to 19.78, your solution is reasonable.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

10. In 2015, an autonomous vehicle drove from the Golden Gate Bridge to New York City at an average speed of 15.7 miles per hour. The journey took 9 days. About how far did the vehicle travel?

11. In the 2012 Olympics, Usain Bolt ran the 200-meter dash at an average speed of about 10.35 meters per second. Was he faster in 2012 or in 2016? By how many seconds?

Laurie's Notes

- Students may quickly reason that you divide distance by time to find the rate. Connecting the steps to the problem-solving plan should not be overlooked.
- Engage students by showing a video of the race. Search the Internet for "Usain Bolt 200m final."
- **MP6 Attend to Precision:** Discuss with students the importance of labeling answers with appropriate units.

THUMBS UP Have students complete the Self-Assessment exercises and then use a thumb signal to show where they are in their learning.

KEY IDEA

Common Problem-Solving Strategies

Use a verbal model.	Guess, check, and revise.
Draw a diagram.	Sketch a graph or number line.
Write an equation.	Make a table.
Look for a pattern.	Make a list.
Work backward.	Break the problem into parts.

GO DIGITAL

EXAMPLE 4 Modeling Real Life

WATCH INFO

On January 22, 1943, the temperature in Spearfish, South Dakota, fell from 54°F at 9:00 A.M. to −4°F at 9:27 A.M. How many degrees did the temperature fall?

SOLUTION

1. **Understand the Problem** You know the temperature before and after the temperature fell. You are asked to find how many degrees the temperature fell.
2. **Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation.
3. **Solve and Check**

Verbal Model Temperature at 9:27 A.M. = Temperature at 9:00 A.M. − Number of degrees the temperature fell

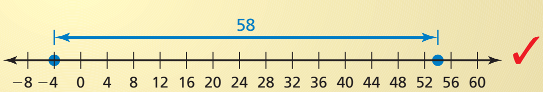
Variable Let T be the number of degrees Fahrenheit the temperature fell.

Equation $-4 \quad = \quad 54 \quad - \quad T$

$-4 = 54 - T$	Write the equation.
$-4 - 54 = 54 - 54 - T$	Subtraction Property of Equality
$-58 = -T$	Simplify.
$58 = T$	Divide each side by −1.

▶ The temperature fell 58°F.

Check The temperature fell from 54 degrees *above* 0 to 4 degrees *below* 0. You can use a number line to check your solution.



SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. You thought the balance in your savings account was $68.33, but you forgot to record a withdrawal. Your statement lists your balance as $26.33. How much was the withdrawal that you forgot to record?

13. In one year, a bluefin tuna releases 300% more eggs than an Atlantic sturgeon. The bluefin tuna releases about 10,000,000 eggs. About how many eggs does the Atlantic sturgeon release?

Extra Example 4

A discounted concert ticket is $14.50 less than the original ticket price. You pay $53.00 for a discounted ticket. What is the original ticket price? $67.50

EVERYDAY CONNECTIONS

Learn more about Step 1 of the Problem-Solving Plan, Understand the Problem.

ELL SUPPORT

Allow students to work in pairs for extra support with the language of word problems. Clarify multiple meaning words, such as *balance* and *statement*. Explain that bluefin tuna and Atlantic sturgeon are types of fish. To check comprehension, have each pair write their answers on a whiteboard to hold up for your review.

ANSWERS

12. $42
13. 2.5 million eggs

Laurie's Notes

- Make a classroom poster to display common problem-solving strategies.
- Discuss how to create the verbal model. Ask students to describe how they would solve the problem without referencing any values.
- **MP4 Model with Mathematics:** Students may suggest that drawing a number line and plotting the two temperatures helps them to quickly see the solution.
- Self-Assessment Exercise 13 involves a percent increase.

Closure

- Describe in words how to solve a one-step equation. *Sample answer:* Use the same inverse operation on each side of the equation to produce an equivalent equation.
- Solve $-13.8 = x - 4.3$. $x = -9.5$

Assignment Guide

Emerging: 1, 3, 9, 11, 13, 19, 23, 24, 25, 26, 31, 33, 38, 40, 43, 46, 50, 57

Proficient: 2, 4, 6, 10, 12, 19, 21, 27, 29, 31, 34, 35, 37, 38, 39, 41, 42, 43, 45, 50, 53, 57

Advanced: 12, 20, 22, 31, 32, 34, 36, 38, 39, 41, 44, 46, 47, 50, 51, 52, 54, 55, 57, 58

- Use the red exercises to check understanding of the learning target.
- Assign Review & Refresh exercises as appropriate for continued spaced practice.

Item Leveling

DOK	Exercises
1	1–10, 13–32
2	11, 12, 33–42, 45–48, 57
3	43, 44, 49–56, 58

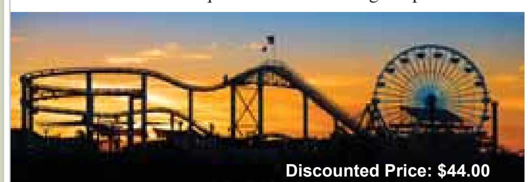
1.1 Practice WITH CalcChat® AND CalcView®

GO DIGITAL

In Exercises 1–10, solve the equation. Justify each step. Check your solution. *Example 1*

1. $x + 5 = 8$
2. $m + 9 = 2$
3. $y - 4 = 3$
4. $s - 2 = 1$
5. $w + 3 = -4$
6. $n - 6 = -7$
7. $5.2 = a - 0.4$
8. $1.7 = 3.1 + c$
9. $\frac{3}{2} + t = \frac{1}{2}$
10. $z - \frac{3}{4} = -\frac{1}{3}$

11. **MODELING REAL LIFE** An amusement park offers a ticket for \$12.95 off the original price p. Write and solve an equation to find the original price.



12. **MODELING REAL LIFE** You and a friend are playing a board game. Your final score x is 12 points less than your friend's final score. Write and solve an equation to find your final score.

	ROUND 9	ROUND 10	FINAL SCORE
Your Friend	22	12	195
You	9	25	?

In Exercises 13–22, solve the equation. Justify each step. Check your solution. *Example 2*

13. $5g = 20$
14. $4q = 52$
15. $p \div 5 = 3$
16. $y \div 7 = 1$
17. $-54 = 9s$
18. $\frac{w}{-3} = 6$
19. $-\frac{x}{6} = 1.4$
20. $-7.8 = -2.6t$
21. $-108\pi = 9\pi r$
22. $5 = \frac{h}{4\pi}$

In Exercises 23–32, solve the equation. Check your solution.

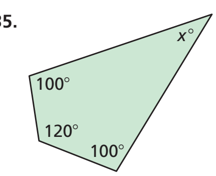
23. $-14 = p - 11$
24. $0 = 4 + q$
25. $-8r = 64$
26. $x \div (-2) = 8$
27. $\frac{3}{7}m = 6$
28. $-\frac{2}{5}y = 4$
29. $-3.8 = d \div 1.5$
30. $2a = \frac{1}{5}$
31. $f + 3\pi = 7\pi$
32. $-3\frac{1}{6} = k - \frac{2}{3}$

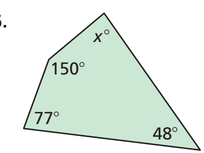
ERROR ANALYSIS In Exercises 33 and 34, describe and correct the error in solving the equation.

33. ✗
$-0.8 + r = 12.6$
$r = 12.6 + (-0.8)$
$r = 11.8$

34. ✗
$-\frac{m}{3} = -4$
$3 \cdot \left(-\frac{m}{3}\right) = 3 \cdot (-4)$
$m = -12$

MP USING TOOLS The sum of the angle measures of a quadrilateral is 360°. In Exercises 35 and 36, write and solve an equation to find the value of x. Use a protractor to check the reasonableness of your answer.

35.


36.


37. **COLLEGE PREP** A baker orders 162 eggs. Each carton contains 18 eggs. Which equation can you use to find the number x of cartons? Explain your reasoning and solve the equation.

Ⓐ $162x = 18$ Ⓑ $\frac{x}{18} = 162$

Ⓒ $18x = 162$ Ⓓ $x + 18 = 162$

ANSWERS

1. $x = 3$; Subtract 5 from each side.
2. $m = -7$; Subtract 9 from each side.
3. $y = 7$; Add 4 to each side.
4. $s = 3$; Add 2 to each side.
5. $w = -7$; Subtract 3 from each side.
6. $n = -1$; Add 6 to each side.
7. $a = 5.6$
8. $c = -1.4$
9. $t = -1$
10. $z = \frac{5}{12}$
11. $p - 12.95 = 44$; \$56.95
12. $x + 12 = 195$; 183 points
13. $g = 4$; Divide each side by 5.
14. $q = 13$; Divide each side by 4.
15. $p = 15$; Multiply each side by 5.
16. $y = 7$; Multiply each side by 7.
17. $s = -6$; Divide each side by 9.
18. $w = -18$; Multiply each side by -3.
19. $x = -8.4$; Multiply each side by -6.
20. $t = 3$; Divide each side by -2.6.
21. $r = -12$; Divide each side by 9π.
22. $h = 20\pi \approx 62.83$; Multiply each side by 4π.
23. $p = -3$; Add 11 to each side.
24. $q = -4$; Subtract 4 from each side.
25. $r = -8$; Divide each side by -8.
26. $x = -16$; Multiply each side by -2.
27. $m = 14$
28. $y = -10$
29. $d = -5.7$
30. $a = \frac{1}{10}$
31. $f = 4\pi$
32. $k = -2\frac{1}{2}$
33. Subtract -0.8 from each side, not add; $r = 12.6 - (-0.8)$; $r = 13.4$
34. Multiply each side by -3, not 3; $-3 \cdot \left(-\frac{m}{3}\right) = -3 \cdot (-4)$; $m = 12$
35. $x + 100 + 120 + 100 = 360$; $x = 40$
36. $x + 150 + 77 + 48 = 360$; $x = 85$
37. C; Multiplying the number of eggs in each carton by the number of cartons will give the total number of eggs; 9 cartons.

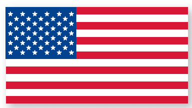
38. **MP REASONING** Are the equations equivalent? Explain.

Equation 1	$x - \frac{1}{2} = \frac{x}{4} + 3$
Equation 2	$4x - 2 = x + 12$

MODELING REAL LIFE **In Exercises 39–42, write and solve an equation to answer the question.**
Examples 3 and 4

39. A swimmer wins the 50-yard freestyle with a time of 24.76 seconds. Find the swimmer's average speed to the nearest hundredth of a yard per second.

40. The length of an American flag is 1.9 times its width. What is the width of the flag?

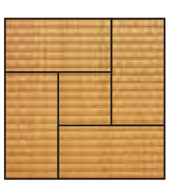


41. The temperature at 5 P.M. is 20°F. The temperature at 10 P.M. is −5°F. How many degrees did the temperature fall?

42. The balance of an investment account is \$308.32 greater than the balance 4 years ago. The current balance is \$4708.57. What was the balance 4 years ago?

43. **MP PROBLEM SOLVING** You spend \$8.64 on 12 cans of cat food. Each can costs the same amount and is on sale for 80% of the original price. The following week, the cans are no longer on sale. You have \$10. Can you buy 12 more cans? Explain your reasoning.

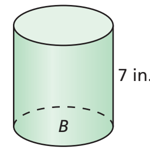
44. **DIG DEEPER** Tatami mats are used as a floor covering in Japan. One possible layout uses four identical rectangular mats and one square mat, as shown. The area of the square mat is half the area of one of the rectangular mats. The length of a rectangular mat is twice the width. Find the dimensions of one rectangular mat. Justify your answer.



CONNECTING CONCEPTS **In Exercises 45–48, find the height h or the area B of the base of the solid.**

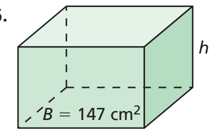
GO DIGITAL

45.



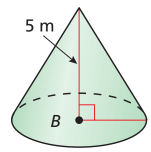
Volume = 84π in.^{3}

46.



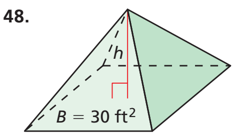
Volume = 1323 cm^3

47.



Volume = 15π m^3

48.



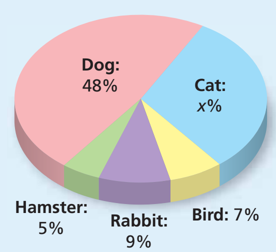
Volume = 35 ft^3

49. **MP STRUCTURE** Use the values −2, 5, 9, and 10 to complete each statement about the equation $ax = b - 5$.

a. When a = ___ and b = ___, x is a positive integer.

b. When a = ___ and b = ___, x is a negative integer.

50. **HOW DO YOU SEE IT?**
The circle graph shows the adoptions from a local animal shelter in 1 year. How does the equation $7 + 9 + 5 + 48 + x = 100$ relate to the circle graph? How can you use this equation to find the percent of adoptions that were cats?



51. **MP REASONING** One-sixth of the girls and two-sevenths of the boys in a school marching band are in the percussion section. The percussion section has 6 girls and 10 boys. How many students are in the marching band? Explain.

Slow Reveal

For Exercise 50, display the graph with the labels but not the percents. Ask students to make at least three observations about the data. For instance, about twice as many dogs were adopted as hamsters, rabbits, and birds combined.

ANSWERS

38. yes; Multiplying each side of Equation 1 by 4 yields Equation 2.

39. $24.76s = 50$; 2.02 yd/sec

40. $9.5 = 1.9w$; 5 ft

41. $-5 = 20 - T$; 25°F

42. $4708.57 = b + 308.32$; \$4400.25

43. no; 12 cans cost $8.64 \div 0.80 = \$10.80$ and $\$10.80 > \$10$.

44. 6 ft by 3 ft; Let w = width and $2w$ = length.
Area $= (w + 2w)^2$
$= (3w)^2$
$= 9w^2$
$= 81$.
So, $w = 3$ and $2w = 6$.

45. $B = 12\pi$ in.^{2}

46. $h = 9$ cm

47. $B = 9\pi$ m^2

48. $h = 3.5$ ft

49. **a.** 5; 10
b. −2; 9

50. It shows the sum of all the parts is 100%; Solve it for x; $x = 31$

51. 71; Because $\frac{1}{6}$ of the girls is 6, there are 36 girls. Because $\frac{2}{7}$ of the boys is 10, there are 35 boys; $36 + 35 = 71$

Mini-Assessment

Solve the equation. Check your solution.

1. $t + 17 = 3$ $t = -14$
2. $-2\pi + d = -3\pi$ $d = -\pi$
3. $-13.5 = 2.7s$ $s = -5$
4. $\frac{2}{3}j = 8$ $j = 12$
5. You earn $9.65 per hour. This week, you earned $173.70 before taxes. Write and solve an equation to find the number of hours you worked this week.
   $9.65x = 173.70$; 18 h

ANSWERS

52. increase; decrease; stay the same; increase
53. $x = -\frac{7}{b}$; $b < 0$
54. $x = \frac{3}{4} - a$; $a < \frac{3}{4}$
55. $x = -6.5c$; $c < 0$
56. $x = -\frac{ab}{c}$; $\frac{ab}{c} < 0$
57. **a.** 132 hits
    b. no; Player B has $132 - 33 = 99$ hits. $\frac{99}{x} = 0.296$, $x \approx 334$ at-bats, and $334 < 446$
58. $N = -1.5$
59. $\frac{4}{15}$
60. $1\frac{5}{12}$
61. $\frac{5}{6}$
62. $3\frac{2}{21}$
63. 7
64. 150, 2, $\frac{1}{4}$; $50 : \frac{1}{2}$, $150 : \frac{3}{2}$, $200 : 2$, $25 : \frac{1}{4}$
65. $-3.6x - 10$
66. $-24m + 28$
67. $2y + 4$
68. 9 arrangements
69. $0.\overline{7}$, $77.\overline{7}\%$
70. $10.99 + 1.5n$; $15.49
71. $4x + 5$
72. $x = -12$; Subtract 7 from each side.
73. $b = -27$; Multiply each side by -9.
74. $t = 2.5$; Divide each side by -1.8.
75. $w = -\frac{7}{12}$; Add $\frac{1}{4}$ to each side.
76. 3.2

52. ANALYZING RELATIONSHIPS As c increases, does the value of x *increase*, *decrease*, or *stay the same* for each equation? Assume c is positive.

Equation	Value of x
$x - c = 0$	
$cx = 1$	
$cx = c$	
$\frac{x}{c} = 1$	

MP REASONING In Exercises 53–56, the letters a, b, and c represent nonzero constants. Solve the equation for x. Then find values of a, b, and c for which the solution is positive.

53. $bx = -7$

54. $x + a = \frac{3}{4}$

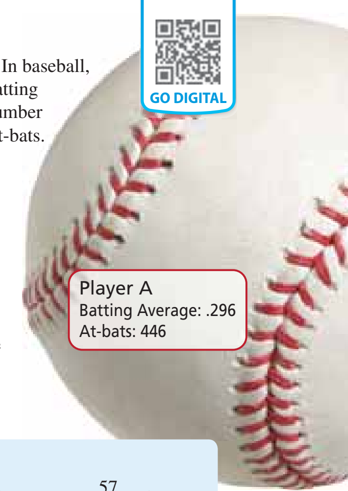
55. $-\frac{x}{c} = 6.5$

56. $\frac{c}{a}x = -b$

57. MAKING AN ARGUMENT In baseball, you calculate a player's batting average by dividing the number of hits by the number of at-bats.

a. How many hits does Player A have?

b. Player B has 33 fewer hits than Player A but has a greater batting average. Your friend concludes that Player B has more at-bats than Player A. Is your friend correct? Explain.



GO DIGITAL

58. THOUGHT PROVOKING

Find the value of N such that $x - N = \frac{57}{10}$ and $\frac{x}{N} = -2.8$ are equivalent equations.

REVIEW & REFRESH

WATCH

In Exercises 59–62, multiply or divide.

59. $\frac{3}{5} \cdot \frac{4}{9}$

60. $2\frac{1}{8} \cdot \frac{2}{3}$

61. $\frac{3}{4} \div \frac{9}{10}$

62. $4\frac{1}{3} \div 1\frac{2}{5}$

63. Evaluate $15 - 6(7 + 5) \div 3^2$.

64. Find the missing values in the ratio table. Then write the equivalent ratios.

Calories	50		200	25
Servings	$\frac{1}{2}$	$\frac{3}{2}$		

In Exercises 65–67, simplify the expression.

65. $-5.9x - 4 + 2.3x - 6$

66. $4(-6m + 7)$

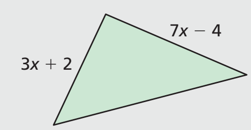
67. $-\frac{1}{3}(9y - 12) + 5y$

68. MODELING REAL LIFE You have 63 red roses and 45 white roses to make floral arrangements. Each arrangement must be identical. What is the greatest number of arrangements you can make using every flower?

69. Write $\frac{7}{9}$ as a decimal and a percent.

70. MODELING REAL LIFE A pizza shop charges $10.99 for a large cheese pizza and $1.50 for each topping. Write an expression that represents the cost (in dollars) of a large pizza with n toppings. How much does a large three-topping pizza cost?

71. The expression $14x + 3$ represents the perimeter of the triangle. What is the length of the third side?



In Exercises 72–75, solve the equation. Justify each step. Check your solution.

72. $7 + x = -5$

73. $-\frac{b}{9} = 3$

74. $-1.8t = -4.5$

75. $w - \frac{1}{4} = -\frac{5}{6}$

76. Find the mean of the data.

Data usage (gigabytes)			
2.5	1.7	3.6	5.4
3.2	1.5	1.8	2.8
4.8	3.5	3.1	4.5

Laurie's Notes

Overview of Section 1.2

Introduction

- **MP1 Make Sense of Problems and Persevere in Solving Them & MP2 Reason Abstractly and Quantitatively:** To make sense of solving multi-step equations, students must understand what the symbols and operations represent. When they evaluate expressions, they follow the order of operations. When they solve equations, they undo that process by performing the inverse operations in a reverse order. For example:

Evaluate $3x - 4$ when $x = 5$.		Solve $3x - 4 = 11$.	
$3(5) - 4$		$3x = 15$	$+ 4$
$15 - 4$	$\times 3$	$x = 5$	$\div 3$
11	$- 4$		

Mathematically proficient students make sense of this relationship and perform symbolic manipulations to solve for the variable.

- ***FOCUS* on Major Work:** Students will have prior experience solving multi-step equations and will recall the basic concept that you must perform the same operations on each side of an equation when solving for the variable. This section refreshes and extends students' understanding of equation solving by asking students to justify the process and use equations to solve real-life problems.
- ***RIGOR* in the Section:** In the exploration, students develop **conceptual understanding** as they compare different methods of solving an equation. The lesson provides opportunities for **procedural fluency** with examples and Self-Assessment exercises on solving multi-step equations using properties of equality. Two **application** examples and additional Self-Assessment exercises provide in-class practice with problem solving before homework.
- The problems students will solve throughout this section may require combining *like terms*. Combining like terms and simplifying expressions, along with solving equations, are foundational skills that can be worked on during the lesson. Do not rush!
- Students often ask what score they need to earn on a quiz or test to have a particular overall average in a course. A related context is presented in Example 4. Be sure to make this connection.

Formative Assessment Tip

TURN AND TALK

- This technique allows all students in the class to have a voice. Using a three-foot voice, students turn and talk to their partners about a problem or discuss a question. There may be different roles that I ask partners to assume, so I refer to Partner A and Partner B. In discussing a procedure or explaining an answer, I might ask Partner A to talk uninterrupted for a fixed period of time. Then Partner B might be asked to repeat back what he or she heard, or to ask a question about what has been shared.
  Example: "*Turn and Talk*, so that Partner A explains how solving $5 - 2x = -8$ differs from solving $2x - 5 = -8$."
- It is important to establish norms: three-foot voices should be expected when students are doing partner work. Discuss with students the difference between *authentic listening* and just being quiet while your partner is speaking.
- I also refer to *elbow partners*, which suggests that students are sitting in close proximity to one another.

Section Resources

PLAN
Chapter at a Glance
Everyday Connections Video Series
Lesson Plans
Pacing Guide
Skills Review Handbook
TEACH
Answer Presentation Tool
CalcChat®
CalcView®
Differentiating the Lesson
Dynamic Classroom
Interactive Tools
Resources by Chapter*
• Warm-Up
• Extra Practice
• Reteach
• Enrichment and Extension
• Puzzle Time
Skills Trainer
Tutorial Video Series
ASSESS
Dynamic Assessment System
• Self-Assessment
• Section Practice
• Section Review & Refresh
• Point-of-use Remediation
• Reports
Formative Check
Homework App
Practice Workbook and Test Prep*
• Extra Practice
• Review & Refresh
• Self-Assessment
Self-Assessment

**Available in print*

Common Core State Standards

HSA-CED.A.1, HSA-REI.A.1, HSA-REI.B.3

Learning Target

Write and solve multi-step linear equations.

Success Criteria

- Apply more than one property of equality to produce equivalent equations.
- Solve multi-step linear equations using inverse operations.
- Write multi-step linear equations that model real-life situations.

Warm-Up

Cumulative and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL SUPPORT

Discuss the meaning of the word *multi-step*. Point out that the word *step* refers to one movement of a foot when walking, or one level of a staircase. It also refers one event or action within a process. The prefix *multi-* means "many." So, a multi-step equation is an equation that requires more than one action (or operation) to solve it.

Laurie's Notes

Launch the Lesson

- Make a card for each student in your class. Write a variable term on each card. Students will walk around to find others with a card containing a like term to the one they are holding.

  Samples: $5x, -13x, 3x^2, 5y, 6xy, x, 3.8x, \frac{1}{2}y, -3.8y, 5x^2, xy$

- **FEEDBACK** Observe and listen as students find their matches. Students may be giving incorrect feedback to one another. For instance, a student may tell another student that $5x$ and $5x^2$ are like terms because they both have a coefficient of 5, or because they both have an x. What understanding of like terms do students have from middle school?

? "What does it mean for terms to be *like terms*?" The terms have the same variables raised to the same exponents.

- "The equations you solve in this section may have like terms."

EXPLORE IT!

- The exploration presents a context for students to reason about solving multi-step problems.
- Most students understand that inverse operations are used to solve equations. When the equation involves more than one operation, students are less confident in knowing where to begin, meaning which operation to undo first. This is explored in part (e).

? Introduce the exploration and tell students to look at the paycheck. "What are you curious about?" Answers will vary. Hopefully, a student will mention how many hours were worked to earn $286.65.

- In part (b), be sure to listen to the reasoning of several students.
- Allow sufficient time for students to decide what each part of the equation represents in part (d). Putting it all together will help students describe a verbal model.
- In part (e), when the first step is to divide each side of the equation by 9.75, students may forget to divide 39 by 9.75. Remind students to divide each term in the equation by 9.75. Alternatively, students may divide each factor in $9.75(h + 6)$ by 9.75. Remind students to only divide the product by 9.75 one time.

Where Are We In Our Learning?

"In parts (a)–(c), you looked at problems that you could solve by reasoning. You understood which operations were necessary to solve the problems. Equations can be written to represent your thinking. In the lesson, you will use properties of equality to solve multi-step equations."

1.2 Solving Multi-Step Equations

GO DIGITAL

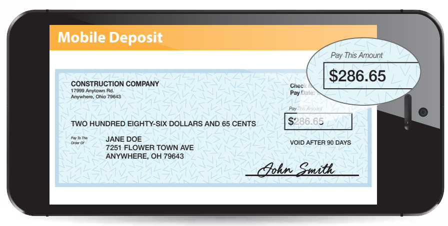
Learning Target Write and solve multi-step linear equations.

Success Criteria

- I can apply more than one property of equality to produce equivalent equations.
- I can solve multi-step linear equations using inverse operations.
- I can write multi-step linear equations that model real-life situations.

EXPLORE IT! Solving a Real-Life Problem

Work with a partner. You earn \$9.75 per hour at your part-time job. Your paycheck for last week is shown.



a. How many hours did you work last week? Explain how you found your answer.

b. This week you earn the same amount as last week, but that amount includes \$39 that you earn babysitting. Without solving, determine whether you work more hours at your part-time job this week than last week. Explain your reasoning.

c. Find the number of hours you work this week at your part-time job. Show two different ways to solve.

d. The equation below represents the amount of money you will earn next week. Let h represent the number of hours you work this week.

$$9.75(h + 6) + 39 = 345.15$$

Explain what each part of the equation represents.

i. $h + 6$ **ii.** $9.75(h + 6)$ **iii.** 39

e. Solve the equation in part (d) three different ways using each of the following as the first step.

i. Subtract 39 from each side.

ii. Subtract 345.15 from each side.

iii. Divide each side by 9.75.

Compare the solution methods. Is one solution method more efficient than the other solution methods? Explain your reasoning.

Math Practice

Maintain Oversight
Does it matter which step you perform first when solving?

ANSWERS

a. 29.4 h; Divide \$286.65 by \$9.75 per hour to find the total hours worked.

b. You work less hours this week because the amount you earn at your part-time job is \$39 less that last week.

c. 25.4 h; Solve the equation $286.65 = 9.75h + 39$ for h. Another way is to find how many fewer hours you work this week: $39 \div 9.75 = 4$ and $29.4 - 4 = 25.4$.

d. i. the number of hours you will work

ii. the amount you earn at your job

iii. the amount you earn babysitting

e. i–iii. $h = 25.4$; *Sample answer:* The first solution method can be solved in fewer steps.

Laurie's Notes

Scaffolding Instruction

- ***EMERGING*** In solving an equation like $5 - 2x = -8$, students see the subtraction sign and conclude that they need to "undo" the operation by adding. Ask probing questions so students recognize that the variable term is being subtracted from the constant versus the constant being subtracted from the variable term. Rewrite a subtraction expression using the related addition problem. For example, $5 - 2x = -8$ can be rewritten as $5 + (-2x) = -8$.
- ***PROFICIENT*** If students recall solving multi-step equations, use Self-Assessment Exercises 1–5 to see where they are in their learning.

Extra Example 1

Solve $15 = 1.5r + 3$. Check your solution.
$r = 8$

Extra Example 2

Solve $5f + 4f - 8 = 19$. Check your solution. $f = 3$

ELL SUPPORT

Demonstrate Examples 1 and 2. Clarify that the phrase *like terms* refers to terms that have the same variables raised to the same exponents. Then have students work in groups to discuss and complete the Self-Assessment. Provide guiding questions such as, "Which terms are like terms? Which properties will you use?" Expect students to perform according to their language levels.

Beginner: Use simple phrases to discuss and write out each solution.

Intermediate: Use simple sentences to discuss the process for solving.

Advanced: Use detailed sentences and help guide discussion.

ANSWERS

1. $n = -3$
2. $c = -21$
3. $x = -\frac{1}{2}$
4. like terms
5. Subtract 15 from each side of the equation $-12 = 9x - 6x + 15$ to yield $-27 = 3x$. Because the equations are equivalent, they have the same solution.

Solving Multi-Step Linear Equations

GO DIGITAL

KEY IDEA

Solving Multi-Step Equations

To solve a multi-step equation, simplify each side of the equation, if necessary. Then use inverse operations to isolate the variable.

EXAMPLE 1 Solving a Two-Step Equation

WATCH

Solve $2.5x - 13 = 2$. Check your solution.

SOLUTION

	$2.5x - 13 = 2$	Write the equation.
Undo the subtraction.	$+13 \quad +13$	Addition Property of Equality
	$2.5x = 15$	Simplify.
Undo the multiplication.	$\frac{2.5x}{2.5} = \frac{15}{2.5}$	Division Property of Equality
	$x = 6$	Simplify.

▶ The solution is $x = 6$.

Check

$2.5x - 13 = 2$

$2.5(6) - 13 \stackrel{?}{=} 2$

$2 = 2$ ✓

EXAMPLE 2 Combining Like Terms to Solve an Equation

WATCH

Solve $-12 = 9x - 6x + 15$. Check your solution.

SOLUTION

	$-12 = 9x - 6x + 15$	Write the equation.
	$-12 = 3x + 15$	Combine like terms.
Undo the addition.	$-15 \quad -15$	Subtraction Property of Equality
	$-27 = 3x$	Simplify.
Undo the multiplication.	$\frac{-27}{3} = \frac{3x}{3}$	Division Property of Equality
	$-9 = x$	Simplify.

▶ The solution is $x = -9$.

Check

$-12 = 9x - 6x + 15$

$-12 \stackrel{?}{=} 9(-9) - 6(-9) + 15$

$-12 = -12$ ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Check your solution.

1. $-2n + 3 = 9$
2. $-21.5 = \frac{1}{2}c - 11$
3. $-2x - 10x + 12 = 18$
4. **COMPLETE THE SENTENCE** To solve the equation $2x + 3x + 5 = 20$, your friend first combines $2x$ and $3x$ because they are __________.
5. **MP REASONING** In Example 2, explain how you know that $-12 = 9x - 6x + 15$ and $-27 = 3x$ have the same solution.

Laurie's Notes

- In solving multi-step equations, the goal is to isolate the variable. Have students *Turn and Talk* to their neighbors. Partner A explains what "isolating the variable" means and how it is done. Select a Partner B to summarize for the class what he or she heard.
- ? Write $2.5x - 13 = 2$ and $2 = 2.5x - 13$ and ask, "Can you solve these equations in the same way? Explain." yes; *Sample answer:* Add 13 to each side and then divide each side by 2.5.
- ? **DIG DEEPER** "Can you solve $-12 = 9x - 6x + 15$ and $-12 = 9x + 15 - 6x$ in the same way? Explain." yes; *Sample answer:* Subtract 15 from each side, combine like terms, and then divide each side by 3.

EXAMPLE 3 Using Structure to Solve a Multi-Step Equation

WATCH

GO DIGITAL

Solve $2(1 - x) + 3 = -8$. Check your solution.

SOLUTION

Method 1 One way to solve the equation is by using the Distributive Property.

$2(1 - x) + 3 = -8$	Write the equation.
$2(1) - 2(x) + 3 = -8$	Distributive Property
$2 - 2x + 3 = -8$	Multiply.
$-2x + 5 = -8$	Combine like terms.
$\underline{-5} \quad \underline{-5}$	Subtraction Property of Equality
$-2x = -13$	Simplify.
$\frac{-2x}{-2} = \frac{-13}{-2}$	Division Property of Equality
$x = 6.5$	Simplify.

▶ The solution is $x = 6.5$.

Check

$2(1 - x) + 3 = -8$

$2(1 - 6.5) + 3 \stackrel{?}{=} -8$

$-8 = -8$ ✓

REMEMBER

The Distributive Property states the following for real numbers a, b, and c.

Sum

$a(b + c) = ab + ac$

Difference

$a(b - c) = ab - ac$

Method 2 Another way to solve the equation is by interpreting the expression $1 - x$ as a single quantity.

$2(1 - x) + 3 = -8$	Write the equation.
$\underline{-3} \quad \underline{-3}$	Subtraction Property of Equality
$2(1 - x) = -11$	Simplify.
$\frac{2(1 - x)}{2} = \frac{-11}{2}$	Division Property of Equality
$1 - x = -5.5$	Simplify.
$\underline{-1} \quad \underline{-1}$	Subtraction Property of Equality
$-x = -6.5$	Simplify.
$\frac{-x}{-1} = \frac{-6.5}{-1}$	Division Property of Equality
$x = 6.5$	Simplify.

▶ The solution is $x = 6.5$.

Math Practice

Look for Structure

Explain why it is convenient to first solve for the expression $1 - x$, and then solve for x. How else could you solve the equation?

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Check your solution.

6. $3(x + 1) + 6 = -9$

7. $17 = 7 + 4(2.2d - 8.5)$

8. $13 = -2(y - 4) + 3y$

9. $2x(5 - 3) - 3x = 5$

10. $-4(2m + 5) - \frac{3}{4}m = 22$

11. $5(3 - x) + 2(3 - x) = 14$

12. MP **REASONING** Solve $2(4x - 11) = 10$ in as many ways as you can. Construct a viable argument to justify each of your solution methods.

Extra Example 3

Solve $4(n - 6) - 1 = 31$. Check your solution. $n = 14$

ANSWERS

6. $x = -6$

7. $d = 5$

8. $y = 5$

9. $x = 5$

10. $m = -\frac{24}{5}$

11. $x = 1$

12. $x = 4$; Use the Distributive Property to simplify, then solve for x; Solve for the expression $4x - 11$, then solve for x.

Laurie's Notes

- **MP7 Look for and Make Use of Structure:** Two methods are shown for solving the equation. Students might not think to divide by 2 as the second step in Method 2 because a decimal is introduced. The focus needs to be on seeing the structure—treating $(1 - x)$ as a term that is being isolated.

? DIG DEEPER "Could the first step in solving the equation be dividing each side by 2? Explain." yes; *Sample answer:* The Division Property of Equality can justify division as the first step. $[2(1 - x) + 3] \div 2 = -8 \div 2$.

- In Method 1, when simplifying $2 - 2x + 3$, students may subtract 3 from 2 and write $2x - 1$. Rewriting $2 - 2x + 3$ as $2 + (-2x) + 3$ will help.

Extra Example 4

Use the table to find what the high temperature on Friday needs to be so that the mean high temperature for the 5 days is 65°F.

Day	High temperature (°F)
Monday	76°F
Tuesday	70°F
Wednesday	68°F
Thursday	55°F

56°F

ANSWERS

13. 68 lb/in.^{2}

GO DIGITAL

Solving Real-Life Problems

EXAMPLE 4 **Modeling Real Life** WATCH

Use the table to find the number of miles you need to bike on Friday so that the mean number of miles biked per day is 5.

Day	Miles
Monday	3.5
Tuesday	5.5
Wednesday	0
Thursday	5

SOLUTION

1. **Understand the Problem** You know how many miles you biked Monday through Thursday. You are asked to find the distance you need to bike on Friday so that the mean number of miles biked per day is 5.
2. **Make a Plan** Use the definition of mean to write an equation that represents the problem. Then solve the equation.
3. **Solve and Check** The mean of a data set is the sum of the data divided by the number of data values. Let x be the number of miles you need to bike on Friday.

$$\frac{3.5 + 5.5 + 0 + 5 + x}{5} = 5 \qquad \text{Write the equation.}$$

$$\frac{14 + x}{5} = 5 \qquad \text{Combine like terms.}$$

$$5 \cdot \frac{14 + x}{5} = 5 \cdot 5 \qquad \text{Multiplication Property of Equality}$$

$$14 + x = 25 \qquad \text{Simplify.}$$

$$-14 \qquad -14 \qquad \text{Subtraction Property of Equality}$$

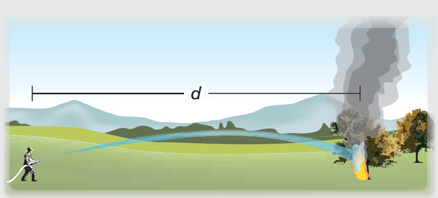
$$x = 11 \qquad \text{Simplify.}$$

▶ You need to bike 11 miles on Friday.

Check Reasonableness Notice that on the days that you did bike, the values are close to the mean. Because you did not bike on Wednesday, you need to bike about twice the mean on Friday. Eleven miles is about twice the mean. So, your solution is reasonable.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

13. The formula $d = \frac{1}{2}n + 26$ relates the nozzle pressure n (in pounds per square inch) of a fire hose and the maximum horizontal distance d (in feet) the water reaches. How much pressure is needed to reach a fire 20 yards away?



Laurie's Notes

- **Note:** This is a classic question. When all of the data are known except for one, what is needed to achieve a particular average? Students often ask this in the context of wanting to know what they have to score on a test to achieve a certain average.
- ? "How do you find the mean of five numbers?" Sum the numbers and divide by 5. Use this response to help students see the importance of understanding the problem, which is the first step of the problem-solving plan.
- It may be helpful to write the third step with parentheses: $5\left(\frac{14 + x}{5}\right)$.

EXAMPLE 5 Modeling Real Life

WATCH INFO

GO DIGITAL

One person buys a used car in Indiana and pays $10,195, including 7% sales tax and $425 in additional fees. Another person buys a used car in Pennsylvania and pays $9995, including 6% sales tax and $420 in additional fees. Compare the list prices of the used cars. (The list price is the price of the car before sales tax and fees.)

SOLUTION

1. **Understand the Problem** You know how much each person pays for a car. You also know the sales tax and additional fees in each state. You are asked to compare the list prices of the cars.
2. **Make a Plan** Use a verbal model to write equations that represent the total amount each person pays. Then solve the equations to find each list price.
3. **Solve and Check**

Verbal Model List price + Sales tax rate (written as a decimal) • List price + Other fees = Total amount paid

Variable Let p be the list price (in dollars) of the used car.

Equations

Indiana:		Pennsylvania:
$p + 0.07p + 425 = 10{,}195$	Write the equation.	$p + 0.06p + 420 = 9995$
$1.07p + 425 = 10{,}195$	Combine like terms.	$1.06p + 420 = 9995$
$-425 \quad -425$	Subtraction Property of Equality	$-420 \quad -420$
$1.07p = 9770$	Simplify.	$1.06p = 9575$
$\frac{1.07p}{1.07} = \frac{9770}{1.07}$	Division Property of Equality	$\frac{1.06p}{1.06} = \frac{9575}{1.06}$
$p \approx 9130.84$	Simplify.	$p \approx 9033.02$

▶ So, the list price of the car in Indiana is about $9130.84 − $9033.02 = $97.82 more than the list price of the car in Pennsylvania.

Check

$p + 0.07p + 425 = 10{,}195$

$9130 + 0.07(9130) + 425 \overset{?}{\approx} 10{,}195$

$10{,}194.10 \approx 10{,}195$ ✓

$p + 0.06p + 420 = 9995$

$9030 + 0.06(9030) + 420 \overset{?}{\approx} 9995$

$9991.80 \approx 9995$ ✓

REMEMBER

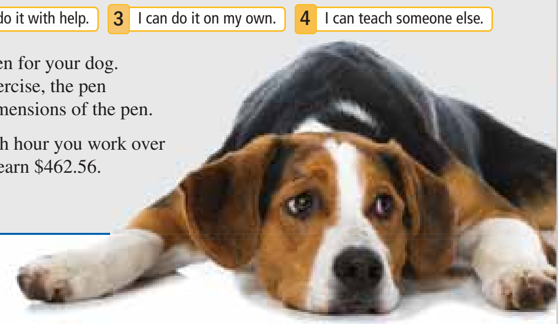
To write a percent as a decimal, remove the percent symbol and divide by 100.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

14. You have 96 feet of fencing to enclose a rectangular pen for your dog. To provide sufficient running space for your dog to exercise, the pen should be three times as long as it is wide. Find the dimensions of the pen.

15. You are paid 1.2 times your normal hourly rate for each hour you work over 40 hours in a week. You work 46 hours this week and earn $462.56. What is your normal hourly rate?



Extra Example 5

You eat a meal at a restaurant in New York and pay $14.15, including 8% sales tax and a $2.00 tip. Your friend eats a meal at a restaurant in Ohio and pays $14.18, including 6% sales tax and a $1.90 tip. Compare the menu prices of the meals. (The menu price is the price of a meal before sales tax and tip.) The menu price of your friend's meal is about $0.33 more than the menu price of your meal.

EVERYDAY CONNECTIONS Learn more about writing a verbal model for Example 5.

ELL SUPPORT

Allow students to work in pairs for extra support with the language of word problems. Clarify that the word *feet* refers to a measurement, not to body parts, and that the word *pen* refers to an enclosure for a dog, not a writing instrument. Have each pair describe the process they used to find each solution.

ANSWERS

14. 36 ft by 12 ft

15. $9.80

Laurie's Notes

? **MP1 Make Sense of Problems and Persevere in Solving Them:** Discuss the meaning of sales tax because not all states have a sales tax. Ask, "Why might there be additional fees when you buy a car?" Some students may be familiar with destination fees or special packages that can be purchased. Students must first understand the context to make sense of the problem.

- Guide students with questions to help them write the verbal model. Do not bypass this step! Students need to see how the verbal model describes the problem, without worrying about the values.

◎ Tell students that Examples 4 and 5 represent the last success criterion. Have students complete the Self-Assessment to see where they are in their learning.

Closure

Solve $8x + 9 - 4x = 25$. Check your solution. $x = 4$

Assignment Guide

Emerging: 1, 4, 5, 10, 13, 17, 23, 31, 33, 36, 39, 42, 44

Proficient: 2, 3, 5, 11, 14, 18, 24, 29, 32, 34, 36, 40, 42, 44, 49

Advanced: 8, 12, 21, 26, 30, 32, 34, 35, 38, 40, 42, 44, 45, 47, 50

- Use the red exercises to check understanding of the learning target.
- Assign Review & Refresh exercises as appropriate for continued spaced practice.

Item Leveling

DOK	Exercises
1	1–12, 15–28, 39, 40
2	13, 14, 29–38, 41, 43
3	42, 44–50

Slow Reveal

For Exercise 14, tell students that the repair bill for a car is \$648.45. Discuss what this means. Do they know the bill represents the sum of the cost of the parts and the labor, and that labor is charged by the hour? This discussion will help them understand how to write the equation. Next, share that the parts cost \$265.95. “What do you know now?” The sum of \$265.95 and the cost of the labor is \$648.45. “Is this enough information to find the number of hours of labor spent repairing the car? Explain.” No, you need to know the hourly rate for labor.

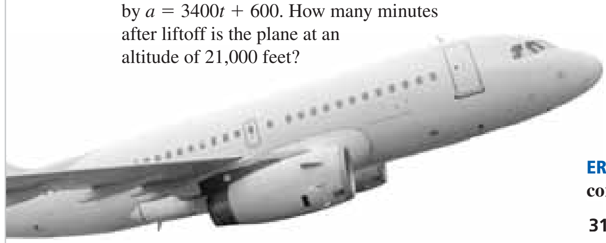
1.2 Practice WITH CalcChat® AND CalcView®

GO DIGITAL

In Exercises 1–12, solve the equation. Check your solution. *Examples 1 and 2*

1. $3w + 7 = 19$
2. $2g - 13 = 3$
3. $11 = 12 - q$
4. $10 = 7 - m$
5. $5 = \frac{z}{-4} - 3$
6. $\frac{a}{3} + 4 = 6$
7. $\frac{h + 6}{5} = 2$
8. $\frac{d - 8}{-2} = 12$
9. $12v + 10v + 14 = 80$
10. $24 = 13n - 4n + 9$
11. $3.8y + 5.6y - 2 = 2.7$
12. $\frac{7}{10}c - 8 - \frac{1}{2}c = -16$

13. **MODELING REAL LIFE** The altitude a (in feet) of a plane t minutes after liftoff is given by $a = 3400t + 600$. How many minutes after liftoff is the plane at an altitude of 21,000 feet?



14. **MODELING REAL LIFE** A repair bill for a car is \$648.45. The parts cost \$265.95. The labor cost is \$85 per hour. Write and solve an equation to find the number of hours of labor spent repairing the car.

In Exercises 15–22, solve the equation. Check your solution. *Example 3*

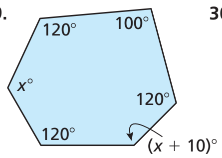
15. $4(z + 5) = 32$
16. $-2(4g - 3) = 30$
17. $6 + 5(m + 1) = 26$
18. $5h + 2(11 - h) = -5$
19. $-15 = -6(3 + x) + 4(x - 6)$
20. $1 = 5(r + 9) - 2(1 - r)$
21. $83.8 = 8.6c - 7.3(6 - 2c)$
22. $3y - 2\frac{3}{4}\left(\frac{1}{2}y - 4\right) = -2$

MP NUMBER SENSE In Exercises 23–28, write and solve an equation to find the number.

23. The sum of twice a number and 13 is 75.
24. The difference of three times a number and 4 is −19.
25. Eight plus the quotient of a number and 3 is −2.
26. The sum of twice a number and half the number is 10.
27. Six times the sum of a number and 15 is −42.
28. Four times the difference of a number and 7 is 12.

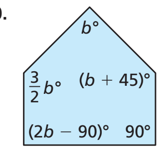
MP USING TOOLS In Exercises 29 and 30, find the value of the variable. Then find the angle measures of the polygon. Use a protractor to check the reasonableness of your answer.

29.



Sum of angle measures: 720°

30.



Sum of angle measures: 540°

ERROR ANALYSIS In Exercises 31 and 32, describe and correct the error in solving the equation.

31. ✗

$$-2(7 - y) + 4 = -4$$
$$-14 - 2y + 4 = -4$$
$$-10 - 2y = -4$$
$$-2y = 6$$
$$y = -3$$

32. ✗

$$\frac{1}{4}(x - 2) + 4 = 12$$
$$\frac{1}{4}(x - 2) = 8$$
$$x - 2 = 2$$
$$x = 4$$

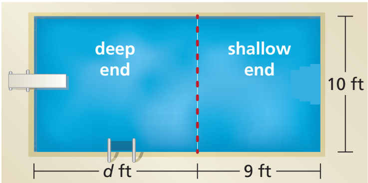
ANSWERS

1. $w = 4$
2. $g = 8$
3. $q = 1$
4. $m = -3$
5. $z = -32$
6. $a = 6$
7. $h = 4$
8. $d = -16$
9. $v = 3$
10. $n = \frac{5}{3}$
11. $y = 0.5$
12. $c = -40$
13. 6 min
14. $648.45 = 265.95 + 85h$; 4.5 h
15. $z = 3$
16. $g = -3$
17. $m = 3$
18. $h = -9$
19. $x = -\frac{27}{2}$
20. $r = -6$
21. $c = 5.5$
22. $y = -8$
23. $2n + 13 = 75$; $n = 31$
24. $3n - 4 = -19$; $n = -5$
25. $8 + \frac{n}{3} = -2$; $n = -30$
26. $2n + \frac{1}{2}n = 10$; $n = 4$
27. $6(n + 15) = -42$; $n = -22$
28. $4(n - 7) = 12$; $n = 10$
29. $x = 125$; 120°, 100°, 120°, 135°, 120°, 125°
30. $b = 90$; 90°, 135°, 90°, 90°, 135°
31. When using the Distributive Property in the second step, the second term should be positive; $-14 + 2y + 4 = -4$; $-10 + 2y = -4$; $2y = 6$; $y = 3$
32. In the third step, the right side should be 8×4, not $8 \div 4$; $x - 2 = 32$; $x = 34$

MODELING REAL LIFE **In Exercises 33–36, write and solve an equation to answer the question.**
Examples 4 and 5

33. During the summer, you work 30 hours per week at a gas station and earn \$8.75 per hour. You also work as a landscaper for \$11 per hour and can work as many hours as you want. You want to earn a total of \$400 per week. How many hours must you work as a landscaper?

34. The area of the surface of the swimming pool is 210 square feet. What is the length of the deep end?

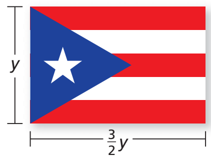


35. Your cell phone has 983.5 MB of free space. You save a 1.4-MB picture and download two songs that are the same size. Your cell phone now has 974.9 MB of free space. What is the size of each song?

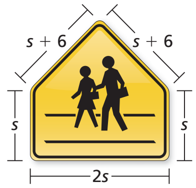
36. You order two tacos and a salad. The salad costs \$2.50. You pay 8% sales tax and leave a \$3 tip. You pay a total of \$13.80. How much does one taco cost?

CONNECTING CONCEPTS **In Exercises 37 and 38, write and solve an equation to answer the question.**

37. The perimeter of the Puerto Rican flag is 150 inches. What are the dimensions of the flag?



38. The perimeter of the school crossing sign is 102 inches. What is the length of each side?



JUSTIFYING STEPS **In Exercises 39 and 40, justify each step of the solution.**

GO DIGITAL

39.

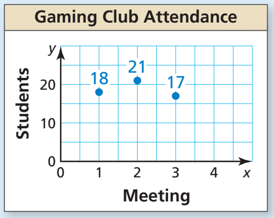
$-\frac{1}{2}(5x - 8) - 1 = 6$	Write the equation.
$-\frac{1}{2}(5x - 8) = 7$	
$5x - 8 = -14$	
$5x = -6$	
$x = -\frac{6}{5}$	

40.

$2(x + 3) + x = -9$	Write the equation.
$2(x) + 2(3) + x = -9$	
$2x + 6 + x = -9$	
$3x + 6 = -9$	
$3x = -15$	
$x = -5$	

41. **COMPARING METHODS** Solve the equation $2(4 - 8x) + 6 = -1$ using two different methods. Which method do you prefer? Explain.

42. **HOW DO YOU SEE IT?**
The scatter plot shows the attendance for each meeting of a gaming club.



a. The mean attendance for the first four meetings is 20. Is the number of students who attended the fourth meeting greater than or less than 20? Explain.

b. Estimate the number of students who attended the fourth meeting. Describe a way you can check your estimate.

ANSWERS

33. $30(8.75) + 11t = 400$; 12.5 h

34. $10d + 10(9) = 210$; 12 ft

35. 3.6 MB

36. $1.08(2t + 2.50) + 3 = 13.80$; \$3.75

37. 30 in. by 45 in.

38. $(s + 6) + (s + 6) + s + 2s + s = 102$; $s = 15$; 21 in., 21 in., 15 in., 30 in., 15 in.

39. Add 1 to each side; Multiply each side by -2; Add 8 to each side; Divide each side by 5.

40. Distributive Property; Simplify; Combine like terms; Subtract 6 from each side; Divide each side by 3.

41. $x = \frac{15}{16}$; *Sample answer:* Distributive Property; There are no fractions until the last step.

42. a. greater than 20; The attendance at the second meeting is only 1 above 20, and the attendance at the other two is more than 1 below 20, so the attendance at the fourth meeting must be greater than 20 to have a mean of 20.

b. *Sample answer:* 24; Use the mean formula to find the attendance at the fourth meeting.

Mini-Assessment

Solve the equation. Check your solution.

1. $-5d + 4 = 12$ $d = -1.6$
2. $-2n + 9n - 8 = 27$ $n = 5$
3. $-18 = 5(2 - x) + 2$ $x = 6$
4. Use the table to find the score you need to receive on Quiz 4 so that your mean quiz score is 90.

Quiz	Score
1	92
2	89
3	95

84

5. You buy a necklace online and pay \$25.06, including 6% sales tax and \$5.99 in shipping fees. What is the list price of the necklace? about \$17.99

43. MP **PROBLEM SOLVING** An online ticket agency charges the amounts shown for basketball tickets. The total cost for an order is \$220.70. How many tickets are purchased?

Charge	Amount
Ticket price	\$32.50 per ticket
Convenience charge	\$3.30 per ticket
Processing charge	\$5.90 per order

44. **MAKING AN ARGUMENT** You have quarters and dimes that total \$2.80. Your friend says it is possible that the number of quarters is 8 more than the number of dimes. Is your friend correct? Explain.

MP **REASONING** **In Exercises 45–48, the letters a, b, and c represent nonzero constants. Solve the equation for x. Then find values of a, b, and c for which the solution is negative.**

45. $ax - b = 12.5$ **46.** $ax + b = c$

47. $2bx - bx = -8$ **48.** $cx - 4b = 5b$

49. DIG DEEPER Find three consecutive even integers that have a sum of 54. Explain your reasoning.

GO DIGITAL

50. **THOUGHT PROVOKING** Your math teacher assigns a weight to each component of the class. The weight of the final exam is half your grade, and the weights of the remaining components are equal. What is the least possible score you can receive on the final exam to earn an A (90%) in the class? Explain your reasoning.

Component	Your score	Weight	Score × Weight
Class participation	92%		
Homework	96%		
Quizzes	88%		
Midterm exam	76%		
Final exam			
Total		1	

REVIEW & REFRESH

WATCH

In Exercises 51–54, find the sum or difference.

51. $-3.37 + 4.135$ **52.** $1\frac{3}{8} - \frac{7}{8}$

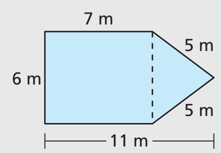
53. $18.36 - (-9.04)$ **54.** $-\frac{5}{12} + \left(-\frac{7}{3}\right)$

55. **MODELING REAL LIFE** About how many times farther from the Sun is Neptune than Mercury?

Planet	Average distance from the Sun (miles)
Mercury	36,000,000
Neptune	2.795×10^9

56. Order the numbers $\frac{11}{20}$, 49%, and 0.5 from least to greatest.

57. Find the perimeter and the area of the figure.



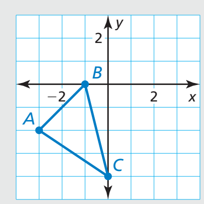
58. MP **NUMBER SENSE** The sum of two-thirds a number and eighteen is twenty-three. What is the number?

In Exercises 59–62, solve the equation. Check your solution.

59. $x + 9 = 7$ **60.** $8.6 = z - 3.8$

61. $3r + 7 = 11$ **62.** $26 = 9p - 6 - p$

63. Translate the triangle 1 unit right and 3 units up. What are the coordinates of the image?



64. **MODELING REAL LIFE** Your friend borrows \$7500 to buy an all-terrain vehicle (ATV). The simple annual interest rate is 6%. She pays off the loan after 5 years of equal monthly payments. How much is each payment?

65. Factor $24x + 32$ using the greatest common factor.

ANSWERS

43. 6 tickets

44. no; Solving the equation $0.25(d + 8) + 0.10d = 2.80$ results in the number of dimes not being a whole number.

45. $x = \frac{12.5 + b}{a}$; $a > b + 12.5$

46. $x = \frac{c - b}{a}$; $a > c - b$

47. $x = -\frac{8}{b}$; $b > 0$

48. $x = \frac{9b}{c}$; $c < 9b$

49. 16, 18, 20; The next consecutive even integers after $2n$ are $2n + 2$ and $2n + 4$. Solve the equation $2n + (2n + 2) + (2n + 4) = 54$. Then substitute the solution into the expressions for the integers.

50. 92%; Use the completed chart to write and solve the equation $11.5 + 12 + 11 + 9.5 + 0.5x = 90$ for x.

51. 0.765

52. $\frac{1}{2}$

53. 27.4

54. $-\frac{11}{4}$

55. 77.6 times

56. 49%, 0.5, $\frac{11}{20}$

57. 30 m; 54 m^2

58. $n = \frac{15}{2}$

59. $x = -2$

60. $z = 12.4$

61. $r = \frac{4}{3}$

62. $p = 4$

63. $A'(-2, 1)$, $B'(0, 3)$, $C'(1, -1)$

64. \$162.50

65. $8(3x + 4)$

Laurie's Notes

Overview of Section 1.3

Introduction

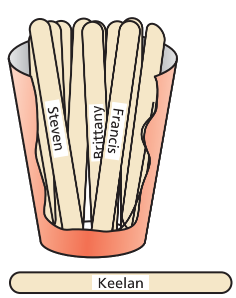
- This section extends the work students have done with ratios, rates, and unit analysis in middle school. The lesson begins with a brief review of ratio terminology. Students must pay attention to the units involved in each problem, which is more than paying attention to labels. The units are giving meaning to the numbers, which is necessary for solving real-life problems.

- ***FOCUS* on Major Work:** Students should be able to reason quantitatively and use units to solve problems. This section refreshes and extends proportional reasoning and unit analysis work students were introduced to in middle school. Now, they will extend ratio language from middle school (e.g., 8 miles per hour) by expressing units as derived units (e.g., 8 mi/h).
- ***RIGOR* in the Section:** In the exploration, students develop **conceptual understanding** as they use unit analysis to estimate the number of cars in a train. The lesson provides opportunities for **procedural fluency** with examples and Self-Assessment exercises on using ratios and rates to solve real-life problems, and converting units and rates. An **application** example and additional Self-Assessment exercises provide in-class practice with problem solving before homework.

Formative Assessment Tip

POPSICLE STICKS

- This technique ensures that *any* student can be called on during questioning time in class. Write the name of each student on a Popsicle stick. Place the sticks in a cup (or can). When questions are posed in class during *No-Hands Questioning**, each student should think and be prepared to answer. If you only call on students who raise their hands, students can then opt out of being engaged. All students think they have an equal chance of being called on when a stick is pulled randomly, so they engage more in the lesson.



- If there are certain students that you want to hear from, the cup could have an inner cylinder where *select* sticks are placed. It will appear that the process is still random, and the voices you have not heard much from will still have the opportunity to prepare and formulate their answers.

*See Section 2.4 for a description of *No-Hands Questioning.*

Section Resources

PLAN

Chapter at a Glance
Everyday Connections Video Series
Lesson Plans
Pacing Guide
Skills Review Handbook

TEACH

Answer Presentation Tool
CalcChat®
CalcView®
Differentiating the Lesson
Dynamic Classroom
Interactive Tools
Resources by Chapter*
- Warm-Up
- Extra Practice
- Reteach
- Enrichment and Extension
- Puzzle Time

Skills Trainer
Tutorial Video Series

ASSESS

Dynamic Assessment System
- Self-Assessment
- Section Practice
- Section Review & Refresh
- Point-of-use Remediation
- Reports

Formative Check
Homework App
Practice Workbook and Test Prep*
- Extra Practice
- Review & Refresh
- Self-Assessment

Self-Assessment

**Available in print*

Common Core State Standards

HSN-Q.A.1, HSN-Q.A.2

Learning Target

Use proportional reasoning and analyze units when solving problems.

Success Criteria

- Use ratios to solve real-life problems.
- Use rates to solve real-life problems.
- Convert units and rates.

Warm-Up

Cumulative and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL SUPPORT

Students may know the word *rate* as a type of measure. For example, the measure of the speed of a heart beating is a heart rate. The total amount charged for fruits or vegetables is determined by their actual weight and the rate that is charged for each unit of weight. In the context of mathematics, a rate is a ratio of two quantities using different units. A ratio compares two quantities.

Laurie's Notes

Launch the Lesson

- Show students a picture of a stack of the same object, such as copies of the same book. The top of the stack should not be visible.

? **TURN AND TALK** "If you know the height of the stack and the height of one book, how can you estimate the number of books in the stack?" Divide the height of the stack by the height of one book.

- The exploration contains a similar problem with a different context.

EXPLORE IT!

- This is a very open-ended exploration. You can limit the range of possible answers by telling students to assume the train cars are between 55 feet 5 inches and 67 feet 11 inches long.
- In this exploration, students will experience modeling a problem in which only some of the necessary information is known (the length of the train is 1.9 kilometers). They will need to make assumptions to estimate the solution.
- There is not one correct answer, though some answers will be more accurate than others. There are many assumptions students may mention, which will influence their estimates.
- Some possible assumptions:
  - The train cars may be different lengths.
  - The engine and caboose have lengths different from the average train car.
  - There is space between each train car.
- **MP5 Use Appropriate Tools Strategically:** You will want students to have access to a calculator. They may also need support in converting from kilometers to feet and inches, or from feet and inches to kilometers.
- **FEEDBACK** Take time for several students to share their processes and reasoning. Students will benefit from hearing the approach and reasoning of different classmates. Students also benefit from listening to classmates critique their reasoning. Even if students do not share their work aloud, they can use the feedback given to others to assess their own work.

Where Are We In Our Learning?

In this exploration, students are developing a conceptual understanding of what it means to use proportional reasoning to solve a real-life problem. In doing so, they are using rates and converting between different units.

1.3 Modeling Quantities

GO DIGITAL

Learning Target Use proportional reasoning and analyze units when solving problems.

Success Criteria
- I can use ratios to solve real-life problems.
- I can use rates to solve real-life problems.
- I can convert units and rates.

EXPLORE IT! Estimating Quantities

Work with a partner. A freight train that is 1.9 kilometers long is traveling on the Cize-Bolozon viaduct in France.

Math Practice
Specify Units
What units of measure did you use in your calculations? Why did you decide to use those units?

a. Research the lengths of different types of train cars.

b. Estimate the number of cars in the train. Explain the assumptions that you make to find your estimate.

c. Compare your results with other pairs. Based on these comparisons, do you think you should revise your estimate? Explain your reasoning.

ANSWERS

1. a–c. Answers will vary. Check students' work.

Laurie's Notes

Scaffolding Instruction

- ***EMERGING*** Students will need support with the computations, as well as the reasoning used to begin solving the problem. They will benefit from scaffolding the work with simpler problems, so computation is not a barrier in thinking about the original problem.
- ***PROFICIENT*** Students may be proficient with the computations, but still need support in reasoning proportionally.

Extra Example 1

Your friend is 5 feet 8 inches tall and casts a shadow 12 feet long. At the same time, a nearby tree casts a shadow 54 feet long. A drone is stuck at the top of the tree. Can you use a ladder that allows you to reach a height of 21 feet to retrieve the drone? Because the height of the tree is 25.5 feet, and 25.5 feet > 21 feet, you cannot retrieve the drone.

ELL SUPPORT

Demonstrate Example 1. Explain that the phrase *cast a shadow* means "make a shadow." Remind students that feet are a measurement, not a body part. Then have students work in groups to discuss and complete the Self-Assessment. Expect students to perform according to their language levels.

Beginner: Write out the steps of each solution and discuss using words or simple phrases.

Intermediate: Use simple sentences to contribute to discussion and write each solution.

Advanced: Use complex sentences and help guide discussion as the group solves each problem.

ANSWERS

1. $x = 3$
2. $y = 4$
3. $z = \frac{24}{5}$
4. $n = \frac{22}{3}$
5. yes; Using the Multiplication Property of Equality, you can obtain the second equation by multiplying each side of the first equation by bd.
6. a. 132 ft
   b. 56.76 lb/in.^{2}

GO DIGITAL

Vocabulary

ratio, *p. 20*
proportion, *p. 20*
rate, *p. 21*

Using Ratios and Proportions

A **ratio** is a comparison of two quantities. A **proportion** is an equation stating that two ratios are equivalent.

Equivalent ratios: $a : b$ and $c : d$

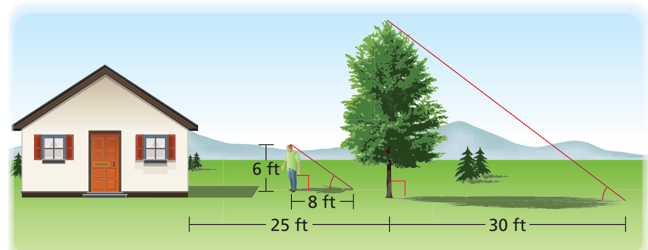
Proportion: $\frac{a}{b} = \frac{c}{d}$

REMEMBER

The *value of a ratio a : b* is $\frac{a}{b}$. The values of equivalent ratios are equivalent.

EXAMPLE 1 Using Ratios

You take the measurements shown in the diagram. The right triangles created by each object and its shadow are similar. Can the tree fall onto the house?



REMEMBER

The triangles created by each object and its shadow have two congruent angles, so the third angles are also congruent and the triangles are similar.

SOLUTION

Ratios of corresponding side lengths in similar triangles are equivalent.

Use a proportion to find the height h (in feet) of the tree.

$\frac{30}{8} = \frac{h}{6}$ — Write a proportion.

$6 \cdot \left(\frac{30}{8}\right) = 6 \cdot \left(\frac{h}{6}\right)$ — Multiplication Property of Equality

$22.5 = h$ — Simplify.

▶ Because 22.5 feet < 25 feet, the tree cannot fall onto the house.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the proportion.

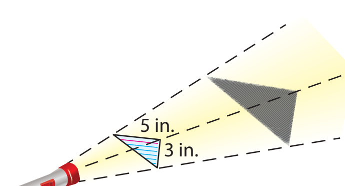
1. $\frac{x}{4} = \frac{9}{12}$
2. $\frac{14}{y} = \frac{7}{2}$
3. $\frac{5}{3} = \frac{8}{z}$
4. $\frac{11}{15} = \frac{n}{10}$

5. **MP REASONING** Are the equations $\frac{a}{b} = \frac{c}{d}$ and $ad = bc$ equivalent? Explain.

6. You are 5 feet 4 inches tall and cast a shadow 3 feet long. At the same time, a nearby water tower casts a shadow 74 feet 3 inches long.
   a. Find the height of the water tower.
   b. Each additional foot in tower height increases the water pressure at the base of the tower by 0.43 pound per square inch. Estimate the water pressure at the base of the tower.

Laurie's Notes

- Students may need to refresh their memories about similarity from middle school. This can be done quickly with a flashlight or a projector and an index card. Cut the index card in half along a diagonal to form a right triangle. Hold the triangle in front of the light source. Discuss how to find the side lengths of the projected image.



DIG DEEPER "If the tree grows at a constant rate of 8 inches each year, when could the tree fall onto the house?" after 3.75 years Students need to use both ratios and rates to answer this question.

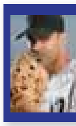
Using Rates

A **rate** is a ratio of two quantities using different units. You can write rates in many ways. For example, 10 meters per second can be written as $\frac{10 \text{ m}}{\text{sec}}$, $10 \frac{\text{m}}{\text{sec}}$, or 10 m/sec. In real-life situations, you may need to choose your own units to solve a problem.

GO DIGITAL

EXAMPLE 2 **Using Rates** WATCH

The diagram shows statistics for a baseball pitcher. Use rates to compare the pitcher's performance in 2019 to his performance in 2020.



SEASON	INNINGS	HITS	EARNED RUNS	WALKS	STRIKEOUTS
2019	70	54	19	29	100
2020	50	42	15	20	73

SOLUTION

There are many rates you can use to make comparisons. Two that you can use are strikeouts per walk and earned runs per inning.

> **REMEMBER**
> A rate $a : b$ has a unit rate of $\frac{a}{b} : 1$.

Method 1: Compare using strikeouts per walk.

2019: 100 strikeouts to 29 walks

rate: $\frac{100}{29} \approx 3.45$ strikeouts/walk

2020: 73 strikeouts to 20 walks

rate: $\frac{73}{20} = 3.65$ strikeouts/walk

▶ The pitcher had fewer strikeouts per walk in 2019.

Method 2: Compare using earned runs per inning.

2019: 19 earned runs in 70 innings

rate: $\frac{19}{70} \approx 0.27$ earned runs/inning

2020: 15 earned runs in 50 innings

rate: $\frac{15}{50} = 0.3$ earned runs/inning

▶ The pitcher had fewer earned runs per inning in 2019.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

7. The table shows enrollment information for two colleges. Use rates to compare the enrollment at the colleges.

College	Students	Athletes	Students with athletic scholarships	Students with academic scholarships
A	4258	288	72	415
B	7120	150	110	826

8. In baseball, a commonly used rate is earned run average (ERA). ERA is a rate of earned runs to a constant number n of innings. In 2020, the pitcher in Example 2 had an ERA of 2.70. Find the value of n. Then explain how to calculate a pitcher's ERA given the number of earned runs and innings pitched.

Extra Example 2

The table shows statistics for a basketball player. Use rates to compare the player's performance in Game 1 to his performance in Game 2.

Game	1	2
Minutes played	42	40
Rebounds	11	5
Assists	7	13
Shot attempts	47	26
Points	37	26

Sample answer: The player had more shot attempts per minutes played in Game 1.

ANSWERS

7. *Sample answer:* College A enrolls more athletes per student. College B enrolls more students with academic scholarships per students with athletic scholarships.

8. $n = 9$; Divide the earned runs by the innings pitched and then multiply by 9.

Laurie's Notes

- In Example 2, students decide which derived units make sense to compare the pitcher's performance.
- ? **TURN AND TALK** "How can you compare the pitcher's performance from one season to the next?" Answers will vary. Students with baseball knowledge may suggest comparing quantities such as walks per inning for each season.
- ? **DIG DEEPER** "The pitcher gave up fewer walks in 2020 than in 2019. Does this mean he had a better season in 2020? Explain." *Sample answer:* You need to compare the walks per inning to compare the two seasons.
- Discuss the *Remember* note.

Extra Example 3

Convert 1.2 tons to ounces.
38,400 oz

Extra Example 4

The average rainfall in City A is about 42 inches per year. The average rainfall in City B is about 13.23 centimeters per month. Which city has a greater average rainfall? City B

ELL SUPPORT

Allow students to work in pairs for extra support with the language of word problems. Clarify the meanings of words that may have multiple meanings, such as *round*, *second*, and *foot*. To check comprehension, have each pair display and explain their answers.

ANSWERS

9. 97.5 in., 98.43 in., or 100 in.

10. 5.53 L/sec, 5.56 L/sec, or 5.64 L/sec

11. speed up

12. **a.** total cost of the wires; dollars

b. cost per foot of wire; dollars per foot

c. total length of wire in yards; yards

Closure

Use the data in Example 2 to compare the pitcher's performance using strikeouts per inning. The pitcher had fewer strikeouts per inning in 2019.

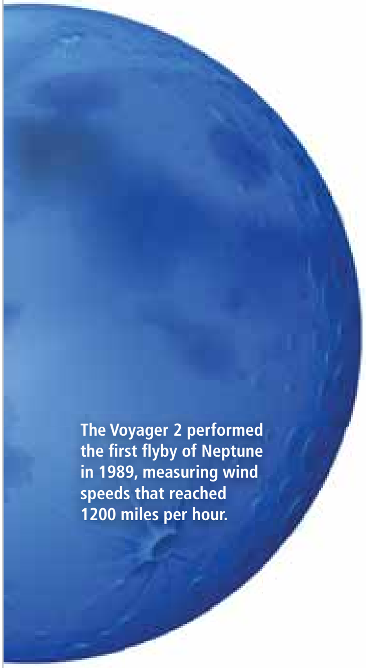
GO DIGITAL

Using Unit Analysis

In Section 1.1 Example 3, you kept track of units when working with rates.

$$200 \text{ m} \div \frac{10 \text{ m}}{1 \text{ sec}} = 200 \cancel{\text{m}} \times \frac{1 \text{ sec}}{10 \cancel{\text{m}}} = 20 \text{ sec}$$

This is called *unit analysis*. You can use unit analysis to help you convert units.



The Voyager 2 performed the first flyby of Neptune in 1989, measuring wind speeds that reached 1200 miles per hour.

EXAMPLE 3 Converting Units of Measure

WATCH

Convert 4.8 gallons to fluid ounces.

SOLUTION

There are 16 cups per gallon and 8 fluid ounces per cup. Use these rates to convert from gallons to fluid ounces.

$$4.8 \text{ gal} = 4.8 \cancel{\text{gal}} \times 16 \frac{\cancel{\text{c}}}{\cancel{\text{gal}}} \times 8 \frac{\text{fl oz}}{\cancel{\text{c}}} = 614.4 \text{ fl oz}$$

▶ So, 4.8 gallons is 614.4 fluid ounces.

EXAMPLE 4 Modeling Real Life

WATCH

Wind speeds on Jupiter can reach 180 meters per second. Which planet has faster winds, Jupiter or Neptune?

SOLUTION

To solve the problem, convert one of the rates so that it has the same units as the other rate. One way is to convert 1200 miles per hour to meters per second.

$$\frac{1200 \text{ mi}}{\text{h}} \approx \frac{1200 \cancel{\text{mi}}}{\cancel{\text{h}}} \times \frac{5280 \cancel{\text{ft}}}{\cancel{\text{mi}}} \times \frac{1 \text{ m}}{3.28 \cancel{\text{ft}}} \times \frac{1 \cancel{\text{h}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \approx \frac{537 \text{ m}}{\text{sec}}$$

▶ Because 180 m/sec < 537 m/sec, Neptune's winds reach higher speeds than Jupiter's winds.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

9. Convert 2.5 meters to inches. Round to the nearest hundredth, if necessary.

10. Convert 88 gallons per minute to liters per second. Round to the nearest hundredth, if necessary.

11. A solar-powered plane travels around Earth. Its cruising speed is 90 kilometers per hour during the day and 1000 meters per minute at night. Does the plane speed up or slow down at sunrise?

12. **MP REASONING** You buy two kinds of wiring for electrical work. The first costs x dollars per foot and the second costs y dollars per foot. You buy A feet of the first wire and B feet of the second wire. What quantities do the following expressions represent? What are the units?

a. $Ax + By$ **b.** $\dfrac{Ax + By}{A + B}$ **c.** $\dfrac{1 \text{ yd}}{3 \text{ ft}} \times (A + B)$

Laurie's Notes

- **MP6 Attend to Precision:** Display the Check Reasonableness in Section 1.1 Example 3. Can students explain the unit analysis? The units of meters are divided out. Do not say, "The units cancel." There is no mathematical definition of the word *cancel*.
- Students will ask how to tell whether to multiply $\frac{1200 \text{ mi}}{\text{h}}$ by $\frac{5280 \text{ ft}}{\text{mi}}$ or $\frac{1 \text{ mi}}{5280 \text{ ft}}$. Explain that they want common units to divide out. Because the initial rate is $\frac{1200 \text{ mi}}{\text{h}}$, you need to multiply by $\frac{5280 \text{ ft}}{\text{mi}}$ so the common units of miles divide out.

? "How have you used unit analysis to solve real-life problems?" Use *Popsicle Sticks* to solicit answers.

1.3 Practice WITH CalcChat® AND CalcView®

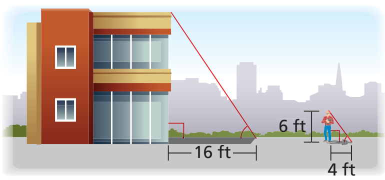
GO DIGITAL

In Exercises 1–4, solve the proportion.

1. $\frac{x}{6} = \frac{10}{12}$ **2.** $\frac{36}{8} = \frac{9}{h}$

3. $\frac{13}{p} = \frac{5}{4}$ **4.** $\frac{4}{15} = \frac{w}{27}$

5. **USING RATIOS** A repairman needs to climb to the top of a building. He takes the measurements shown. The right triangles created by each object and its shadow are similar. Can he use a ladder that reaches heights of up to 28 feet? *Example 1*



6. **USING RATIOS** An entrepreneur wants to rent a billboard at least 30 feet tall to display an advertisement for her business. She is 5 feet 6 inches tall and casts a shadow 7 feet long. At the same time, a billboard casts a shadow 35 feet long. Is the billboard tall enough?

7. **USING RATES** The table shows the numbers of students and staff at two high schools. Use rates to compare the two schools. *Example 2*

School	Students	Teachers	Support staff
A	2308	144	34
B	1522	85	23

8. **USING RATES** The table shows sales data for two salespeople at a car dealership. Use rates to compare the performances of the salespeople.

Person	Months employed	Sales attempted	Sales made	Sales (millions)
A	10	167	109	\$3.3
B	8	163	97	\$2.7

In Exercises 9–12, complete the statement. Round to the nearest hundredth, if necessary. *Example 3*

9. 160 fl oz = ___ qt **10.** 3.2 km = ___ cm

11. 30.9 mm ≈ ___ in. **12.** 4.1 kg ≈ ___ oz

In Exercises 13 and 14, complete the statement. Round to the nearest hundredth, if necessary.

13. $\frac{7\text{ gal}}{\text{min}} \approx \frac{\_\_\_\text{ qt}}{\text{sec}}$ **14.** $\frac{8\text{ km}}{\text{min}} \approx \frac{\_\_\_\text{ mi}}{\text{h}}$

15. **MODELING REAL LIFE** Roller coaster A can reach a top speed of 110 feet per second. Roller coaster B can reach a top speed of 85 miles per hour. Which roller coaster has a greater top speed? *Example 4*

16. **MODELING REAL LIFE** Faucet A leaks at a rate of 21 liters per day. Faucet B leaks at a rate of 30 drips per minute. Which faucet leaks at a faster rate? (1 L ≈ 4000 drips)

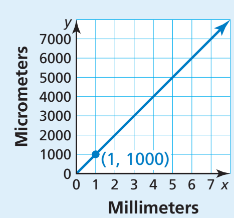
17. **ERROR ANALYSIS** Describe and correct the error in converting 3.5 feet to centimeters.

✗ $3.5\text{ ft} \approx 3.5\text{ ft} \times 3.28\frac{\text{m}}{\text{ft}} \times 100\frac{\text{cm}}{\text{m}}$
$= 1148\text{ cm}$

18. **MP PROBLEM SOLVING** You travel on the highway at a constant speed of 70 miles per hour for 1 hour 45 minutes. Your vehicle travels 25 miles per gallon and gasoline costs \$2.90 per gallon. How much do you spend on fuel for the trip? Explain your reasoning.

19. **MAKING AN ARGUMENT** Your friend says that when you convert a measurement from yards to meters, the number of meters is greater than the number of yards. Is your friend correct? Explain.

20. **HOW DO YOU SEE IT?**
The graph shows the relationship between millimeters and micrometers. Use the graph to convert 5 millimeters to micrometers.



Assignment Guide

Emerging: 1, 5, 7, 9, 11, 13, 14, 15, 17, 18, 20, 22

Proficient: 3, 6, 8, 10, 11, 13, 16, 17, 20, 21, 22, 23, 24

Advanced: 4, 6, 8, 11, 12, 15, 17, 18, 20, 21, 22, 23, 24

Exercise 25 is a performance task.

- Use the red exercises to check understanding of the learning target.
- Assign Review & Refresh exercises as appropriate for continued spaced practice.

Item Leveling

DOK	Exercises
1	1–4, 9–12, 14
2	5–8, 13, 15–20, 24
3	21–23
4	25

Slow Reveal

For Exercise 16, tell students that a faucet leaks at a rate of 21 liters per day. Then ask, “What are you curious about? What do you wonder?” There are many answers students may give, including why someone has not fixed the faucet! Ask students, “Is this a slow leak? If a stopper is in a bathroom sink and the faucet is leaking at this rate, will it overflow in one day?”

ANSWERS

1. $x = 5$

2. $h = 2$

3. $p = \frac{52}{5}$

4. $w = \frac{36}{5}$

5. yes

6. no

7. *Sample answer:* School A has fewer students per teacher. School B has more support staff per teacher.

8. *Sample answer:* Person A has more sales per sales attempted. Person B has more money in sales per months employed.

9. 5

10. 320,000

11. 1.21, 1.22, or 1.24

12. 143.5, 144.32, 144.88, or 145.78

13. 0.47

14. 298.14, or 297.6

15. roller coaster B

16. faucet A

17. $3.28\frac{\text{m}}{\text{ft}}$ should be $\frac{1\text{ m}}{3.28\text{ ft}}$; 106.71 cm

18. \$14.21; $1.75\cancel{\text{h}} \times 70\frac{\cancel{\text{mL}}}{\cancel{\text{h}}} \times 1\frac{\cancel{\text{gal}}}{25\cancel{\text{mL}}} \times \frac{\$2.90}{1\cancel{\text{gal}}} = \$14.21$

19. no; *Sample answer:* One meter is longer than one yard, so the number of yards would be greater than the number of meters.

20. 5000 micrometers

EVERYDAY CONNECTIONS
Learn more about using indirect measurement.

Mini-Assessment

1. Solve $\frac{6}{x} = \frac{9}{15}$. $x = 10$
2. The table shows the numbers of students and adults for two grades attending a field trip. Use rates to compare the two grades.

Grade	9	10
Students	126	142
Teachers	6	8
Other chaperones	3	2

Sample answer: Grade 9 has fewer students per adult than grade 10.

3. Convert 3.5 pints to fluid ounces. 56 fl oz
4. The typical takeoff speed of a jetliner is about 160 miles per hour. Can the jetliner take off when it reaches a speed of 75 meters per second? yes

ANSWERS

21. no; $\left(\frac{1 \text{ yd}}{36 \text{ in.}}\right)^2 = \frac{1 \text{ yd}^2}{1296 \text{ in.}^2}$,
$3240 \text{ in.}^2 \times \frac{1 \text{ yd}^2}{1296 \text{ in.}^2}$
$= 2.5 \text{ yd}^2 < 3 \text{ yd}^2$

22. *Sample answer:* Player A scored more points per game; Player B made more field goals per attempt.

23. *Sample answer:* 100 people; The distance between two people standing in a line is about 2 feet.

24. D

25. **a–c.** Answers will vary.

26. 18 m^2

27. 105 cm^2

28. $100\pi \text{ in.}^2$

29. $m = 8$

30. $a = -0.6$

31. $x = 14$

32. increase; 50%

33. decrease; 25%

34. 7.55, 7.58, 7.6, or 7.69

35. 720, 724.48, 727.27, or 731.43

36. 10 sec

37. $\frac{27}{50}$, or 54%

21. **DIG DEEPER** The surface area of a solid is 3240 square inches. Is the surface area greater than 3 square yards? Explain.

22. **THOUGHT PROVOKING** The table shows statistics for two basketball players during a season. Write an argument supporting that Player A performed better than Player B. Then write an argument supporting that Player B performed better than Player A.

	Player A	Player B
Games played	72	80
Points	1432	1465
Field goals made	505	515
Field goals attempted	1136	1023
Rebounds	490	641
Assists	375	483

23. **MP REASONING** You are standing in a line that is about 200 feet long for a movie premier. Estimate the number of people in the line. Explain your reasoning.

24. **COLLEGE PREP** You make blueberry muffins and banana bread for a bake sale. One batch of blueberry muffins requires m cups of flour, and one batch of banana bread requires n cups of flour. You make A batches of blueberry muffins and B batches of banana bread. Which expression represents the total amount of flour (in cups) you need?

Ⓐ $m + n$ Ⓑ $A + B$
Ⓒ $\frac{A}{m} + \frac{B}{n}$ Ⓓ $Am + Bn$

GO DIGITAL

25. **PERFORMANCE TASK** Choose an object for which you cannot directly measure its height.

a. Visually estimate the height of the object.

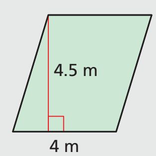
b. Indirectly measure the height of the object. Explain your procedure.

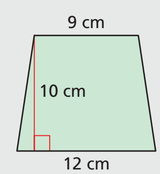
c. Compare your result from part (b) with your estimate in part (a).

REVIEW & REFRESH

WATCH

In Exercises 26 and 27, find the area of the figure.

26.


27.


28. The circumference of a circle is 20π inches. What is the area of the circle?

In Exercises 29–31, solve the equation. Check your solution.

29. $2m - 3 = 13$

30. $-21a + 28a - 6 = -10.2$

31. $68 = \frac{1}{5}(20x + 50) + 2$

In Exercises 32 and 33, identify the percent of change as an increase or a decrease. Then find the percent of change.

32. 80 customers to 120 customers

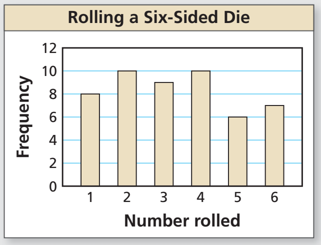
33. 24 points to 18 points

In Exercises 34 and 35, complete the statement. Round to the nearest hundredth, if necessary.

34. 32 c ≈ ___ L **35.** 1.6 lb ≈ ___ g

36. **MODELING REAL LIFE** To estimate how many miles you are from a thunderstorm, count the seconds between when you see lightning and when you hear thunder. Then divide by 5. Determine how many seconds you count for a thunderstorm that is 2 miles away.

37. The bar graph shows the results of rolling a six-sided die 50 times. Find the experimental probability of rolling an even number.



Laurie's Notes

Overview of Section 1.4

Introduction

- In middle school, students worked with quantities, numbers with units that relate to measurement. The quantities measured were often attributes, such as height, length, perimeter, and area. In high school, students become more specific in their use of derived units and their abbreviations.
- ***FOCUS* on Major Work:** This section provides further practice in reasoning quantitatively and using units to solve real-life problems.
- ***RIGOR* in the Section:** In the exploration, students develop **conceptual understanding** as they choose tools and measure objects. This leads to a discussion of precision and accuracy. The lesson provides opportunities for **procedural fluency** with examples and Self-Assessment exercises on estimating measurements and results. An **application** example and additional Self-Assessment exercises provide in-class practice with problem solving before homework.
- The concepts of *precision* and *accuracy* are introduced in this section. Students may view these concepts as the same, but precision is *not* equivalent to accuracy and accuracy is *not* equivalent to precision. A measurement may be precise but not accurate, accurate but not precise, both precise and accurate, or neither precise nor accurate. Be sure to draw attention to the definitions and discuss what each word means. Students must consider the context when choosing an appropriate level of accuracy for a calculation. How precisely can you write the result? How does the accuracy of measurements affect how you state an answer?

Formative Assessment Tip

I USED TO THINK...BUT NOW I KNOW...

- This technique asks students to consider how their thinking about a concept or skill has changed from the beginning of instruction to the end of instruction. This can be done orally or in writing. It is important for students to be able to self-assess and reflect on their own learning.
- Use this technique at the end of the formal lesson. If time permits, have students discuss with one another or the whole class how their understanding developed and/or changed. They may be able to identify the example or class discussion that helped to promote the learning.

Section Resources

PLAN
Chapter at a Glance Everyday Connections Video Series Lesson Plans Pacing Guide Skills Review Handbook
TEACH
Answer Presentation Tool CalcChat® CalcView® Differentiating the Lesson Dynamic Classroom Interactive Tools Resources by Chapter* • Warm-Up • Extra Practice • Reteach • Enrichment and Extension • Puzzle Time Skills Trainer Tutorial Video Series
ASSESS
Assessment Book* • Mid-Chapter Quiz Dynamic Assessment System • Self-Assessment • Section Practice • Section Review & Refresh • Mid-Chapter Quiz • Point-of-use Remediation • Reports Formative Check Homework App Practice Workbook and Test Prep* • Extra Practice • Review & Refresh • Self-Assessment Self-Assessment

**Available in print*

Common Core State Standards

HSN-Q.A.3

Learning Target

Choose an appropriate level of accuracy when calculating with measurements.

Success Criteria

- Choose an appropriate level of accuracy when measuring to solve real-life problems.
- Determine where to round numbers when finding estimates.

Warm-Up

Cumulative and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL SUPPORT

Discuss the meaning of the word *precise*. When you are asked to be precise in your description of something, you must use specific details that describe exactly, or describe with precision. When measuring, precision is the level of detail of the measurement.

Laurie's Notes

Launch the Lesson

? "How do astronomers know how far Mars is from Earth?" Some students may know that astronomers use the speed of light, bouncing radar off Mars and measuring the time it takes for the signal to return. Some students may mention that the distance varies due to each planet's orbit.

- **FYI:** The closest Mars is to Earth is about 54.6 million kilometers and the farthest is about 401 million kilometers.
- Share with students that this section is about choosing an appropriate level of accuracy when calculating with measurements.

EXPLORE IT!

- This is a very open-ended exploration. It allows for a discussion of measurement and accuracy.
- **Note:** Parts (a)–(c) could be done as homework the previous night.
- For the classroom measurements, encourage students to use different tools and different units of measure. Measurements should be measured using only one unit. Be sure students do not use mixed measurements, such as 2 feet 3 inches. They should use either 2.25 feet or 27 inches.
- In part (d), have more than one group use the same tool and unit to measure. One group may measure to the nearest inch while another group measures to the nearest quarter of an inch.

? "The context does not suggest a degree of accuracy. How does the accuracy of the measurements affect the calculated perimeter? area?"

? "For what contexts would accuracy be important? For what contexts would an estimate be sufficient?"

Where Are We In Our Learning?

"In this exploration, no context was suggested. Did your measurements have an appropriate level of accuracy? Did any groups measure to the nearest tenth of a centimeter? Is that degree of accuracy needed to find the perimeter and area of the classroom? How would the perimeter and area you calculated compare to those of a group that measured to the nearest centimeter?"

1.4 Accuracy with Measurements

Learning Target Choose an appropriate level of accuracy when calculating with measurements.

Success Criteria
- I can choose an appropriate level of accuracy when measuring to solve real-life problems.
- I can determine where to round numbers when finding estimates.

GO DIGITAL

EXPLORE IT! Measuring Objects

Work with a partner.

a. MP **CHOOSE TOOLS** Measure an object in your classroom or at home. Choose two tools and two different units to find your measurement.

You can measure any object you want. Consider some of the following.

- the height of a doorway
- the length of a table
- the height of a sibling
- the width of a book
- the arm span of a student

b. Which tools did you choose to find your measurements? Which units of measure did you choose? Explain your choices.

c. Is one measurement more accurate than the other? How can you take another measurement that is more accurate? Explain.

d. Measure the dimensions of the floor of your classroom. Then find the perimeter and area of the floor of your classroom.

e. If someone asks you what the perimeter and area of the floor of your classroom are, how would you answer? Explain your reasoning. Then compare your results with your classmates.

Math Practice

Evaluate Results
What may cause the perimeter and area measurements of your classroom to differ among you and your classmates?

ANSWERS

a–e. Answers will vary. Check students' work.

Laurie's Notes

Scaffolding Instruction

- ***EMERGING*** Students will need support with the calculations, as well as the reasoning required to understand the contexts. The contexts related to calculated measurements may not be familiar.
- ***PROFICIENT*** Students may need guidance with the contexts presented in the lesson.

Extra Example 1

You use a centimeter ruler to measure the diameter and height of the juice can. Estimate the volume of the juice can.

about 450 cm^3

ELL SUPPORT

After demonstrating Example 1, have students work in pairs to discuss and complete the Self-Assessment. For Exercise 2, have one student ask another, "What formula will you use? What is your estimate?"

Beginner: Write out the steps of each solution and state the answer.

Intermediate: Discuss each solution using simple sentences.

Advanced: Explain each solution using detailed sentences.

ANSWERS

1. *Sample answer:* That answer implies that the volume is accurate to the thousandths place.
2. 24 cm^2
3. 5 oz; $75.6 \div 15 = 5.04$ and the weight is measured to the tenth of an ounce so estimate should not be as precise.

Vocabulary

precision, *p. 26*
accuracy, *p. 27*

GO DIGITAL

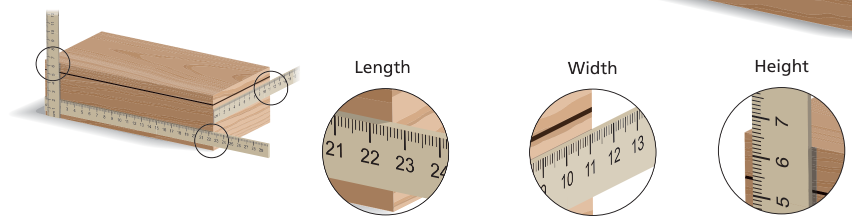
Calculating with Measurements

When measuring, **precision** is the level of detail of the measurement. When performing calculations with measurements, the calculated value is no more *precise* than the original measurements.

EXAMPLE 1 Estimating Measurements

WATCH

You use a centimeter ruler to measure the dimensions of the jewelry box shown. Estimate the volume of the jewelry box.



SOLUTION

The length of the jewelry box is about 22.5 centimeters, the width is about 11.3 centimeters, and the height is about 6.5 centimeters. Substitute these values into the formula for the volume of a rectangular prism.

$V = \ell wh$	Volume of a rectangular prism
$= (22.5 \text{ cm})(11.3 \text{ cm})(6.5 \text{ cm})$	Substitute 22.5 for ℓ, 11.3 for w, and 6.5 for h.
$= 1652.625 \text{ cm}^3$	Multiply.

Because the dimensions are measured in tenths of a centimeter, you should not state the volume beyond tenths of a cubic centimeter. The magnitude of the volume is large enough that rounding to 1650 cm^3 is precise enough in this context.

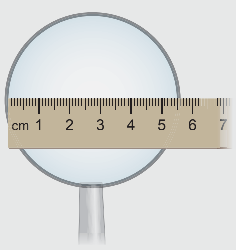
▶ So, the volume is about 1650 cubic centimeters.

SELF-ASSESSMENT

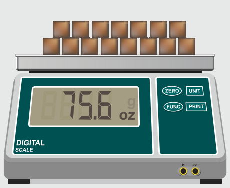
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. **MP REASONING** Explain why it is not reasonable to say the volume of the jewelry box in Example 1 is 1652.625 cubic centimeters.

2. You use a centimeter ruler to measure the diameter of the lens of the magnifying glass. Estimate the area of the lens.

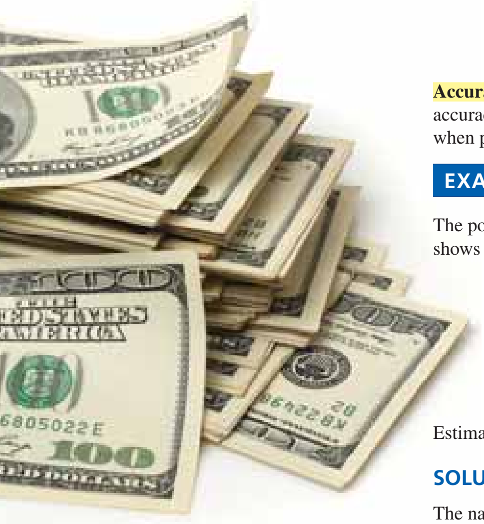


3. A chemist is measuring the weight (in ounces) of 15 equal samples of a substance on the electronic balance at once. Estimate the weight of each sample. Explain your reasoning.



Laurie's Notes

- Discuss the definition of *precision*. In the example, students are reading measurements from a ruler. Because the measurements are read to the nearest tenth of a centimeter, any calculation that uses those measurements cannot be more precise than the nearest tenth of a centimeter, square centimeter, or cubic centimeter.
- ? "Is 1653 cubic centimeters an appropriate estimate for the volume? Explain." yes; *Sample answer:* Because the dimensions are measured in tenths of a centimeter, you can state the volume in whole cubic centimeters.
- ? **DIG DEEPER** "How would estimating the width to be 11.2 centimeters affect your answer?" *Sample answer:* The volume will be about 1638 cubic centimeters.
- ◎ **DIG DEEPER** "For what context might you keep tenths in the answer?" *Sample answer:* measurements of plane engine components



The phrase "per capita" means for each person. When you write an amount per capita, you are writing the amount per person.

Accuracy refers to how close a measured value is to the actual value. The accuracy of measurements may affect how you decide to state answers when performing calculations with them.

GO DIGITAL

EXAMPLE 2 Estimating Results

WATCH

The population of the United States in 2018 was about 328,181,510. The figure below shows the national debt for the United States as reported in 2018.

Estimate the United States national debt per capita.

SOLUTION

The national debt per capita can be thought of as the amount of money each person in the United States would have to pay in order to pay off the entire national debt.

Because the population of the United States and the national debt are constantly increasing, the measured values were accurate for only a moment in time. So, you can round to greater values before making your calculation.

21,867,237,827,643 ≈ 22,000,000,000,000 Round to nearest trillion.

328,181,510 ≈ 330,000,000 Round to nearest ten million.

To find the national debt per capita, divide the national debt by the population. Use technology.

22,000,000,000,000 ÷ 330,000,000 ≈ 66,666.6666...

▶ Because the measured values are constantly increasing and were rounded, it is not reasonable to express the answer to the nearest cent or nearest dollar. So, you can estimate the national debt per capita to be about $67,000.

STUDY TIP

Due to unknown levels of accuracy in the population and debt figures, it would also be acceptable to estimate this amount as $66,700 or even $70,000 in this context.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. MP **REASONING** In Example 2, does the place to which you round affect the accuracy of your results? Explain your reasoning.

5. Repeat Example 2 using current statistics. Compare your results. What conclusions can you make?

6. The number of student loan borrowers in the United States is about 44,532,700. The amount of student loan debt held by the borrowers in 2018 is shown. Estimate the student loan debt per student loan borrower.

7. The land area of the entire United States is 3,531,905 square miles. Use the information you found in Exercise 5 to estimate the population per square mile.

Extra Example 2

A hotel had a total of 6974 guests and staff staying at the hotel in June. The water bill for that month shows a water usage of 682,057 gallons. Estimate the water usage per person. about 97 gallons per person

ANSWERS

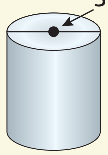
4. yes; Rounding to lesser place values produces estimates that are closer to the actual values, so your calculations will be more accurate.
5. Check students' work; The population and national debt are increasing.
6. $36,000
7. 90 people

Laurie's Notes

- Discuss the definition of *accuracy*. Students will have a better understanding of the concept after working through the example.
- **Note:** You do not have to round and you can round in different ways. The context does not require an exact answer. Because both quantities are constantly changing, an exact answer is only true for one moment in time.
- Population clocks and national debt clocks can be found online.
- ◎ Relate the second success criterion to the decisions made in working through this example.
- **FEEDBACK** "Use *Thumbs Up* to show where you are in your learning."

Extra Example 3

You measure a cylindrical water tank as shown.



a. Estimate the amount of water the tank can hold. about 36 ft^3

b. A water cooler can hold about 1155 cubic inches of water. How many water coolers can you fill when the water tank is full? 53 water coolers

ELL SUPPORT

Discuss the scenario of using mulch in a playground, which may be unfamiliar. You may want to discuss the measurements meters and pounds. One is metric and the other is customary. Allow students to work in groups for extra support with the language of word problems. Have each group present their explanations for Self-Assessment Exercises 8 and 9 to the class, and then display their answer for Exercise 10 for your review.

ANSWERS

8. no; The amount of mulch needed to cover the surface of the playground is the same.

9. See Additional Answers.

10. \$6600

Closure

I USED TO THINK...BUT NOW I KNOW... "How does *accuracy* relate to measurement and calculated measures?"

? "In Example 3, if the recommended depth of mulch is 1 foot, can you double the number of bags of mulch needed for a depth of 6 inches? Explain."
yes; *Sample answer:*
$V = (66.75\text{ ft})(40.5\text{ ft})(1\text{ ft})$
$= 2703.375\text{ ft}^3$, which is twice the volume needed for a depth of 6 inches.

Solving Real-Life Problems

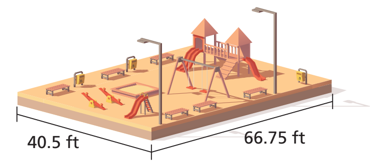
GO DIGITAL

EXAMPLE 3 **Modeling Real Life** WATCH

The surface of a city playground is being covered with rubber mulch, which is shredded rubber made from recycled tires. A city worker measures the dimensions of the playground as shown. The recommended depth of the mulch is 6 inches.

a. Estimate the volume of mulch needed to cover the surface.

b. A 1-ton bag of mulch contains about 3 cubic yards of mulch. How many bags of mulch are needed to cover the surface of the playground?



SOLUTION

a. You can use the formula for the volume of a rectangular prism to estimate the volume V (in cubic feet) of mulch needed to cover the surface.

$V = \ell wh$ — Volume of a rectangular prism

$= (66.75\text{ ft})(40.5\text{ ft})(0.5\text{ ft})$ — Substitute 66.75 for ℓ, 40.5 for w, and 0.5 for h.

$= 1351.6875\text{ ft}^3$ — Multiply.

▶ You can estimate the volume of mulch needed to be about 1350 cubic feet. You are calculating with measured values and 1.6875 cubic feet is a relatively small amount in this context.

b. Convert the amount of mulch in a bag to cubic feet.

$$3\text{ yd}^3 \times \left(\frac{3\text{ ft}}{1\text{ yd}}\right)^3 = 3\text{ yd}^3 \times \frac{27\text{ ft}^3}{1\text{ yd}^3} = 81\text{ ft}^3$$

You need about 1350 cubic feet of mulch and each bag contains 81 cubic feet. Determine how many bags are needed to fill 1350 cubic feet.

$$1350\text{ ft}^3 \times \frac{1\text{ bag}}{81\text{ ft}^3} = \frac{50}{3}, \text{ or } 16\frac{2}{3}\text{ bags}$$

▶ Because mulch is sold in whole bags, 17 bags are needed to cover the entire surface.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

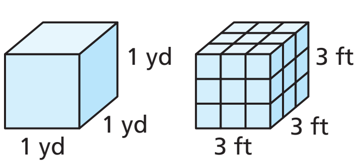
8. **MP REASONING** Another worker measures the dimensions of the playground in meters. Does this change the amount of mulch needed? Explain.

9. **WHAT IF?** How many bags of rubber mulch are needed in Example 3 when the recommended depth of the mulch is 4 inches? Explain.

10. In Example 3, the company's delivery truck can haul at most 8000 pounds. One bag of rubber mulch costs \$375. There is a \$45 delivery charge for each trip the truck makes. What is the total cost for purchasing and delivering the bags of rubber mulch needed for the playground?

Laurie's Notes

- Remind students that *depth* and *height* are interchangeable in the formula for the volume of a rectangular prism.
- **Math Misconception:** Students may incorrectly think that 1 yd^3 = 3 ft^3. A quick sketch can help them see
$1\text{ yd}^3 = (1\text{ yd})(1\text{ yd})(1\text{ yd}) = (3\text{ ft})(3\text{ ft})(3\text{ ft}) = 27\text{ ft}^3$.

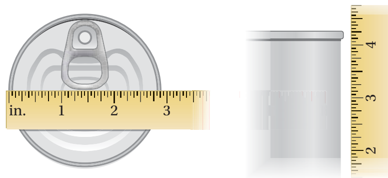


? **DIG DEEPER** "Is it possible to spread the mulch exactly 6 inches high?" You want students to understand the feasibility of spreading the mulch exactly 6 inches high, and that 16 bags may be sufficient because the mulch will be spread around objects. 17 bags definitely meets the need, but 16 bags may as well.

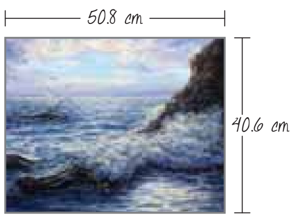
1.4 Practice with CalcChat® and CalcView®

GO DIGITAL

1. ESTIMATING MEASUREMENTS You use an inch ruler to measure the dimensions of a cylindrical can, as shown. Estimate the volume of the can. *Example 1*



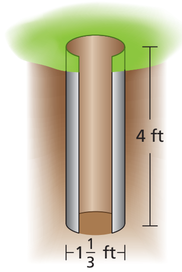
2. ESTIMATING MEASUREMENTS You use a tape measure to measure the dimensions of a canvas, as shown. Estimate the area of the canvas.



3. ESTIMATING RESULTS The figures show the number of taxpayers and the federal tax revenue for the United States in 2018. Estimate the United States federal tax revenue per taxpayer. *Example 2*

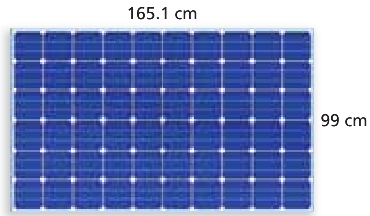
4. ESTIMATING RESULTS The circumference of Mercury is about 15,329 kilometers. The circumference of Jupiter is about 439,263 kilometers. About how many times larger is Jupiter than Mercury?

5. MODELING REAL LIFE You want to install an in-ground basketball post. You dig a hole with the dimensions shown for the concrete. An 80-pound bag of concrete mix yields 0.6 cubic feet. How many bags do you need to install the basketball post? *Example 3*



6. MODELING REAL LIFE A farmer fills a field with solar panels. The area of the field is 32,374.9 square meters.

a. About how many solar panels of the size shown can fit in the field? Explain.



b. One solar panel of this size can produce about 1.06 kilowatt hours of electricity per day. On average, a house uses about 1000 kilowatt hours of electricity per month. Can the field produce enough electricity for 500 houses each month? Explain.

7. COLLEGE PREP You measure the dimensions of a gift box as 8.75 inches, 4.75 inches, and 2.5 inches. Which of the following is *not* an appropriate approximation of the volume of the gift box?

Ⓐ 103 in.^{3}

Ⓑ 100 in.^{3}

Ⓒ 103.9 in.^{3}

Ⓓ 103.906 in.^{3}

Assignment Guide

Emerging: 1, 3, 5, 7, 8, 9, 10, 11

Proficient: 2, 4, 6, 7, 8, 9, 10, 11, 13

Advanced: 2, 4, 6, 7, 8, 9, 10, 12, 13

- Use the red exercises to check understanding of the learning target.
- Assign Review & Refresh exercises as appropriate for continued spaced practice.

Item Leveling

DOK	Exercises
1	1–4
2	5–11
3	12, 13

Slow Reveal

For Exercise 5, say, "You want to install an in-ground basketball post. You need to dig a hole to set the post in and then fill the space around the post with concrete. What information do you need to determine how many bags of concrete mix to buy?" Students should recognize that much information needs to be known: the shape and dimensions of the post, the depth and shape of the hole, and the volume of concrete a bag of mix yields. When students recognize what information needs to be known, slowly reveal the information.

ANSWERS

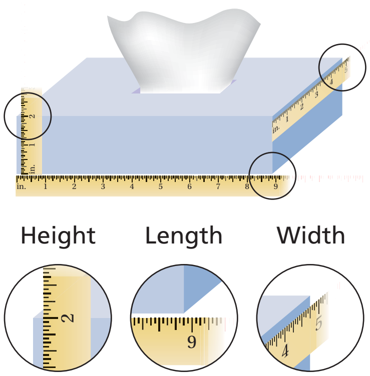
1. 30 in.^{3}
2. 2060 cm^2
3. $27,000
4. 29 times
5. 10 bags
6. **a.** 19,860 panels; $(1.651 \text{ m})(0.99 \text{ m}) \approx 1.63 \text{ m}^2$ and $32{,}374.9 \text{ m}^2 \div 1.63 \text{ m}^2 \approx 19{,}860$

   b. yes; $19{,}860 \times 1.06 \times 30 \approx 630{,}000$ kwh/month, $500 \times 1000 = 500{,}000$ kwh/month, and $630{,}000 > 500{,}000$
7. D

EVERYDAY CONNECTIONS
Learn more about determining the accuracy of a thermometer.

Mini-Assessment

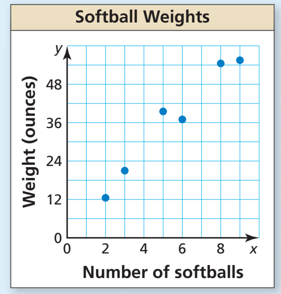
1. You use an inch ruler to measure the dimensions of the tissue box, as shown. Estimate the volume of the tissue box.



about 84 in.^{3}

2. The area of Texas is about 268,596 square miles. The area of Maryland is about 12,406 square miles. About how many times larger is Texas than Maryland? 23 times larger
3. You want to install a fence post. You dig a cylindrical hole with a diameter of 1 foot and a depth of 1.5 feet for the concrete. A 40-pound bag of concrete mix yields 0.3 cubic feet. How many bags do you need to install the fence post? 4 bags

8. **HOW DO YOU SEE IT?** A softball weighs about 6.5 ounces. You weigh different numbers of softballs and record the results in the graph.



a. Explain how graphing $y = 6.5x$ can help you reason about the accuracy of your results.

b. Which measurement appears to be the least accurate? Why? Explain how this may have occurred.

9. **MP NUMBER SENSE** A substance weighs exactly 1 pound. You use three different scales to measure the weight of the substance. The results are shown in the table. Which measurement is the most accurate?

Scale	1	2	3
Weight (pounds)	1.019	0.9	1.01

10. **MAKING AN ARGUMENT** Your friend measures the circumference of a fitness ball as 2.04 meters and says, "The volume of the ball is 0.1433638009 cubic meters." Do you agree with your friend's statement? Explain.

GO DIGITAL

11. **ANALYZING RELATIONSHIPS** Explain how the choice of a unit of measure can impact the accuracy of the measurement of an object.

12. **THOUGHT PROVOKING** Explain one way you can determine the accuracy of a thermometer.

13. **MP PROBLEM SOLVING** You are comparing three different bottles of liquid bleach with the same active ingredient. The concentration of active ingredient in each bottle is labeled to the nearest tenth of a percent.

Bleach type	Concentration	Size of bottle (fluid ounces)	Price
Ultra	6.0%	64	\$2.49
Regular	5.3%	128	\$3.99
Concentrated	8.3%	121	\$6.99

a. About how many fluid ounces of active ingredient are in each bottle? Explain the level of accuracy you used in your results.

b. Which bottle is the best buy? the worst buy? Explain.

REVIEW & REFRESH

WATCH

In Exercises 14 and 15, write the number in scientific notation.

14. 96,400,000 15. 0.00035

16. Evaluate $26 + 9(17 - 3^2) \div 4$.

MP NUMBER SENSE In Exercises 17 and 18, write and solve an equation to find the number.

17. The sum of three times a number and 12 is 45.

18. Nine minus the quotient of a number and 7 is −5.

19. **MODELING REAL LIFE** The distance from Earth to the moon is about 238,855 miles. The distance from Earth to Mars is about 140 million miles. About how many trips to the moon are equal to the distance from Earth to Mars?

In Exercises 20 and 21, solve the equation.

20. $z - 5.2 = -3.4$ 21. $56t = 16$

22. **MODELING REAL LIFE** Which sign shows a greater speed limit? How much greater?

23. The vertices of a trapezoid are $W(1, 4)$, $X(4, 4)$, $Y(4, 1)$, and $Z(-1, 1)$. Draw the figure and its reflection in the x-axis. Identify the coordinates of the image.

ANSWERS

8. a. The closer the points are to the line, the more accurate they are.
   b. the weight of 5 softballs; the point is the farthest from the line; *Sample answer:* The balls could have absorbed water which would make them weigh more.
9. 1.01 lb
10. no; The volume should not be stated beyond hundredths of a cubic meter.
11. *Sample answer:* To obtain accurate measurements, the unit of measure used should be decided based on the size of the object.
12. *Sample answer:* Use a thermometer to take the temperature of the same object several times. If the temperatures are all close to each other, the thermometer is probably accurate. If the temperatures are all spread out, the thermometer is probably not accurate.
13. a. Ultra: 3.8 fl oz, Regular: 6.8 fl oz, Concentrated: 10.0 fl oz; *Sample answer:* Because the concentration is measured to the nearest tenth of a percent, the same level of accuracy was used.
    b. regular; concentrated; Ultra: $2.49/3.8 \approx \$0.66$ per fl oz, Regular: $3.99/6.8 \approx \$0.59$ per fl oz, Concentrated: $\$6.99/10.0 \approx \$0.70$ per fl oz
14. 9.64×10^7 15. 3.5×10^{-4}
16. 44 17. $3n + 12 = 45$; $n = 11$
18. $9 - \frac{n}{7} = -5$; $n = 98$
19. 580 trips 20. $z = 1.8$
21. $t = \frac{2}{7}$
22. 30 mi/h; about 5.2 mi/h, or about 8.4 km/h
23. See Additional Answers.

Laurie's Notes

Overview of Section 1.5

Introduction

- ***FOCUS* on Major Work:** The symbolic manipulation used to solve equations with variables on both sides is presented in this lesson. Students will also solve equations with grouping symbols. This lesson refreshes and extends equation solving students did in middle school.
- ***RIGOR* in the Section:** In the exploration, students develop **conceptual understanding** as they interpret, write, and solve equations with variables on both sides. The lesson provides opportunities for **procedural fluency** with examples and Self-Assessment exercises on solving equations with variables on both sides. An **application** example related to the Distance Formula and the impact of water current and additional Self-Assessment exercises provide in-class practice with problem solving before homework.
- Students can generally solve visual equations on a balance scale. (See *Launch the Lesson* on the next page.) Applying this to equation solving, explain that either variable terms or constant terms can be added to (or removed from) each side of the equation. For at least one equation, demonstrate solving the equation by first collecting variable terms on one side and then solve the same equation again by collecting constant terms first.

  Example:

$5x - 4 = 3x + 12$	$5x - 4 = 3x + 12$
$2x - 4 = 12$	$5x = 3x + 16$
$2x = 16$	$2x = 16$
$x = 8$	$x = 8$

- As teachers, we tend to use a lot of familiar vocabulary when solving equations. This vocabulary may not be familiar to students. Be sure to check for understanding when using phrases such as: "isolate the variable," "collect variable terms," and "collect constant terms."

Making Math Visible

- Use a table of values to help students see how each expression changes as x (the independent variable) changes. You can use technology to make the tables.
- The following equations are from Example 3.

a. $3(5x + 2) = 15x$

x	$3(5x + 2)$	$15x$
−2	−24	−30
−1	−9	−15
0	6	0
1	21	15
2	36	30
3	51	45
4	66	60

b. $\frac{n}{6} = -\frac{n}{6} + \frac{1}{2}$

n	$\frac{n}{6}$	$-\frac{n}{6} + \frac{1}{2}$
$-\frac{1}{2}$	$-\frac{1}{12}$	$\frac{7}{12}$
0	0	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{12}$	$\frac{5}{12}$
1	$\frac{1}{6}$	$\frac{1}{3}$
$\frac{3}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

c. $-2(4y + 1) = -8y - 2$

y	$-2(4y + 1)$	$-8y - 2$
−2	14	14
−1	6	6
0	−2	−2
1	−10	−10
2	−18	−18
3	−26	−26
4	−34	−34

- In part (a), as x increases, both expressions increase and they always differ by 6. In part (b), as n increases, the first expression increases and the second expression decreases. They are equal when $n = \frac{3}{2}$. In part (c), the expressions are equal for all values of y.

Section Resources

PLAN

Chapter at a Glance
Everyday Connections Video Series
Lesson Plans
Pacing Guide
Skills Review Handbook

TEACH

Answer Presentation Tool
CalcChat®
CalcView®
Differentiating the Lesson
Dynamic Classroom
Interactive Tools
Resources by Chapter*
- Warm-Up
- Extra Practice
- Reteach
- Enrichment and Extension
- Puzzle Time

Skills Trainer
Tutorial Video Series

ASSESS

Dynamic Assessment System
- Self-Assessment
- Section Practice
- Section Review & Refresh
- Point-of-use Remediation
- Reports

Formative Check
Homework App
Practice Workbook and Test Prep*
- Extra Practice
- Review & Refresh
- Self-Assessment

Self-Assessment

*Available in print

Common Core State Standards

HSA-CED.A.1, HSA-REI.A.1, HSA-REI.B.3

Learning Target

Write and solve equations with variables on both sides.

Success Criteria

- Apply properties of equality using variable terms.
- Solve equations with variables on both sides.
- Recognize when an equation has zero, one, or infinitely many solutions.

Warm-Up

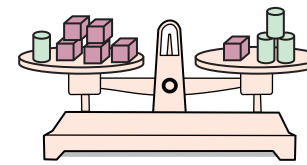
Cumulative and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL SUPPORT

Explain that an equation that has infinitely many solutions is an *identity*. In everyday language, the word *identity* is used to describe how someone is seen or recognized (identified). A person's identity describes his or her essential, basic or core, self. While characteristics that describe one person are unique and limited, in math, an identity has infinitely many solutions.

Laurie's Notes

Launch the Lesson



? "What balances with the cylinder? Explain." 2 cubes; *Sample answer:* Remove one cube and one cylinder from each side. 2 cylinders balance with 4 cubes, so 1 cylinder will balance with 2 cubes.

- The balance problem is equivalent to $x + 5 = 3x + 1$, where a cylinder represents x and the cubes represent the whole numbers. This is an example of an equation with variables on both sides, the type students will solve in this section.
- If students are familiar with algebra tiles, you can model the problem using the tiles. Replace the cylinders with variable-tiles, and replace the cubes with integer-tiles.

EXPLORE IT!

- Each of the situations in this exploration can be represented as an equation with variables on both sides. Students will reason about the context and whether the context has a possible solution. You want students to use reasoning when solving equations such as $x + 2 = x + 7$. There is no value of x that you can add to 2 and add to 7 that will make the expressions equal. Similarly, when students reason about $3x + 5 = 7x + 5$, they should realize the expressions are equal only when $x = 0$.
- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students need to read and understand the information: you and your friend have *different* hourly pay rates, and you each receive a weekly allowance. Ask questions to assess student understanding of the given information.
- In part (a), you could divide the class into four groups and assign one scenario to each group. Have each group share their reasoning with the class. Remind students that the question is asking whether it is possible for you and your friend to work the same number of hours and earn the same total amount in a given week.
- The focus is on the reasoning. Students are comparing $ax + b$ to $cx + d$ when they know a, b, c, and d. How do the expressions compare as x increases?

? DIG DEEPER "How do you know that the equation for the third situation in part (a) has no solution?" *Sample answer:* If the hourly pay rates are the same and your friend has a greater allowance, you and your friend will never earn the same amount. You can also ask students for an example of an equation with no solution.

Where Are We In Our Learning?

"In this exploration, you explored several situations that involved comparing two expressions. Each expression has a term that varies (e.g., $9.75p$) and a term that is constant (e.g., 10). In the lesson, you will solve equations with variables on both sides of the equal sign (e.g., $9.75p + 10 = 9.35p + 20$). Can you look at an equation and recognize whether it has zero, one, or infinitely many solutions?"

1.5 Solving Equations with Variables on Both Sides

GO DIGITAL

Learning Target Write and solve equations with variables on both sides.

Success Criteria

- I can apply properties of equality using variable terms.
- I can solve equations with variables on both sides.
- I can recognize when an equation has zero, one, or infinitely many solutions.

EXPLORE IT! Solving a Real-Life Problem

Work with a partner. You earn \$9.75 per hour at a part-time job. Your friend earns \$9.35 per hour at a part-time job. The only other income you and your friend earn is a weekly allowance.

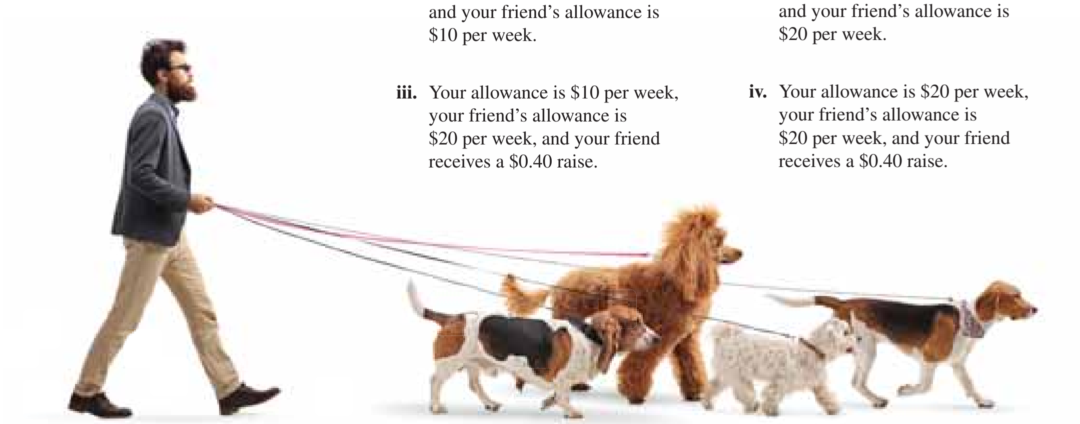
a. Determine whether it is possible for you and your friend to work the same number of hours and earn the same total amount in a given week for the following situations. Explain your reasoning.

i. Your allowance is \$20 per week and your friend's allowance is \$10 per week.

ii. Your allowance is \$10 per week and your friend's allowance is \$20 per week.

iii. Your allowance is \$10 per week, your friend's allowance is \$20 per week, and your friend receives a \$0.40 raise.

iv. Your allowance is \$20 per week, your friend's allowance is \$20 per week, and your friend receives a \$0.40 raise.



b. The following equation represents one of the situations in part (a).

$$9.75p + 10 = 9.35p + 20$$

Interpret each term and each side of the equation. Which situation does it represent?

c. Solve the equation in part (b). Explain how you solved the equation and what the solution represents. Can you start with a different first step?

d. Write and solve an equation for each of the other three situations in part (a). Compare your solutions to your answers in part (a). What do you notice?

Math Practice

Look for Structure
What do you notice about the coefficients of the variable terms in each equation? What do they tell you about the number of solutions an equation may have?

ANSWERS

a. i. not possible; You earn more per hour and have a greater allowance, so you will always earn more than your friend.

ii. possible; You earn more per hour, but your friend has a greater allowance, so your totals could be the same.

iii. not possible; You both earn the same per hour, and your friend has a greater allowance, so your friend will always earn more than you.

iv. possible; You both earn the same per hour and earn the same allowance, so your totals will always be the same.

b. left side: total amount you earn, $9.75p$: amount you earn at your job, 10: your allowance; right side: total amount your friend earns, $9.35p$: amount your friend earns at his or her job, 20: your friend's allowance; part(ii)

c. $p = 25$; *Sample answer:* Subtract $9.35p$ and 10 from each side, then divide each side by 0.4; yes

d. i. $9.75p + 20 = 9.35p + 10$, $p = -25$

ii. $9.75p + 10 = (9.35 + 0.4)p + 20$, no solution

iii. $9.75p + 20 = (9.35 + 0.4)p + 20$, all real numbers; *Sample answer:* When the equation has a negative or no solution, the situation is not possible. When the solutions of an equation are all real numbers, the situation is possible.

Laurie's Notes

Scaffolding Instruction

- ***EMERGING*** Students are still developing number sense. Encourage them to explain which operations are represented in each expression before solving an equation.
- ***PROFICIENT*** Students may be able to perform the symbolic manipulation of solving an equation but cannot recognize when a linear equation has zero, one, or infinitely many solutions. They will benefit from using a table of values to see how each expression changes as the value of the variable increases.

Extra Example 1

Solve $5p = 24 - 3p$. Check your solution. $p = 3$

Extra Example 2

Solve $2(3n - 4) = 0.5(10 - n)$. $n = 2$

ELL SUPPORT

Demonstrate Examples 1 and 2. Then have students work in groups to discuss and complete the Self-Assessment. Provide questions to guide their work: Which property will you use in each step? How can you simplify? Expect students to perform according to their language levels.

Beginner: Write out each solution and state the answer.

Intermediate: Use simple sentences to discuss the process for solving each equation.

Advanced: Use detailed sentences and help guide the discussion for solving each equation

ANSWERS

1. $x = -2$
2. $h = 0.375$
3. $n = 0$
4. Use the Distributive Property to get $9x - 24 = 4x + 6$. Subtract $4x$ from each side to get $5x - 24 = 6$. Add 24 to each side to get $5x = 30$. Divide each side by 5 to get $x = 6$; The properties of equality were used.
5. *Sample answer:* to eliminate the fractions before using the Distributive Property

Solving Equations with Variables on Both Sides

GO DIGITAL

Vocabulary VOCAB

identity, *p. 33*

KEY IDEA

Solving Equations with Variables on Both Sides

To solve an equation with variables on both sides, use inverse operations to collect the variable terms on one side and the constant terms on the other side. Then isolate the variable.

EXAMPLE 1 Solving an Equation with Variables on Both Sides

Solve $10 - 4x = -9x$. Check your solution.

WATCH

SOLUTION

$10 - 4x = -9x$	Write the equation.
$+4x \quad +4x$	Addition Property of Equality
$10 = -5x$	Simplify.
$\frac{10}{-5} = \frac{-5x}{-5}$	Division Property of Equality
$-2 = x$	Simplify.

The solution is $x = -2$.

Check

$10 - 4x = -9x$

$10 - 4(-2) \stackrel{?}{=} -9(-2)$

$18 = 18$ ✓

EXAMPLE 2 Solving an Equation with Grouping Symbols

WATCH

Solve $3(3x - 4) = \frac{1}{4}(32x + 24)$.

SOLUTION

$3(3x - 4) = \frac{1}{4}(32x + 24)$	Write the equation.
$9x - 12 = 8x + 6$	Distributive Property
$+12 \quad +12$	Addition Property of Equality
$9x = 8x + 18$	Simplify.
$-8x \quad -8x$	Subtraction Property of Equality
$x = 18$	Simplify.

The solution is $x = 18$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Check your solution.

1. $-2x = 3x + 10$
2. $0.5(6h - 4) = -5h + 1$
3. $-\frac{3}{4}(8n + 12) = 3(n - 3)$
4. **WRITING** Describe the steps in solving the linear equation $3(3x - 8) = 4x + 6$. Explain why the steps produce a valid solution.
5. **MP REASONING** Your friend first multiplies each side of $\frac{1}{2}(x + 9) = \frac{3}{4}(5x + 1)$ by 4 when solving the equation. Why might your friend do this?

Laurie's Notes

- Write the *Key Idea* and then ask students to *Turn and Talk*. Partner A explains his or her understanding of the *Key Idea*. After one minute, select a Partner B to explain the *Key Idea* to the class.
- In Example 1, students may want to add $9x$ to each side so that the variable term is on the left side of the equation. Explain that you can solve for the variable on either side of the equation.

FEEDBACK After students complete the Self-Assessment, have them check their work and solutions with their elbow partners. Then say, "These exercises are related to the first two success criteria. Where are you in your learning?"

Identifying the Number of Solutions

GO DIGITAL

Equations do not always have one solution. An equation that is true for all values of the variable is an **identity** and has *infinitely many solutions*. All real numbers are solutions of any identity. An equation that is not true for any value of the variable has *no solution*.

WORDS AND MATH

Think about the meaning of *identity* in everyday life. Your identity is who you are. It can be the name, distinguishing characteristics, or personality that identifies you. Your identity is the same no matter what changes in your day.

EXAMPLE 3 Solving Equations with Variables on Both Sides

WATCH

Solve each equation.

a. $3(5x + 2) = 15x$ **b.** $\frac{n}{6} = -\frac{n}{6} + \frac{1}{2}$ **c.** $-2(4y + 1) = -8y - 2$

SOLUTION

a.

$3(5x + 2) = 15x$	Write the equation.
$15x + 6 = 15x$	Distributive Property
$-15x \quad -15x$	Subtraction Property of Equality
$6 = 0$ ✗	Simplify.

▶ The statement $6 = 0$ is never true. So, the equation has no solution.

b.

$\frac{n}{6} = -\frac{n}{6} + \frac{1}{2}$	Write the equation.
$6 \cdot \frac{n}{6} = 6 \cdot \left(-\frac{n}{6} + \frac{1}{2}\right)$	Multiplication Property of Equality
$n = -n + 3$	Simplify.
$+n \quad +n$	Addition Property of Equality
$2n = 3$	Simplify.
$\frac{2n}{2} = \frac{3}{2}$	Division Property of Equality
$n = \frac{3}{2}$	Simplify.

▶ The only solution is $n = \frac{3}{2}$.

c.

$-2(4y + 1) = -8y - 2$	Write the equation.
$-8y - 2 = -8y - 2$	Distributive Property
$+8y \quad +8y$	Addition Property of Equality
$-2 = -2$	Simplify.

▶ Because the statement $-2 = -2$ is always true, the original equation is an identity and has infinitely many solutions. So, the solution is all real numbers.

Math Practice

Look for Structure

Why is it helpful to multiply each side by 6? How else could you begin to solve this equation?

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation.

6. $6m - m = \frac{5}{6}(-6m - 30)$ **7.** $10k + 7 = -3 + 10k$ **8.** $3(2a - 2) = 2(3a - 3)$

9. VOCABULARY Is the equation $-2(4 - x) = 2x + 8$ an identity? Explain your reasoning.

10. WRITING In Example 3, your friend says that there is enough information to determine the numbers of solutions of the equations in parts (a) and (c) once you obtain $15x + 6 = 15x$ and $-8y - 2 = -8y - 2$. Is your friend correct? Explain.

Extra Example 3

Solve each equation.

a. $4(3d - 1) = 12d$ no solution

b. $\frac{n}{4} = -\frac{n}{4} - 3$ $n = -6$

c. $21 - 7w = 7(3 - w)$ all real numbers

ANSWERS

6. $m = -\frac{5}{2}$
7. no solution
8. all real numbers
9. no; Solving the equation gives a statement that is never true, not one that is always true.
10. yes; In part (a), the equation is never true and in part (b), $-8y - 2$ will equal itself, for all real values of y.

Laurie's Notes

? FEEDBACK "Can you think of an equation that has infinitely many solutions? no solution?" Allow time for students to think independently before discussing with a partner. Elicit responses and list them on the board in two columns without commenting. Point to the first column (infinitely many solutions) and ask, "How do you know these equations have infinitely many solutions?" Repeat for the second column (no solution).

- **MP2 Reason Abstractly and Quantitatively:** At the point of simplifying the equation in part (a) to $15x + 6 = 15x$, students should reason that there is no solution because the number $15x$ cannot be equal to 6 more than itself. Similarly, in part (c), $-8y - 2 = -8y - 2$ is always true, so there are infinitely many solutions.
- For any of these equations, use a table to represent each expression.

Extra Example 4

You bike for 3 hours to reach the beach. Using the same route, the return trip takes only 2.5 hours because you travel 2 miles per hour faster. How far did you bike going to the beach? 30 mi

EVERYDAY CONNECTIONS
Learn more about river currents.

ELL SUPPORT

You may want to talk about travel by boat, currents, and the directions of upstream and downstream. Allow students to work in groups for extra support with the language of word problems. Have each group display their answers for your review.

ANSWERS

11. 17.5 mi

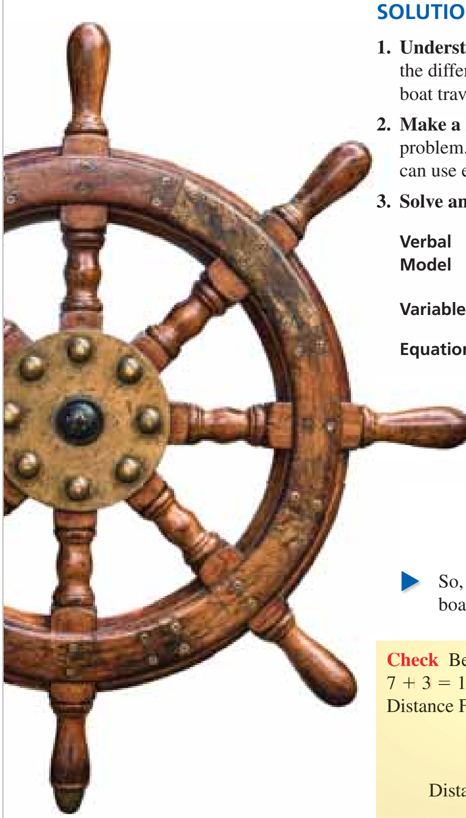
12. $11.70

GO DIGITAL

Solving Real-Life Problems

EXAMPLE 4 **Modeling Real Life** WATCH STEM

A boat leaves New Orleans and travels upstream on the Mississippi River for 4 hours. The return trip takes only 2.8 hours because the boat travels 3 miles per hour faster downstream due to the current. How far does the boat travel upstream?



SOLUTION

1. **Understand the Problem** You are given the amounts of time the boat travels and the difference in speeds for each direction. You are asked to find the distance the boat travels upstream.
2. **Make a Plan** Use the Distance Formula to write expressions that represent the problem. Because the distance the boat travels in both directions is the same, you can use expressions for the distance to write an equation.
3. **Solve and Check** The distance is equal to the product of speed and time.

Verbal Model Distance upstream = Distance downstream

Variable Let x be the speed (in miles per hour) of the boat traveling upstream.

Equation

$x \cdot 4 = (x + 3) \cdot 2.8$	Write the equation.
$4x = 2.8x + 8.4$	Distributive Property
$-2.8x \qquad -2.8x$	Subtraction Property of Equality
$1.2x = 8.4$	Simplify.
$\frac{1.2x}{1.2} = \frac{8.4}{1.2}$	Division Property of Equality
$x = 7$	Simplify.

▶ So, the boat travels 7 miles per hour upstream. To determine how far the boat travels upstream, multiply 7 miles per hour by 4 hours to obtain 28 miles.

Check Because the speed upstream is 7 miles per hour, the speed downstream is $7 + 3 = 10$ miles per hour. When you substitute each speed and time into the Distance Formula, you get the same distance upstream and downstream.

Upstream

$$\text{Distance} = \frac{7\text{ mi}}{1\text{ h}} \cdot 4\text{ h} = 28\text{ mi}$$

Downstream

$$\text{Distance} = \frac{10\text{ mi}}{1\text{ h}} \cdot 2.8\text{ h} = 28\text{ mi} \checkmark$$

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. A boat travels upstream on the Missouri River for 3.5 hours. The return trip only takes 2.5 hours because the boat travels 2 miles per hour faster downstream due to the current. How far does the boat travel downstream?

12. You ask a deli clerk for x pounds of ham and x pounds of cheese. You end up getting 4 extra ounces of ham and 3 fewer ounces of cheese. The ham costs $6.24 per pound and the cheese costs $4.80 per pound. You spend twice as much on ham as you do on cheese. How much do you spend in total?

Closure

- **I USED TO THINK...BUT NOW I KNOW...** Take time for students to reflect on their current understanding of solving equations.
- Solve $6 - 2x = 4x - 9$. $x = 2.5$

Laurie's Notes

- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students must first understand that the distances are the same, but the times are different because the rates change.

? "How do you find the distance traveled when you know the rate and time?" multiply; $d = rt$

MP6 Attend to Precision: Discuss with students how unit analysis is used to verify that the equation written makes sense.

1.5 Practice WITH CalcChat® AND CalcView®

GO DIGITAL

In Exercises 1–14, solve the equation. Check your solution. *Examples 1 and 2*

1. $15 - 2x = 3x$
2. $26 - 4s = 9s$
3. $5p - 9 = 2p + 12$
4. $8g + 10 = 35 + 3g$
5. $5t + 16 = 6 - 5t$
6. $-3r + 10 = 15r - 8$
7. $7 + 3x - 12x = 3x + 1$
8. $w - 2 + 2w = 6 + 5w$
9. $10(g + 5) = 2(g + 9)$
10. $-9(t - 2) = 4(t - 15)$
11. $\frac{2}{3}(3x + 9) = -2(2x + 6)$
12. $2(2t + 4) = \frac{3}{4}(24 - 8t)$
13. $1.5(3y + 2) - y = 2(8y - 6)$
14. $\frac{1}{2}(4x + 5) = 9x - 12(x - 1)$

In Exercises 15–22, solve the equation. *Example 3*

15. $3t + 4 = 12 + 3t$
16. $6d + 8 = 14 + 3d$
17. $2(h + 1) = 5h - 7$
18. $12y + 6 = 6(2y + 1)$
19. $-\frac{w}{5} = \frac{w}{5} - \frac{1}{10}$
20. $\frac{x}{12} + 1 = \frac{x}{3} - \frac{x}{4}$
21. $3(4g + 6) = 2(6g + 9)$
22. $5(1 + 2m) = \frac{1}{2}(8 + 20m)$

23. **MODELING REAL LIFE** You and your friend drive toward each other. The equation $50h = 190 - 45h$ represents the number h of hours until you and your friend meet. After how many hours will you meet?

24. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

    ✗
    $6(2y + 6) = 4(9 + 3y)$
    $12y + 36 = 36 + 12y$
    $12y = 12y$
    $0 = 0$
    The equation has no solution.

25. **MODELING REAL LIFE** A cheetah that is running 90 feet per second is 120 feet behind an antelope that is running 60 feet per second. How long will it take the cheetah to catch up to the antelope? *Example 4*

26. **MAKING AN ARGUMENT** A cheetah can run at top speed for only about 20 seconds. If an antelope is too far away for a cheetah to catch it in 20 seconds, the antelope is probably safe. Your friend claims the antelope in Exercise 25 will not be safe if the cheetah starts running 650 feet behind it. Is your friend correct? Explain.

27. **MODELING REAL LIFE** You want to create a piece of pottery at an art studio. The total cost is the cost of the piece plus an hourly studio fee. The costs at two studios are shown.

    a. After how many hours are the total costs the same at both studios? Justify your answer.
    b. Studio B increases its hourly studio fee by $1.50. How does this affect your answer in part (a)? Explain.

28. **MP PROBLEM SOLVING** One serving of granola provides 4% of the protein you need daily. You must get the remaining 48 grams of protein from other sources. How many grams of protein do you need daily?

MP REASONING In Exercises 29 and 30, find the value of *a* for which the equation is an identity. Explain your reasoning.

29. $a(2x + 3) = 9x + 15 + x$
30. $8x - 8 + 3ax = 5ax - 2a$

31. **DIG DEEPER** Two times the greater of two consecutive integers is 9 less than three times the lesser integer. What are the integers?

Assignment Guide

Emerging: 1, 3, 7, 9, 15, 17, 19, 20, 23, 24, 25, 28, 32

Proficient: 2, 10, 15, 16, 21, 22, 23, 24, 26, 28, 29, 31, 32, 34, 35

Advanced: 7, 11, 20, 21, 22, 24, 27, 30, 31, 32, 33, 34, 35, 36

- Use the red exercises to check understanding of the learning target.
- Assign Review & Refresh exercises as appropriate for continued spaced practice.

Item Leveling

DOK	Exercises
1	1–22
2	23–30, 32–35
3	31, 36

Slow Reveal

For Exercise 23, say, "You and your friend drive toward each other. What does this mean and what are you curious about?" You may hear several of the following responses. The distance between you and your friend is decreasing. Are you both traveling at the same rate? When will you meet? How far apart are you at the beginning? These responses help students make sense of the equation and should be discussed *before* sharing the equation.

ANSWERS

1. $x = 3$
2. $s = 2$
3. $p = 7$
4. $g = 5$
5. $t = -1$
6. $r = 1$
7. $x = \frac{1}{2}$
8. $w = -4$
9. $g = -4$
10. $t = 6$
11. $x = -3$
12. $t = 1$
13. $y = 1.2$
14. $x = \frac{19}{10}$
15. no solution
16. $d = 2$
17. $h = 3$
18. all real numbers
19. $w = \frac{1}{4}$
20. no solution
21. all real numbers
22. no solution
23. 2 h
24. $0 = 0$ is always true, so it is an identity; The solution is all real numbers.
25. 4 sec
26. no; The cheetah would have to maintain top speed for about 21.7 seconds, which is greater than 20 seconds.
27. a. 3 h; The cost at Studio A is $10.49 + 8h$ and the cost at Studio B is $14.99 + 6.5h$. To find when the costs are the same, set these two expressions equal and solve for the time.
    b. The costs will never be the same; *Sample answer:* The cost for Studio B changes to $14.99 + 8h$, and the new equation has no solution.
28. 50 g
29. $a = 5$; Both sides simplify to $10x + 15$.
30. $a = 4$; Both sides simplify to $20x - 8$.
31. 11, 12

Mini-Assessment

Solve the equation.

1. $8a + 96 = -24a$ $a = -3$
2. $0.2(10b + 60) = -4(b - 12)$ $b = 6$
3. $3(2m + 4) = 2(3m + 6)$ all real numbers
4. $4y = -2(6 - 2y)$ no solution
5. You drive 4 hours to reach a campground. Driving home along the same route, you increase your speed by 15 miles per hour, reducing your driving time by 1 hour. How fast were you driving to the campground? 45 mi/h

ANSWERS

32. **a.** 6 yr

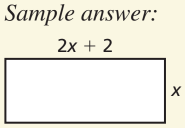
b. The left side shows the Spanish enrollment after x years, and the right side shows the French enrollment after x years; Solving the equation will indicate after how many years the enrollments will be equal.

33. no solution; 5 more than a number will never be equal to 5 less than the same number.

34. Cylinder A: 1 ft, Cylinder B: 6.5 ft

35. C, D

36. *Sample answer:*



The perimeters are both $6x + 4$.

37. 8.0 cm^2

38. $v = 5$

39. $k = -9$

40. $x = -\frac{3}{5}$

41. $n = 31.5$

42. $t = -2$

43. $c = \frac{1}{3}$

44. 27 yd^2

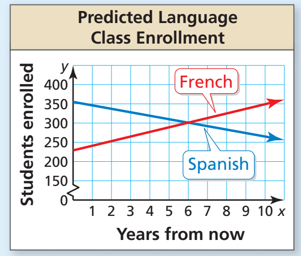
45. 48 words per minute

46. $-|21|, -16, |-10|, 22, |-32|$

47. 140 ft

32. HOW DO YOU SEE IT? The table and the graph show information about students at a high school.

	Students enrolled this year	Average rate of change
Spanish	355	9 fewer students each year
French	229	12 more students each year



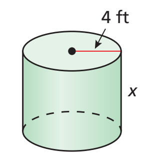
a. Use the graph to determine after how many years there will be equal enrollment in both classes.

b. How does the equation $355 - 9x = 229 + 12x$ relate to the table and the graph? How can you use this equation to determine whether your answer in part (a) is reasonable?

33. MP **REASONING** Without solving, determine whether the equation $3n + 5 = 3n - 5$ has *one solution*, *no solution*, or *infinitely many solutions*. Explain your reasoning.

GO DIGITAL

34. CONNECTING CONCEPTS Cylinder A has a radius of 6 feet and a height that is 5.5 feet less than Cylinder B. The cylinders have the same surface area. Find the height of each cylinder.

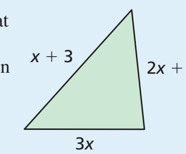


35. COLLEGE PREP For which of the following values of c and d does the equation $cx - 2 = 7x + d$ have no solution? Select all that apply.

Ⓐ $c = -7, d = -2$ Ⓑ $c = 7, d = -2$

Ⓒ $c = 7, d = 0$ Ⓓ $c = 7, d = 2$

36. THOUGHT PROVOKING Draw a different figure that has the same perimeter as the triangle shown. Explain why your figure has the same perimeter.



REVIEW & REFRESH

WATCH

37. You measure the diameter of a circular watch face to be 3.2 centimeters. Estimate the area of the watch face.

In Exercises 38–43, solve the equation. Check your solution.

38. $5 = 10 - v$

39. $2k - 3(2k - 3) = 45$

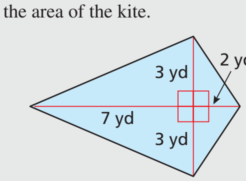
40. $x - \frac{1}{5} = -\frac{4}{5}$

41. $\frac{n}{7} = 4.5$

42. $7t - 20 = -50 - 8t$

43. $\frac{1}{2}(6c + 2) = -3(c - 1)$

44. Find the area of the kite.

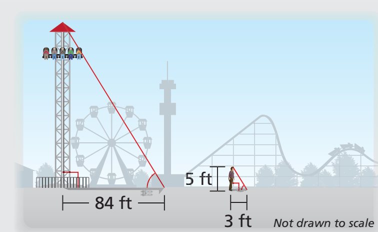


45. You type 168 words in $3\frac{1}{2}$ minutes. How many words do you type per minute?

46. Order the values from least to greatest.

$|-32|, 22, -16, -|21|, |-10|$

47. You want to find the height of a drop tower at an amusement park. You take the measurements shown in the diagram. The right triangles created by each object and its shadow are similar. How tall is the drop tower?



Laurie's Notes

Overview of Section 1.6

Introduction

- ***FOCUS* on Major Work:** This section applies the equation solving skills that students developed in previous sections to solving absolute value equations.
- ***RIGOR* in the Section:** In the exploration, students develop **conceptual understanding** as they solve absolute value equations graphically, algebraically, and numerically. The lesson provides opportunities for **procedural fluency** with examples and Self-Assessment exercises on solving absolute value equations. An **application** example and additional Self-Assessment exercises provide in-class practice with problem solving before homework.
- Students are taught that the absolute value of a number is always positive, or 0. Make sure students realize that this is not the same as asking the question, "What value or values of x will make the equation $|x| = 4$ true?"
- I have a long number line (8 feet long) above one of my whiteboards. The number line is a helpful visual reference for students in this section.

Math Misconception

- Students may believe that absolute value equations must have two solutions, one positive and one negative. Address this possible misconception when working through Examples 2 and 3.

Making Math Visible

- When absolute value is first introduced in middle school, students think of the absolute value of a number as the distance a number is from 0. So, $|4| = 4$ and $|-4| = 4$. This is the geometric definition of absolute value.
- In this lesson, students need to be secure with the algebraic definition of absolute value: $|x| = x$ when $x \geq 0$ or $|x| = -x$ when $x < 0$.
- I like to write an equation such as $|x| = 6$ on the board and then place my hand over the x and ask, "What could be hidden under my hand that would make this equation true?" Students quickly answer 6 or -6. This same technique is then used for the equation $|x - 4| = 6$. I place my hand over the expression $x - 4$ and say, "Whatever is under my hand must be equal to 6 or -6." This leads to the two linear equations, $x - 4 = 6$ and $x - 4 = -6$.
- Placing my hand over the expression inside the absolute value symbols is a way of drawing attention to the input of the absolute value function. This is not language I use at this time, but it is an approach that will occur again in Algebra 2 as students study quadratics [$(x - 4)^2$], logarithms [$\log(x - 4)$], and trigonometry [$\sin(x - 4)$]. In each case, there is a transformation of the parent function 4 units to the right.

Section Resources

PLAN

Chapter at a Glance
Everyday Connections Video Series
Lesson Plans
Pacing Guide
Skills Review Handbook

TEACH

Answer Presentation Tool
CalcChat®
CalcView®
Differentiating the Lesson
Dynamic Classroom
Interactive Tools
Resources by Chapter*
- Warm-Up
- Extra Practice
- Reteach
- Enrichment and Extension
- Puzzle Time

Skills Trainer
Tutorial Video Series

ASSESS

Dynamic Assessment System
- Self-Assessment
- Section Practice
- Section Review & Refresh
- Point-of-use Remediation
- Reports

Formative Check
Homework App
Practice Workbook and Test Prep*
- Extra Practice
- Review & Refresh
- Self-Assessment

Self-Assessment

******Available in print*

Common Core State Standards

HSA-CED.A.1, HSA-REI.A.1, HSA-REI.B.3

Learning Target

Write and solve equations involving absolute value.

Success Criteria

- Write the two linear equations related to a given absolute value equation.
- Solve equations involving one or two absolute values.
- Identify special solutions of absolute value equations.

Warm-Up

Cumulative and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL SUPPORT

Review the meaning of the phrase *absolute value*. Students may know the word *absolute* in the context of adding emphasis or describing infinite power. Explain that absolute value is the distance between a number and 0 on a number line.

Laurie's Notes

Launch the Lesson

? "Can you think of an equation that has more than one answer?" Students may suggest something like $x + 4 = 4 + x$, which has infinitely many solutions.

? "What does $|5|$ mean?" absolute value of 5 "What does $|-5|$ mean?" absolute value of -5

? "What is the solution of $|x| = 5$?" 5 and -5

? DIG DEEPER "Will absolute value equations always have two solutions?" Students may not have a sense about this yet, but it is likely that at least one student will answer *no* because absolute value cannot be negative.

EXPLORE IT!

- Students will very quickly see that $x = 1$ is a solution. Many students will then think that $x = -1$ is also a solution. Students incorrectly reason that if you take the absolute value of -1 and then add 2, the solution is 3. Point out that the addition occurs first, and then the absolute value of the sum is taken. You can also suggest that students substitute $x = -1$ and evaluate correctly to see that their reasoning is incorrect.
- Have students continue to work with their partners as you circulate and listen to conversations.
- ***COHERENCE*** In part (d), the first step of finding the point for which $|x + 2| = 0$ is a translation. When solving $|x| = 3$, you are finding the two values that are 3 units from 0, making 0 the midpoint. When solving $|x + 2| = 3$, you are still finding two values that are 3 units from a midpoint, but the midpoint has been translated 2 units to the left, where $|x + 2| = 0$. Making the connection to the translation $\left(|x| = 3 \rightarrow |x + 2| = 3\right)$ is a big idea that students will study throughout algebra.
- The spreadsheet (or table) helps students see and understand how the values of $|x + 2|$ decrease and then increase as x increases. Students are developing an understanding of the behavior of absolute value functions.
- After students work through the exploration, use *Popsicle Sticks* to hear a sampling of explanations of the different methods of solving absolute value equations.

Where Are We In Our Learning?

In this exploration, students developed a conceptual understanding of solving absolute value equations. They are now ready to make sense of the procedural approach to solving absolute value equations.

1.6 Solving Absolute Value Equations

GO DIGITAL

Learning Target Write and solve equations involving absolute value.

Success Criteria
- I can write the two linear equations related to a given absolute value equation.
- I can solve equations involving one or two absolute values.
- I can identify special solutions of absolute value equations.

EXPLORE IT! Solving an Absolute Value Equation

Work with a partner. Consider the absolute value equation

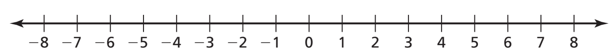
$$|x + 2| = 3.$$

a. Explain what you think this equation means.

b. Can you find a number that makes the equation true? If so, what is the number?

c. Do you think there is another number that makes the equation true? If so, find that number. Compare your answer with your classmates.

d. On the real number line below, locate the point for which the expression $|x + 2|$ is equal to 0.



Then locate the numbers you found in parts (b) and (c) on the real number line. What do you notice?

e. Complete the two linear equations below so that the solutions are the values you found in parts (b) and (c).

$x + 2 =$ _____ $x + 2 =$ _____

f. Describe how to find the solutions of the absolute value equations algebraically. Then find the solutions.

i. $|x + 2| = 5$

ii. $|x - 3| = 1$

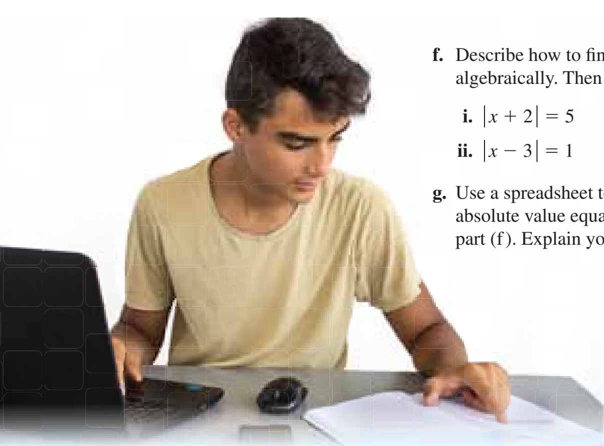
g. Use a spreadsheet to solve the absolute value equations in part (f). Explain your method.

	A	B
1	x	|x + 2|
2	−8	6
3	−7	
4	−6	
5	−5	
6	−4	
7	−3	
8	−2	
9	−1	
10	0	

=abs(A2 + 2)

Math Practice

Construct Arguments
Construct a viable argument as to why you think there is or is not more than one solution to the absolute value equation.



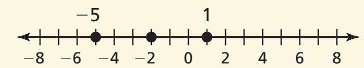
ANSWERS

a. *Sample answer:* The distance between a number x and -2 on a number line is 3 units.

b. yes; *Sample answer:* $x = 1$

c. yes; *Sample answer:* $x = -5$

d.



1 and -5 are the same distance from -2.

e. $x + 2 = 3, x + 2 = -3$

f. i. Solve $x + 2 = 5$ and $x + 2 = -5$; $x = 3$ and $x = -7$

ii. Solve $x - 3 = 1$ and $x - 3 = -1$; $x = 4$ and $x = 2$

g. i. $x = -7, x = 3$

ii. $x = 2, x = 4$; *Sample answer:* When the value of a cell in column B is equal to the right side of the equation, the corresponding value in column A is a solution of the equation.

Laurie's Notes

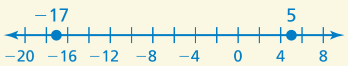
Scaffolding Instruction

- ***EMERGING*** Students may need support with the symbolic manipulation necessary for solving absolute value equations. Refer back to the exploration and remind students of the geometric definition of absolute value. Using a number line is also helpful.
- ***PROFICIENT*** Students are able to make sense of the algebraic definition of absolute value and apply it to equation solving. They may not need the support of a number line. While they may be able to use mental math to reason about the solutions of equations like $|x - 2| = 4$, can they use mental math to solve equations like $|3x + 4| = 13$?

Extra Example 1

Solve each equation. Graph the solutions, if possible.

a. $|x + 6| = 11$ $x = 5$ and $x = -17$;



b. $|2x - 1| = -4$ no solution

ELL SUPPORT

Demonstrate Example 1. Then have students work in groups to complete the Self-Assessment. Have them discuss Exercise 4 and then present their reasoning to another group. Expect students to perform according to their language levels.

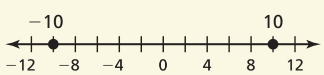
Beginner: Write and graph each solution.

Intermediate: Write and graph each solution. Use simple sentences to describe the steps of the solution.

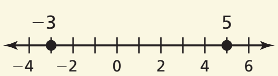
Advanced: Write and graph each solution. Use detailed sentences to help guide discussion.

ANSWERS

1. $x = -10, x = 10$



2. $x = -3, x = 5$



3. no solution
4. The absolute value of an expression is always positive.

GO DIGITAL

Solving Absolute Value Equations

Vocabulary AZ VOCAB

absolute value equation, *p. 38*

extraneous solution, *p. 41*

An **absolute value equation** is an equation that contains an absolute value expression. You can solve these types of equations by solving two related linear equations.

KEY IDEAS

Properties of Absolute Value

Let a and b be real numbers. Then the following properties are true.

1. $|a| \geq 0$ **2.** $|-a| = |a|$

3. $|ab| = |a||b|$ **4.** $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

Solving Absolute Value Equations

To solve $|ax + b| = c$ when $c \geq 0$, solve the related linear equations

$$ax + b = c \quad \text{or} \quad ax + b = -c.$$

When $c < 0$, the absolute value equation $|ax + b| = c$ has no solution because absolute value represents a distance and cannot be negative.

EXAMPLE 1 Solving Absolute Value Equations

WATCH

Solve each equation. Graph the solutions, if possible.

a. $|x - 4| = 6$ **b.** $|3x + 1| = -5$

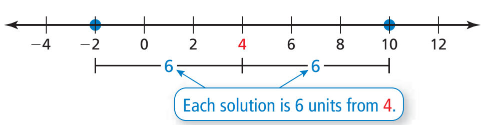
SOLUTION

a. Write the two related linear equations for $|x - 4| = 6$. Then solve.

$x - 4 = 6$ *or* $x - 4 = -6$ Write related linear equations.

$x = 10$ $x = -2$ Add 4 to each side.

▶ The solutions are $x = 10$ and $x = -2$.



b. The absolute value of an expression must be greater than or equal to 0. The expression $|3x + 1|$ cannot equal -5.

▶ So, the equation has no solution.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Graph the solutions, if possible.

1. $|x| = 10$ **2.** $|x - 1| = 4$ **3.** $|3 + x| = -\frac{1}{2}$

4. **MP REASONING** How do you know that the equation $|4x - 7| = -1$ has no solution?

Laurie's Notes

- Write the *Key Ideas* and discuss each property. Substitute values for *a* and *b* to help students make sense of each property. Be sure to connect the algebraic definition of *absolute value* to how these equations are solved.
- ? "What are the two related linear equations for $|x - 4| = 6$?" $x - 4 = 6$ *or* $x - 4 = -6$
- ***COHERENCE*** Connect the graphical representation of the solution to transformations. The solutions of the equation $|x| = 6$ have been translated 4 units to the right.
- ? **DIG DEEPER** "How are the solutions of $|x| = 10$ and $|x + 2| = 10$ alike?" The difference between each pair of solutions is 20 units (-10 and 10; -12 and 8).
- **FEEDBACK** Example 1 is related to the first two success criteria. Where are students in their learning? Can they write the two linear equations related to a given absolute value equation?

EXAMPLE 2 Solving a Multi-Step Absolute Value Equation

Solve $|3x + 9| - 10 = -4$.

SOLUTION

First isolate the absolute value expression on one side of the equation.

$|3x + 9| - 10 = -4$ Write the equation.

$|3x + 9| = 6$ Add 10 to each side.

Now write the two related linear equations for $|3x + 9| = 6$. Then solve.

$3x + 9 = 6$	*or*	$3x + 9 = -6$	Write related linear equations.
$3x = -3$		$3x = -15$	Subtract 9 from each side.
$x = -1$		$x = -5$	Divide each side by 3.

▶ The solutions are $x = -1$ and $x = -5$.

ANOTHER WAY

Using the Product Property of Absolute Value, $|ab| = |a||b|$, you can first rewrite the equation as

$3|x + 3| - 10 = -4$

and then solve.

EXAMPLE 3 Modeling Real Life

You are driving on a highway and are about 250 miles from your state's border. You set your cruise control at 60 miles per hour and plan to turn it off within 30 miles of the border on either side. Find the minimum and maximum numbers of hours you will have cruise control on.

SOLUTION

One way to solve is to write an absolute value equation that models the number x of hours you will have cruise control on. You know that the distance you travel will be within 30 miles of 250 miles.

distance you travel → $60x$; 30 miles from the state border → 30

$$|60x - 250| = 30$$

distance from state border → $60x - 250$

Write the two related linear equations for $|60x - 250| = 30$. Then solve.

$60x - 250 = 30$	*or*	$60x - 250 = -30$	Write related linear equations.
$60x = 280$		$60x = 220$	Add 250 to each side.
$x = 4\frac{2}{3}$		$x = 3\frac{2}{3}$	Divide each side by 60.

The solutions are $x = 3\frac{2}{3}$ and $x = 4\frac{2}{3}$.

▶ So, you will travel at least $3\frac{2}{3}$ hours and at most $4\frac{2}{3}$ hours with cruise control on.

Check

Minimum

$60\left(3\frac{2}{3}\right) - 250 \stackrel{?}{=} -30$

$-30 = -30$ ✓

Maximum

$60\left(4\frac{2}{3}\right) - 250 \stackrel{?}{=} 30$

$30 = 30$ ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Check your solutions.

5. $|x - 2| + 5 = 9$

6. $4|2x + 7| = 16$

7. $-2|5x - 1| - 3 = -11$

8. A plane is flying at a speed of 150 miles per hour. The pilot plans on flying at this speed for the next 160 miles, plus or minus 25 miles. Write an absolute value equation to find the minimum and maximum number of hours the plane will travel at that speed.

Extra Example 2

Solve $|3x - 6| - 9 = -3$.

$x = 0$ and $x = 4$

Extra Example 3

You are riding your bike and are about 15.5 miles from your friend's house. The distance varies by up to 1.5 miles, depending on the route you take. You are traveling at a constant speed of 8 miles per hour. Find the minimum and maximum numbers of hours it will take you to ride to your friend's house.

at least $1\frac{3}{4}$ hours and at most $2\frac{1}{8}$ hours

ELL SUPPORT

In Example 3, students may see the word *cruise* and think of a trip taken on a ship (or boat). Explain that cruise control is a device in a car that allows the driver to maintain a constant speed without using the accelerator. Have students work independently on Self-Assessment Exercises 5–7 and then compare answers with a partner. If answers differ, have partners work together to find the correct solution. Have each pair display their answers on a whiteboard for your review. For Exercise 8, allow students to work in pairs for extra language support. Then have each pair display their equation for your review.

ANSWERS

5. $x = -2, x = 6$

6. $x = -5.5, x = -1.5$

7. $x = -0.6, x = 1$

8. $|150x - 160| = 25$; $\frac{9}{10}$ h, $1\frac{7}{30}$ h

Laurie's Notes

- **MP7 Look for and Make Use of Structure:** In Example 2, students should recognize that isolating the *absolute value term* in $|3x + 9| - 10 = -4$ is the same as *isolating the variable term* in the equation $2x - 10 = -4$.
- **POPSICLE STICKS** Once the absolute value term has been isolated, write the two related linear equations. Allow time for students to finish the example with their elbow partners. Draw a Popsicle stick to ask for a student's solutions.
- **MP2 Reason Abstractly and Quantitatively & MP4 Model with Mathematics:** Pose the problem in Example 3 and give students time with their partners to consider a method of solution.
- **Self-Assessment:** As students begin to check with their neighbors, ask volunteers to put solutions on the board.

Extra Example 4

Solve (a) $|2m + 6| = |m|$ and (b) $|3x - 8| = 2|4x + 1|$.

a. $m = -6$ and $m = -2$

b. $x = -2$ and $x = \frac{6}{11}$

ANSWERS

9. $x = -3, x = 7$

10. $x = 1.4, x = 17$

11. $x = -\frac{2}{3}, x = \frac{10}{7}$

GO DIGITAL

Solving Equations with Two Absolute Values

If the absolute values of two algebraic expressions are equal, then they must either be equal to each other or be opposites of each other.

KEY IDEA

Solving Equations with Two Absolute Values

To solve $|ax + b| = |cx + d|$, solve the related linear equations

$$ax + b = cx + d \quad or \quad ax + b = -(cx + d).$$

EXAMPLE 4 Solving Equations with Two Absolute Values

WATCH

Solve (a) $|3x - 4| = |x|$ and (b) $|4x - 10| = 2|3x + 1|$.

SOLUTION

a. Write the two related linear equations for $|3x - 4| = |x|$. Then solve.

$3x - 4 = x$	*or*	$3x - 4 = -x$
$-x \quad -x$		$+x \quad +x$
$2x - 4 = 0$		$4x - 4 = 0$
$+4 \quad +4$		$+4 \quad +4$
$2x = 4$		$4x = 4$
$\frac{2x}{2} = \frac{4}{2}$		$\frac{4x}{4} = \frac{4}{4}$
$x = 2$		$x = 1$

▶ The solutions are $x = 2$ and $x = 1$.

Check

$$|3x - 4| = |x|$$
$$|3(2) - 4| \stackrel{?}{=} |2|$$
$$|2| \stackrel{?}{=} |2|$$
$$2 = 2 \ ✓$$

$$|3x - 4| = |x|$$
$$|3(1) - 4| \stackrel{?}{=} |1|$$
$$|-1| \stackrel{?}{=} |1|$$
$$1 = 1 \ ✓$$

b. Write the two related linear equations for $|4x - 10| = 2|3x + 1|$. Then solve.

$4x - 10 = 2(3x + 1)$	*or*	$4x - 10 = 2[-(3x + 1)]$
$4x - 10 = 6x + 2$		$4x - 10 = 2(-3x - 1)$
$-6x \quad -6x$		$4x - 10 = -6x - 2$
$-2x - 10 = 2$		$+6x \quad +6x$
$+10 \quad +10$		$10x - 10 = -2$
$-2x = 12$		$+10 \quad +10$
$\frac{-2x}{-2} = \frac{12}{-2}$		$10x = 8$
$x = -6$		$\frac{10x}{10} = \frac{8}{10}$
		$x = 0.8$

▶ The solutions are $x = -6$ and $x = 0.8$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Check your solutions.

9. $|x + 8| = |2x + 1|$

10. $3|x - 4| = |2x + 5|$

11. $\frac{1}{2}|x + 8| = |4x - 1|$

Laurie's Notes

? "Consider the equation $|a| = |b|$. If $a = 4$, what could b equal?" 4 or −4
"If $a = -8$, what could b equal?" 8 or −8

- Write the *Key Idea* and discuss the need for parentheses around the expression $(cx + d)$ in the second related equation.

◎ This example is related to the first two success criteria.

- **FEEDBACK** Students are solving equations with variables on both sides. As you circulate, you may notice students with the same solutions but the steps in different orders. As time permits, have students share different methods of solution. Students can gain new insights from seeing a problem solved using a different but correct method.
- Be sure to model checking the solutions.

Identifying Special Solutions

When you solve an absolute value equation, it is possible for a solution to be *extraneous*. An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

GO DIGITAL

WORDS AND MATH

The word *extraneous* contains the prefix *extra-*, which means "more than is necessary." Extraneous information is information that is not relevant or essential.

EXAMPLE 5 Identifying Extraneous Solutions

WATCH

Solve $|2x + 12| = 4x$. Check your solutions.

SOLUTION

Write the two related linear equations for $|2x + 12| = 4x$. Then solve.

$2x + 12 = 4x$	*or*	$2x + 12 = -4x$	Write related linear equations.
$12 = 2x$		$12 = -6x$	Subtract $2x$ from each side.
$6 = x$		$-2 = x$	Solve for x.

Check the apparent solutions to see if either is extraneous.

▶ The solution is $x = 6$. Reject $x = -2$ because it is extraneous.

Check

$|2x + 12| = 4x$

$|2(6) + 12| \stackrel{?}{=} 4(6)$

$|24| \stackrel{?}{=} 24$

$24 = 24$ ✓

$|2x + 12| = 4x$

$|2(-2) + 12| \stackrel{?}{=} 4(-2)$

$|8| \stackrel{?}{=} -8$

$8 \neq -8$ ✗

When solving equations of the form $|ax + b| = |cx + d|$, it is possible that one of the related linear equations will not have a solution.

EXAMPLE 6 Solving an Equation with Two Absolute Values

WATCH

Solve $|x + 5| = |x + 11|$.

SOLUTION

By equating the expression $x + 5$ and the opposite of $x + 11$, you obtain

$x + 5 = -(x + 11)$	Write related linear equation.
$x + 5 = -x - 11$	Distributive Property
$2x + 5 = -11$	Add x to each side.
$2x = -16$	Subtract 5 from each side.
$x = -8.$	Divide each side by 2.

However, by equating the expressions $x + 5$ and $x + 11$, you obtain

$x + 5 = x + 11$	Write related linear equation.
$x = x + 6$	Subtract 5 from each side.
$0 = 6$ ✗	Subtract x from each side.

which is a false statement. So, the original equation has only one solution.

▶ The solution is $x = -8$.

REMEMBER

Always check your solutions in the original equation to make sure they are not extraneous.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the equation. Check your solutions.

12. $|x + 6| = 2x$

13. $|3x - 2| = x$

14. $|2 + x| = |x - 8|$

15. $|5x - 2| = |5x + 4|$

16. WRITING How is solving an absolute value equation similar to solving an equation without an absolute value? How is it different?

Extra Example 5

Solve $|3n + 18| = 6n$. Check your solutions. $n = 6$

EVERYDAY CONNECTIONS

Learn more about extraneous solutions.

Extra Example 6

Solve $|2r - 4| = |2r + 10|$. $r = -1.5$

ANSWERS

12. $x = 6$

13. $x = \frac{1}{2}, x = 1$

14. $x = 3$

15. $x = -\frac{1}{5}$

16. *Sample answer:* The two related linear equations can be solved using the properties of equality; An absolute value equation must be rewritten as two related linear equations before solving.

Closure

Describe in words the first step in solving each equation.

a. $|x - 4| = 8$ *Sample answer:* Write the two related linear equations, $x - 4 = 8$ *or* $x - 4 = -8$.

b. $|x + 5| = |x - 3|$ *Sample answer:* Write the two related linear equations, $x + 5 = x - 3$ *or* $x + 5 = -(x - 3)$.

Laurie's Notes

- ? "What does the word *extraneous* mean?" extra, or not needed
- Define *extraneous solution.*
- Students who have been successful in solving absolute value equations will ask why they did not have extraneous solutions before. It is when there are variables on both sides of the absolute value equation that it is possible to have an extraneous solution.

Assignment Guide

Emerging: 1, 5, 9, 11, 22, 23, 25, 27, 29, 31, 35, 39, 43, 45, 47, 48

Proficient: 2, 5, 12, 16, 20, 23, 26, 28, 30, 33, 35, 40, 44, 45, 47, 48, 49, 55, 59

Advanced: 14, 19, 21, 24, 32, 34, 35, 41, 44, 45, 46, 47, 48, 50, 52, 54, 56, 57, 58, 60

- Use the red exercises to check understanding of the learning target.
- Assign Review & Refresh exercises as appropriate for continued spaced practice.

Item Leveling

DOK	Exercises
1	1–22, 29–38
2	23, 25–28, 39–55, 57, 60
3	24, 56, 58, 59

1.6 Practice WITH CalcChat® AND CalcView®

GO DIGITAL

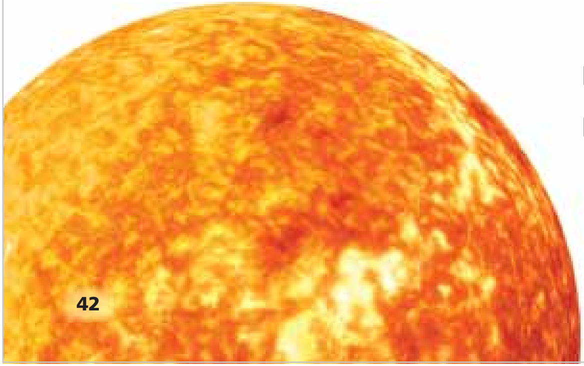
In Exercises 1–8, simplify the expression.

1. $|-9|$
2. $-|15|$
3. $|14| - |-14|$
4. $|-3| + |3|$
5. $-|-5 \cdot (-7)|$
6. $|-0.8 \cdot 10|$
7. $\left|\frac{27}{-3}\right|$
8. $\left|-\frac{-12}{4}\right|$

In Exercises 9–22, solve the equation. Graph the solution(s), if possible. *Examples 1 and 2*

9. $|r| = -2$
10. $|x| = 13.4$
11. $|m + 3| = 7$
12. $|q - 8| = 14$
13. $\left|\frac{t}{2}\right| = 6$
14. $|-3.5d| = 15.4$
15. $|4b - 5| = 19$
16. $|x - 1| + 5 = 2$
17. $2|-8w + 6| = 76$
18. $\left|\frac{1}{3}y - 2\right| - 7 = 3$
19. $-4|8 - 5n| = 13$
20. $-3\left|1 - \frac{2}{3}v\right| = -9$
21. $3 = -2\left|\frac{1}{4}s - 5\right| + 3$
22. $9|4p + 2| + 8 = 35$

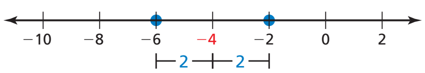
23. **MODELING REAL LIFE** The average distance from Earth to the Sun is 92.95 million miles. The actual distance varies from the average by up to 1.55 million miles. Write and solve an absolute value equation to find the minimum and maximum distance from Earth to the Sun. *Example 3*

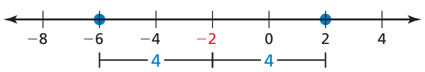


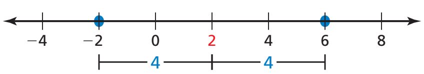
24. **MODELING REAL LIFE** The recommended mass of a soccer ball is 0.43 kilogram. The actual mass is allowed to vary by up to 20 grams.
    a. Write and solve an absolute value equation to find the minimum and maximum acceptable soccer ball masses.
    b. A soccer ball has a mass of 423 grams. The soccer ball loses 0.016 kilogram of mass over time. Is the mass now acceptable? Explain.

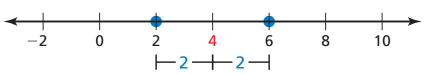
MP STRUCTURE In Exercises 25–28, match the absolute value equation with its graph without solving the equation.

25. $|x + 2| = 4$
26. $|x - 4| = 2$
27. $|x - 2| = 4$
28. $|x + 4| = 2$

A. 

B. 

C. 

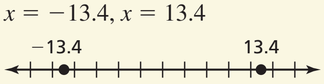
D. 

In Exercises 29–38, solve the equation. Check your solutions. *Examples 4, 5, and 6*

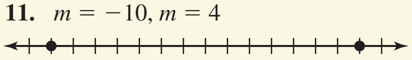
29. $|4n - 15| = |n|$
30. $|2c + 8| = |10c|$
31. $|3k - 2| = 2|k + 2|$
32. $\left|\frac{1}{2}b - 8\right| = \left|\frac{1}{4}b - 1\right|$
33. $4|p - 3| = |2p + 8|$
34. $2|4w - 1| = 3|4w + 2|$
35. $|3h + 1| = 7h$
36. $|6a - 5| = 4a$
37. $\left|f - \frac{4}{3}\right| = \left|f + \frac{1}{6}\right|$
38. $|3x - 4| = |3x - 5|$

ANSWERS

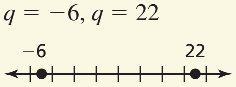
1. 9
2. −15
3. 0
4. 6
5. −35
6. 8
7. 9
8. 3
9. no solution
10. $x = -13.4, x = 13.4$



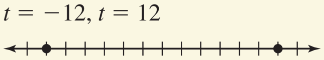
11. $m = -10, m = 4$



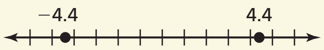
12. $q = -6, q = 22$



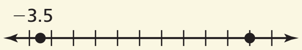
13. $t = -12, t = 12$



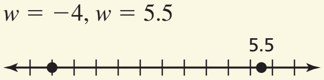
14. $d = -4.4, d = 4.4$



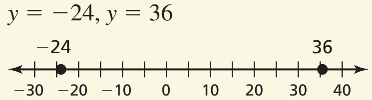
15. $b = -3.5, b = 6$



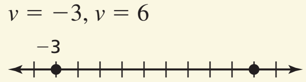
16. no solution
17. $w = -4, w = 5.5$



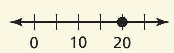
18. $y = -24, y = 36$



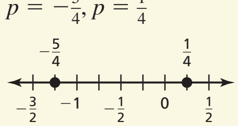
19. no solution
20. $v = -3, v = 6$



21. $s = 20$



22. $p = -\frac{5}{4}, p = \frac{1}{4}$



23. $|d - 92.95| = 1.55$; 91.4 million mi, 94.5 million mi
24. a. $|x - 430| = 20$; 410 g, 450 g
    b. no; 407 grams is below the minimum acceptable weight.
25. B
26. D
27. C
28. A

29–38. See Additional Answers.

MP **REASONING** **In Exercises 39–42, write an absolute value equation that has the given solutions.**

39. $x = 8$ and $x = 18$

40. $x = -6$ and $x = 10$

41. $x = 1.5$ and $x = 8.5$

42. $x = -10$ and $x = -5$

ERROR ANALYSIS **In Exercises 43 and 44, describe and correct the error in solving the equation.**

43. ✗

$$|2x - 1| = -9$$
$$2x - 1 = -9 \quad or \quad 2x - 1 = -(-9)$$
$$2x = -8 \qquad 2x = 10$$
$$x = -4 \qquad x = 5$$

The solutions are $x = -4$ and $x = 5$.

44. ✗

$$|5x + 8| = x$$
$$5x + 8 = x \quad or \quad 5x + 8 = -x$$
$$4x + 8 = 0 \qquad 6x + 8 = 0$$
$$4x = -8 \qquad 6x = -8$$
$$x = -2 \qquad x = -\frac{4}{3}$$

The solutions are $x = -2$ and $x = -\frac{4}{3}$.

45. **MODELING REAL LIFE** Starting from 300 feet away, a car drives toward you. It then passes by you at a constant speed of 48 feet per second. The distance d (in feet) of the car from you after t seconds is given by the equation $d = |300 - 48t|$.

a. Explain what $48t$ represents in the given equation.

b. At what times is the car 60 feet from you?

46. MP **REASONING** Without solving completely, place each equation into one of the three categories. Explain your reasoning.

No solution	One solution	Two solutions

$|x - 2| + 6 = 0$ $\quad$ $|x + 3| - 1 = 0$

$|x + 8| + 2 = 7$ $\quad$ $|x - 1| + 4 = 4$

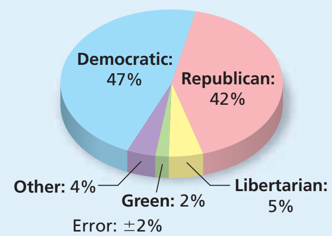
$|x - 6| - 5 = -9$ $\quad$ $|x + 5| - 8 = -8$

47. **MAKING AN ARGUMENT** Your friend says that the absolute value equation $|3x + 8| - 9 = -5$ has no solution because the constant on the right side of the equation is negative. Is your friend correct? Explain.

GO DIGITAL

48. **HOW DO YOU SEE IT?** The circle graph shows the results of a survey of registered voters the day of an election.

Which Party's Candidate Will Get Your Vote?



The error given in the graph means that the actual percent could be 2% more or 2% less than the percent reported by the survey.

a. What does the survey predict are the minimum and maximum percents of voters who will vote Republican? Green?

b. Write absolute value equations to represent your answers in part (a).

c. One candidate receives 44% of the vote. Which party do you think the candidate belongs to? Explain.

ABSTRACT REASONING **In Exercises 49–52, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

49. If $x^2 = a^2$, then $|x|$ is ________ equal to $|a|$.

50. If a and b are real numbers, then $|a - b|$ is ________ equal to $|b - a|$.

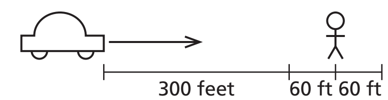
51. For any real number p, the equation $|x - 4| = p$ will ________ have two solutions.

52. For any real number p, the equation $|x - p| = 4$ will ________ have two solutions.

53. **WRITING** Explain why absolute value equations can have no solution, one solution, or two solutions. Give an example of each case.

Slow Reveal

For Exercise 45, draw a picture to show a car 300 feet from a person. Use an arrow to indicate that the car is driving towards the person. Mark the two points at which the car will be 60 feet from the person. You want students to understand that there will be two places where the car will be 60 feet from you, one on either side.



ANSWERS

39. $|x - 13| = 5$

40. $|x - 2| = 8$

41. $|x - 5| = 3.5$

42. $|x + 7.5| = 2.5$

43. The absolute value cannot be negative. So, there is no solution.

44. Both solutions are extraneous. So, there is no solution.

45. **a.** the distance the car travels after t seconds

b. 5 sec, 7.5 sec

46. No solution: $|x - 2| + 6 = 0$, $|x - 6| - 5 = -9$
One Solution: $|x - 1| + 4 = 4$, $|x + 5| - 8 = -8$
Two solutions: $|x + 8| + 2 = 7$, $|x + 3| - 1 = 0$

47. no; The absolute value has to be isolated first, which makes the constant on the right positive.

48. **a.** 40%, 44%; 0%, 4%

b. Set the absolute value of x minus the percentage of the party interested in equal to 2, then solve for the minimum and maximum values.

c. Republican; 44% is within the error range of 42%.

49. always; Square roots of the same number have the same absolute value.

50. always; $a - b$ and $b - a$ are opposites, so they will have the same absolute values.

51. sometimes; The equation will only have two solutions if p is positive.

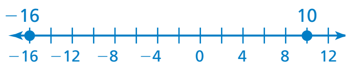
52. always; The solutions are $p + 4$ and $p - 4$.

53. Absolute value equations will have no solution when the absolute value is equal to a negative number, one solution when the absolute value is equal to zero, and two solutions when the absolute value is equal to a positive number; *Sample answer:* $|x + 12| = -2$ has no solution, $|x + 12| = 0$ has one solution, and $|x + 12| = 2$ has two solutions.

Mini-Assessment

Solve the equation. Graph the solution(s), if possible.

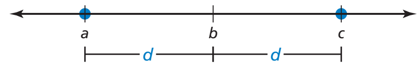
1. $|x + 3| = 13$ $x = 10$ and $x = -16$;



2. $|3x - 1| + 6 = 4$ no solution
3. $|6x + 8| = |10x|$ $x = 2$ and $x = -0.5$;

4. Solve $2x - 3 = |x - 2|$. Check your solutions. $x = \frac{5}{3}$
5. A truck driver is driving on an interstate and is about 150 miles from his planned exit. He is traveling at a speed of 50 miles per hour and plans to stop for a rest within 30 miles of the exit on either side. Find the minimum and maximum numbers of hours the truck driver will drive before stopping for a rest. at least 2.4 hours and at most 3.6 hours

54. MP **STRUCTURE** Complete the equation $|x - \square| = \square$ with a, b, c, or d so that the equation is graphed correctly.



55. **COLLEGE PREP** Which values are solutions of the equation $5 = -\frac{2}{3}|4x - 7| + 11$? Select all that apply.

Ⓐ $x = -\frac{1}{2}$ Ⓑ $x = \frac{1}{2}$

Ⓒ $x = \frac{3}{4}$ Ⓓ $x = \frac{11}{4}$

Ⓔ $x = 4$ Ⓕ no solution

56. **CRITICAL THINKING** Solve the equation shown. Explain how you found your solution(s).

$$8|x + 2| - 6 = 5|x + 2| + 3$$

57. **OPEN-ENDED** Describe a real-life situation that can be modeled by an absolute value equation with the solutions $x = 62$ and $x = 72$.

GO DIGITAL

58. **THOUGHT PROVOKING** What is the maximum number of solutions an equation of the form $||ax - b| + c| = d$ can have? Justify your reasoning with an example.

59. DIG DEEPER The minimum normal glucose level for a fasting adult is 70 mg/dL. The maximum normal level is 99 mg/dL. Write an absolute value equation that represents the minimum and maximum normal glucose levels.

60. **ABSTRACT REASONING** How many solutions does the equation $a|x + b| + c = d$ have when $a > 0$ and $c = d$? when $a < 0$ and $c > d$? Explain your reasoning.

REVIEW & REFRESH

WATCH

In Exercises 61–64, solve the equation. Check your solution(s).

61. $3c + 1 = c + 1$

62. $4(6k + 9) = 8(3k - 2)$

63. $-10 - 12g = -4(3g + 2.5)$

64. $|y - 4| = |y + 10|$

65. **MODELING REAL LIFE** An outdoor music festival provides 4000 square yards of land for the audience. Attendees are permitted to reserve a section using a rectangular tarp with a length of 12 feet and a width of 10 feet. About how many sections can be reserved at the music festival?

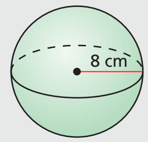
66. Simplify $12 + 5h - 3.5 + 8h$.

In Exercises 67 and 68, write the number in standard form.

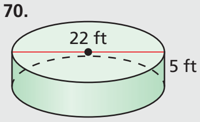
67. 7×10^{-8} 68. 2.59×10^3

In Exercises 69 and 70, find the volume of the figure. Round your answer to the nearest tenth.

69.



70.



JUSTIFYING STEPS **In Exercises 71 and 72, identify the property of equality that makes Equation 1 and Equation 2 equivalent.**

71.

Equation 1	$3x + 8 = x - 1$
Equation 2	$3x + 9 = x$

72.

Equation 1	$4y = 28$
Equation 2	$y = 7$

73. A circle has an area of 36π square inches. Find the radius.

74. A triangle has a height of 8 feet and an area of 48 square feet. Find the base.

75. **MODELING REAL LIFE** You are driving your moped to school. The drive is about 12.5 miles, but the distance varies by up to 1.25 miles, depending on the route you take. You drive at a constant speed of 25 miles per hour. Find the minimum and maximum number of minutes it will take you to travel to school.

In Exercises 76–79, complete the statement. Round to the nearest hundredth, if necessary.

76. 9900 sec = $\square$ h 77. 0.25 T = $\square$ oz

78. 11.5 qt ≈ $\square$ mL 79. 49.6 cm ≈ $\square$ ft

ANSWERS

54. b; d
55. A, E
56. $x = 1$, $x = -5$; Interpret $|x + 2|$ as a single quantity and solve for it, then solve the resulting absolute value equation.
57. *Sample answer:* In a round of a trivia game, you lose 5 points for an incorrect answer and win 5 points for a correct answer. You have 67 points. x represents your score after your next answer.
58. 4; *Sample answer:* When $a = 2$, $b = -5$, $c = -4$, and $d = 3$, the solutions are $x = -6$, $x = -3$, $x = -2$, and $x = 1$.
59. $|x - 84.5| = 14.5$
60. 1; If $c = d$, then the absolute value expression will be equal to 0; 2; If $c > d$, then $d - c$ will be negative. When this value is divided by a negative value of a, the result will be positive.
61. $c = 0$
62. no solution
63. all real numbers
64. $y = -3$
65. 300 sections
66. $13h + 8.5$
67. 0.00000007
68. 2590
69. 2144.7 cm^3
70. 1900.7 ft^3
71. Addition Property of Equality
72. Division Property of Equality
73. 6 in.
74. 12 ft
75. 27 min, 33 min
76. $2\frac{3}{4}$
77. 8000
78. 10,849.06, or 10,925
79. 1.61, 1.63, or 1.65

Laurie's Notes

Overview of Section 1.7

Introduction

- ***FOCUS* on Major Work:** In the study of high school mathematics, students will work with equations involving more than one variable. In Algebra 1, this occurs when students start to represent linear equations in two variables, *x* and *y*. Previous sections presented equation-solving techniques, often referred to as symbolic manipulation, that students will now apply to equations involving more than one variable.
- ***RIGOR* in the Section:** In the exploration, students develop **conceptual understanding** as they use multiple equations to relate quantities. The lesson provides opportunities for **procedural fluency** with examples and Self-Assessment exercises on rewriting literal equations and formulas. **Application** examples related to three common contexts (temperature, simple interest, and the Distance Formula) and additional Self-Assessment exercises provide in-class practice with problem solving before homework.
- The lesson uses formulas, which should be familiar to students, to practice the skill of rewriting equations. In addition, students will rewrite an equation in two variables, *x* and *y*, to solve for *y*. Rewriting an equation in function form is an important skill that you want students to be secure with now. Any weaknesses in rational-number operations will surface, so it is important to differentiate where students are having a problem in this lesson. Is the challenge with using inverse operations correctly, or is it problems with rational numbers, or both?
- New vocabulary is introduced. You want students to understand that the formulas they are familiar with are also referred to as *literal equations.*

Formative Assessment Tip

WHITEBOARDS

- Whiteboards can be used to provide individual responses, or used with small groups to encourage student collaboration and consensus on a problem or solution method. Whiteboards can be used at the beginning of class for the Warm-Up or throughout the lesson to elicit student responses. Unlike writing on scrap paper (individual response) or chart paper (group response), responses can be erased and modified easily. As understanding progresses, responses can reflect this growth.
- Use whiteboards for more than quick responses. Sizable whiteboards can be used to communicate thinking, providing evidence of how a problem was solved. When students display their whiteboards in the front of the room, classmates can critique their reasoning or methods of solution.
- **FEEDBACK** When students hold up their whiteboards, you receive valuable feedback about their thinking. The process can be private and informative. Students can also swap whiteboards to receive peer feedback on their work.

Section Resources

PLAN
Chapter at a Glance Everyday Connections Video Series Lesson Plans Pacing Guide Skills Review Handbook
TEACH
Answer Presentation Tool CalcChat® CalcView® Differentiating the Lesson Dynamic Classroom Interactive Tools Resources by Chapter* • Warm-Up • Extra Practice • Reteach • Enrichment and Extension • Puzzle Time Skills Trainer Tutorial Video Series
ASSESS
Assessment Book* • End-of-Chapter Quiz Dynamic Assessment System • Self-Assessment • Section Practice • Section Review & Refresh • End-of-Chapter Quiz • Point-of-use Remediation • Reports Formative Check Homework App Practice Workbook and Test Prep* • Extra Practice • Review & Refresh • Self-Assessment Self-Assessment

**Available in print*

Common Core State Standards

HSA-CED.A.4

Learning Target

Solve literal equations for given variables.

Success Criteria

- Identify a literal equation.
- Use properties of equality to rewrite literal equations.
- Use rewritten formulas to solve problems.

Warm-Up

Cumulative and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL SUPPORT

Students may be familiar with the word *literal* as referring to something that is real versus something that is figurative. Explain that the term *literal equation* has a very specific meaning. It is an equation that has two or more variables.

Laurie's Notes

Launch the Lesson

? Distribute whiteboards and ask students to answer the following questions.

- "I drove 2.1 hours at 65 miles per hour. How far did I drive?" 136.5 mi
- "It took me 32 minutes to walk 4 kilometers. How fast was I walking on average?" $\frac{1}{8}$ kilometer per minute, or 7.5 kilometers per hour
- "At 8 miles per hour, how long will it take me to bike 6 miles?" $\frac{3}{4}$ hour, or 45 minutes

? "How are these three problems alike?" They all involve distance, rate, and time.

- Write the three equations relating distance, rate, and time.

$$d = rt \qquad r = \frac{d}{t} \qquad t = \frac{d}{r}$$

- Explain that, in the lesson, students will be rewriting familiar formulas and solving them for different variables, just as the familiar formula $d = rt$ can be solved for r or t.

EXPLORE IT!

- Pose the situation. Then have students discuss part (a) in small groups. On a whiteboard, each group can write how they think the quantities are related. It is unlikely that all groups will say that the number of gallons of gasoline times the price per gallon of gasoline equals the total cost of the gasoline. Like the quantities in the launch, there are different ways to represent the relationship.
- Allow time for students to work with their partners to complete the exploration.
- **FEEDBACK** Students can display their whiteboards under a document camera to quickly share their work. Feedback should be provided to the student who is sharing his or her work. This feedback should help answer, "Where am I in my learning?" or "Where am I going next in my learning?"

Where Are We In Our Learning?

In this exploration, students are working with literal equations without giving a formal name to the equations. Say, "You were able to write and use an equation to solve a real-life application. More specifically, the equation you wrote had three different forms. The equations showed how two of the three quantities (or variables) were related to the third. In the lesson, you will learn how to rewrite literal equations, a skill that is frequently used in mathematics."

1.7 Rewriting Equations and Formulas

GO DIGITAL

Learning Target Solve literal equations for given variables.

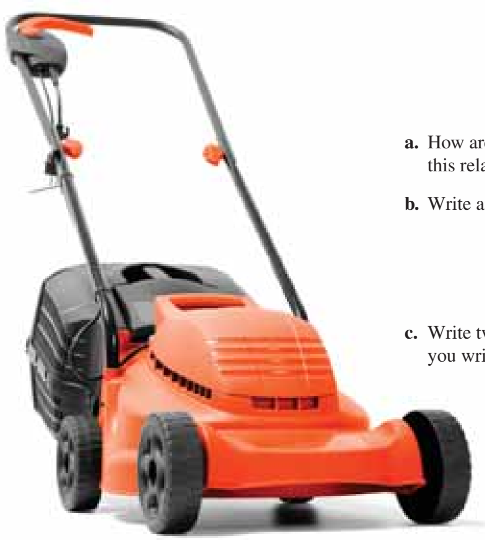
Success Criteria
- I can identify a literal equation.
- I can use properties of equality to rewrite literal equations.
- I can use rewritten formulas to solve problems.

EXPLORE IT! Using Multiple Equations to Relate Quantities

Work with a partner. A landscaper purchases gasoline for various lawn care equipment. Consider the following quantities involved in this situation.

Number of gallons of gasoline	Price per gallon of gasoline

Total cost of gasoline



a. How are these quantities related in this situation? How can you represent this relationship?

b. Write an equation that represents the relationship among the three quantities.

Total cost of gasoline = ?

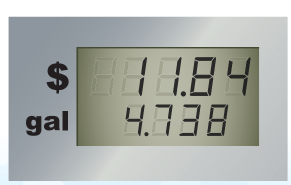
c. Write two more equations that represent the same relationship. How did you write these equations?

Price per gallon of gasoline = ?

Number of gallons of gasoline = ?

d. Why do you think it is beneficial to have solved an equation for each quantity?

e. After pumping gasoline, the landscaper sees the screen shown on the pump. Which quantity is missing? Find the missing quantity and explain how you found it.



Math Practice
Calculate Accurately
How many decimal places did you use in your answer in part (e)? Explain your reasoning.

ANSWERS

1. **a.** The total cost of gasoline is equal to the product of the number of gallons of gasoline and the price per gallon of gasoline.

b. [Number of gallons of gasoline] × [Price per gallon of gasoline]

c. [Total cost of gasoline] ÷ [Number of gallons of gasoline];

[Total cost of gasoline] ÷ [Price per gallon of gasoline]

d. *Sample answer:* Substitute values for each quantity you know to find the unknown value.

e. price per gallon of gasoline; \$2.50; *Sample answer:* Solve the first equation from part (c).

Laurie's Notes

Scaffolding Instruction

- ***EMERGING*** Students may need support with the symbolic manipulation of rewriting literal equations.
- ***PROFICIENT*** If students are confident with equation solving, have them complete Self-Assessment Exercises 1–6.

Extra Example 1

Solve the literal equation $7x - 4y = 12$ for y. $y = \frac{7}{4}x - 3$

Extra Example 2

Solve the literal equation $3w + 4pw = a$ for w. $w = \dfrac{a}{3 + 4p}$

ELL SUPPORT

Demonstrate Examples 1 and 2. Then have students work in groups to discuss and complete the Self-Assessment. Provide questions to guide their work: Which property will you use in each step? How can you simplify? Expect students to perform according to their language levels.

Beginner: Write out each solution and state the answer.

Intermediate: Use simple sentences to discuss the process for solving each equation.

Advanced: Use detailed sentences and help guide the discussion for solving each equation.

ANSWERS

1. $y = 3 + \frac{1}{3}x$
2. $y = x - \frac{5}{2}$
3. $y = 5 - 2x$
4. $x = y$
5. $x = \dfrac{m}{2 + k}$
6. $x = \dfrac{y - 3}{5 - k}$
7. Solve $6y = 24 - 3x$ for y in terms of x; $y = 4 - \frac{1}{2}x$; $x = 8 - 2y$

GO DIGITAL

Vocabulary

literal equation, *p. 46*
formula, *p. 47*

Rewriting Literal Equations

An equation that has two or more variables is called a **literal equation**. To rewrite a literal equation, solve for one variable in terms of the other variable(s).

EXAMPLE 1 Rewriting a Literal Equation

Solve the literal equation $3y + 4x = 9$ for y.

SOLUTION

$3y + 4x = 9$	Write the equation.
$3y + 4x - 4x = 9 - 4x$	Subtraction Property of Equality
$3y = 9 - 4x$	Simplify.
$\frac{3y}{3} = \frac{9 - 4x}{3}$	Division Property of Equality
$y = 3 - \frac{4}{3}x$	Simplify.

▶ The rewritten literal equation is $y = 3 - \frac{4}{3}x$.

EXAMPLE 2 Rewriting a Literal Equation

Solve the literal equation $y = 3x + 5xz$ for x.

SOLUTION

$y = 3x + 5xz$	Write the equation.
$y = x(3 + 5z)$	Distributive Property
$\frac{y}{3 + 5z} = \frac{x(3 + 5z)}{3 + 5z}$	Division Property of Equality
$\frac{y}{3 + 5z} = x$	Simplify.

▶ The rewritten literal equation is $x = \dfrac{y}{3 + 5z}$.

REMEMBER
Division by 0 is undefined.

In Example 2, you must assume that $z \neq -\frac{3}{5}$ in order to divide by $3 + 5z$. In general, when dividing by a variable or variable expression to rewrite a literal equation, assume that the variable or variable expression does not equal 0.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the literal equation for *y*.

1. $3y - x = 9$
2. $2x - 2y = 5$
3. $20 = 8x + 4y$

Solve the literal equation for *x*.

4. $y = 5x - 4x$
5. $2x + kx = m$
6. $3 + 5x - kx = y$

7. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Solve $3x + 6y = 24$ for x.	Solve $24 - 3x = 6y$ for x.
Solve $6y = 24 - 3x$ for y in terms of x.	Solve $24 - 6y = 3x$ for x in terms of y.

Laurie's Notes

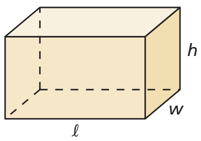
- Write the definition of a *literal equation*. Solicit examples of familiar formulas and explain that each formula is a type of literal equation. This will help students identify literal equations.
- **MP7 Look for and Make Use of Structure:** When solving literal equations, you want students to verbalize the operations represented. For example, when solving $3y + 4x = 9$, students should understand there are two variable terms. To isolate the $3y$-term, subtract $4x$. In other words, approach solving $3y + 4x = 9$ for y in the same way as you would solve $3y + 4 = 9$ for y.
- ? "In Example 1, can 9 and $4x$ be combined? Explain." No, they are not like terms.
- In Example 2, use color to highlight each x: $y = 3x + 5xz$. Using the Distributive Property is not obvious to most students because they do not think of the property in a factoring context.

Rewriting and Using Formulas for Area

GO DIGITAL

A **formula** shows how one variable is related to one or more other variables. A formula is a type of literal equation.

EXAMPLE 3 **Rewriting a Formula for Surface Area** WATCH



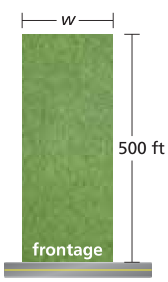
The formula for the surface area S of a rectangular prism is $S = 2\ell w + 2\ell h + 2wh$. Solve the formula for the length ℓ.

SOLUTION

$S = 2\ell w + 2\ell h + 2wh$	Write the equation.
$S - 2wh = 2\ell w + 2\ell h + 2wh - 2wh$	Subtraction Property of Equality
$S - 2wh = 2\ell w + 2\ell h$	Simplify.
$S - 2wh = \ell(2w + 2h)$	Distributive Property
$\dfrac{S - 2wh}{2w + 2h} = \dfrac{\ell(2w + 2h)}{2w + 2h}$	Division Property of Equality
$\dfrac{S - 2wh}{2w + 2h} = \ell$	Simplify.

▶ When you solve the formula for ℓ, you obtain $\ell = \dfrac{S - 2wh}{2w + 2h}$.

EXAMPLE 4 **Modeling Real Life** WATCH INFO



You own a rectangular lot that is 500 feet deep. It has an area of 100,000 square feet. To pay for a new water system, you are assessed \$5.50 per foot of lot frontage. How much are you assessed for the new water system?

SOLUTION

To find the amount assessed, first find the frontage of your lot. In the formula for the area of a rectangle, let the width w represent the lot frontage.

$A = \ell w$	Write the formula for area of a rectangle.
$\dfrac{A}{\ell} = w$	Divide each side by ℓ to solve for w.
$\dfrac{100{,}000}{500} = w$	Substitute 100,000 for A and 500 for ℓ.
$200 = w$	Simplify.

Your frontage is 200 feet. Multiply this by the cost of frontage, \$5.50 per foot.

$$\frac{\$5.50}{1\ \cancel{\text{ft}}} \cdot 200\ \cancel{\text{ft}} = \$1100$$

▶ So, you are assessed \$1100 for the new water system.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the formula for the indicated variable.

8. Area of a triangle: $A = \frac{1}{2}bh$; Solve for h.

9. Surface area of a cone: $S = \pi r^2 + \pi r\ell$; Solve for ℓ.

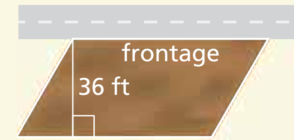
10. You want to enclose a rectangular region with an area of 1200 square feet and a length of 40 feet, 50 feet, or 60 feet. Find the perimeter for each possible region. Explain why you might rewrite the area formula to find the solutions.

Extra Example 3

The formula for the sum S of the interior angle measures of a polygon with n sides is $S = (n - 2)180°$. Solve the formula for n. $n = \dfrac{S}{180°} + 2$

Extra Example 4

You own a parallelogram-shaped lot that is 36 feet deep. It has an area of 2592 square feet. To replace the sewer line, you are assessed \$106 per foot of lot frontage. How much are you assessed for replacing the sewer line?



\$7632

ANSWERS

8. $h = \dfrac{2A}{b}$

9. $\ell = \dfrac{S - \pi r^2}{\pi r}$

10. 140 ft, 148 ft, 160 ft; *Sample answer:* It is easier to solve for w.

EVERYDAY CONNECTIONS Learn more about rewriting formulas.

Laurie's Notes

- In Example 3, use color to highlight each ℓ: $S = 2\ell w + 2\ell h + 2wh$.
- **POPSICLE STICKS** Pose the problem in Example 4. Allow time for elbow partners to work through the example on whiteboards. As you circulate, observe solution methods. Determine if there are unique and/or particularly clear solutions that should be seen by the class. Either ask for those solutions to be shared, or make it appear random by placing particular Popsicle sticks in the center of the can.

Extra Example 5

Solve the perimeter of a rectangle formula $P = 2\ell + 2w$ for ℓ. $\ell = \dfrac{P - 2w}{2}$

Extra Example 6

On a July day, the temperature at the peak of Mount Everest is −16°C. Is that temperature warmer or colder than 0°F? warmer

ANSWERS

11. no; yes

Rewriting and Using Other Common Formulas

GO DIGITAL

KEY IDEA

Common Formulas

Temperature F = degrees Fahrenheit, C = degrees Celsius

$C = \frac{5}{9}(F - 32)$

Simple Interest I = interest, P = principal,

r = annual interest rate (decimal form),

t = time (years)

$I = Prt$

Distance d = distance traveled, r = rate, t = time

$d = rt$

EXAMPLE 5 Rewriting the Formula for Temperature

WATCH

Solve the temperature formula for F.

SOLUTION

$C = \frac{5}{9}(F - 32)$ Write the temperature formula.

$\frac{9}{5}C = F - 32$ Multiply each side by $\frac{9}{5}$.

$\frac{9}{5}C + 32 = F$ Add 32 to each side.

▶ The rewritten formula is $F = \frac{9}{5}C + 32$.

EXAMPLE 6 Modeling Real Life

WATCH

Which has the greater surface temperature: Mercury or Venus?

SOLUTION

Convert the Celsius temperature of Mercury to degrees Fahrenheit.

$F = \frac{9}{5}C + 32$ Write the rewritten formula from Example 5.

$= \frac{9}{5}(427) + 32$ Substitute 427 for C.

$= 768.6 + 32$ Multiply.

$= 800.6$ Add.

▶ The surface temperature of Mercury is 800.6°F. Because 864°F is greater than 800.6°F, Venus has the greater surface temperature.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. A fever is generally considered to be a body temperature greater than 100°F. Your friend has a temperature of 37°C. Does your friend have a fever when the temperature increases by 1°F? 1°C?

Laurie's Notes

- **MP6 Attend to Precision:** In the simple interest formula, students should note that both rate and time are in terms of years.
- Students may know the formula for converting Celsius to Fahrenheit. If so, that should be the common formula stated and change Example 5 to rewriting the formula to solve for Celsius.
- **Extension:** Students have likely heard about the Kelvin temperature scale. Have students research a formula for converting Kelvin to Fahrenheit or Celsius and then rewrite the formula.
- Ask students to discuss where they are in their learning. What is challenging about rewriting literal equations? Why is it helpful to rewrite literal equations? You want students to understand where they are going in their learning.

EXAMPLE 7 Modeling Real Life

WATCH

GO DIGITAL

You deposit \$5000 in an account that earns simple interest. After 6 months, the account earns \$162.50 in interest. What is the annual interest rate?

SOLUTION

One way to find the annual interest rate is to solve the simple interest formula for r.

$I = Prt$	Write the simple interest formula.
$\frac{I}{Pt} = r$	Divide each side by Pt to solve for r.
$\frac{162.50}{(5000)(0.5)} = r$	Substitute 162.50 for I, 5000 for P, and 0.5 for t.
$0.065 = r$	Simplify.

▶ The annual interest rate is 0.065, or 6.5%.

COMMON ERROR

The unit of t is years. Be sure to convert months to years.

$$\frac{1 \text{ yr}}{12 \text{ mo}} \cdot 6 \text{ mo} = 0.5 \text{ yr}$$

EXAMPLE 8 Modeling Real Life

WATCH

A truck driver averages 60 miles per hour while delivering freight to a customer. On the return trip, the driver averages 50 miles per hour due to construction. The total driving time is 6 hours and 36 minutes. How long does each trip take?

SOLUTION

Step 1 Rewrite the Distance Formula to write expressions that represent the two trip times. Solving the formula $d = rt$ for t, you obtain $t = \frac{d}{r}$. So, $\frac{d}{60}$ represents the delivery time, and $\frac{d}{50}$ represents the return trip time.

Step 2 Use these expressions and the total driving time to write and solve an equation to find the distance one way.

$\frac{d}{60} + \frac{d}{50} = 6.6$	Total driving time: $6 \text{ h} + \frac{36}{60} \text{ h} = 6.6 \text{ h}$
$\frac{11d}{300} = 6.6$	Add the fractions using a common denominator of 300.
$11d = 1980$	Multiply each side by 300.
$d = 180$	Divide each side by 11.

The distance one way is 180 miles.

Step 3 Use the expressions from Step 1 to find the two trip times.

▶ So, the delivery takes $180 \text{ mi} \div \frac{60 \text{ mi}}{1 \text{ h}} = 3$ hours and the return trip takes $180 \text{ mi} \div \frac{50 \text{ mi}}{1 \text{ h}} = 3.6$ hours, or 3 hours and 36 minutes.

Check Check that the sum of the driving times is equal to the total driving time.

$$\begin{array}{r} 3 \text{ h} \\ + \; 3 \text{ h} \;\; 36 \text{ min} \\ \hline 6 \text{ h} \;\; 36 \text{ min} \end{array}$$ ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

12. How much money must you deposit in a simple interest account to earn \$500 in interest in 5 years at 4% annual interest?

13. A truck driver averages 60 miles per hour while delivering freight and 45 miles per hour on the return trip. The total driving time is 7 hours. How much longer does the return trip take than the delivery?

Extra Example 7

You deposit \$3500 in an account that earns simple interest at an annual rate of 4%. How long must you leave the money in the account to earn \$350 in interest? 2.5 yr

Extra Example 8

Your cousin averages 60 miles per hour while making a work delivery. On the return trip, your cousin averages 45 miles per hour due to weather. The total driving time is 5 hours and 15 minutes. How long does each trip take? delivery: 2.25 h, or 2 h 15 min; return trip: 3 h

ELL SUPPORT

Discuss the context of a truck delivering freight. Before assigning the Self-Assessment, clarify any difficult vocabulary. Allow students to work in pairs for extra support with the language of word problems. Check comprehension by having each pair write their equations on a whiteboard to display for your review. Monitor discussions and provide support as needed.

ANSWERS

12. \$2500

13. 1 h

Laurie's Notes

- In solving the simple interest formula for r in Example 7, students often divide by P and then divide by t. By doing so, they end up with a compound fraction on the left side of the equation.
- **DIG DEEPER** "Why can you divide by P and t in one step?" *Sample answer:* Using the Commutative Property of Multiplication, $Prt = Ptr$. Then the product Pt can be considered one of the factors of Ptr.
- **WHITEBOARDS** Pose the question in Example 8 and allow time for elbow partners to work through the example. Circulate and ask deeper questions to assist students in making progress.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Have a pair of students share their solution.

Closure

Solve $2x + 4y = 11$ for y.
$y = \frac{11}{4} - \frac{1}{2}x$

Assignment Guide

Emerging: 3, 11, 21, 23, 25, 30, 31, 33, 34, 35, 38

Proficient: 6, 13, 22, 24, 26, 27, 29, 31, 33, 34, 35, 38, 39, 41

Advanced: 10, 18, 24, 27, 29, 32, 33, 36, 37, 38, 40, 41, 42, 43

- Use the red exercises to check understanding of the learning target.
- Assign Review & Refresh exercises as appropriate for continued spaced practice.

Item Leveling

DOK	Exercises
1	1–20, 25–28
2	21–24, 29–38, 41, 42
3	39, 40, 43, 44

ANSWERS

1. $y = 13 + 3x$

2. $y = 7 - 2x$

3. $y = -13 + 9x$

4. $y = 3 - 4x$

5. $y = 9x - 45$

6. $y = 2x + 2$

7. $y = x - 3$

8. $y = 2x + 1$

9. $y = 18x + 12$

10. $y = 16 - 12x$

11. $x = \frac{1}{12}y$

12. $x = \frac{1}{9}m$

13. $x = \dfrac{a}{2 + 6z}$

14. $x = \dfrac{y}{3b - 7}$

15. $x = \dfrac{y - 6}{4 + r}$

16. $x = \dfrac{z - 8}{6 - p}$

17. $x = \dfrac{r}{s + t}$

18. $x = \dfrac{a - d}{b + c}$

19. $x = \dfrac{y - 12}{-5 - 4k}$

20. $x = \dfrac{y + 9}{1 + 2w}$

1.7 Practice WITH CalcChat® AND CalcView®

GO DIGITAL

In Exercises 1–10, solve the literal equation for *y*. *Example 1*

1. $y - 3x = 13$

2. $2x + y = 7$

3. $2y - 18x = -26$

4. $20x + 5y = 15$

5. $9x - y = 45$

6. $6x - 3y = -6$

7. $4x - 5 = 7 + 4y$

8. $16x + 9 = 9y - 2x$

9. $2 + \frac{1}{6}y = 3x + 4$

10. $11 - \frac{1}{2}y = 3 + 6x$

In Exercises 11–20, solve the literal equation for *x*. *Example 2*

11. $y = 4x + 8x$

12. $m = 10x - x$

13. $a = 2x + 6xz$

14. $y = 3bx - 7x$

15. $y = 4x + rx + 6$

16. $z = 8 + 6x - px$

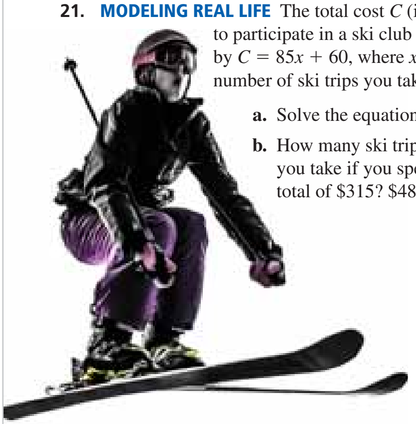
17. $sx + tx = r$

18. $a = bx + cx + d$

19. $12 - 5x - 4kx = y$

20. $x - 9 + 2wx = y$

21. MODELING REAL LIFE The total cost C (in dollars) to participate in a ski club is given by $C = 85x + 60$, where x is the number of ski trips you take.



a. Solve the equation for x.

b. How many ski trips did you take if you spent a total of \$315? \$485?

22. MODELING REAL LIFE The penny size of a nail indicates the length of the nail. The penny size d of a nail that is 1 to 3 inches long is given by $d = 4n - 2$, where n is the length (in inches) of the nail.

a. Solve the equation for n.

b. Find the lengths of nails with the following penny sizes: 3, 6, and 10.

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in solving the equation for *x*.**

23. ✗

$12 - 2x = -2(y - x)$

$-2x = -2(y - x) - 12$

$x = (y - x) + 6$

24. ✗

$10 = ax - 3b$

$10 = x(a - 3b)$

$\dfrac{10}{a - 3b} = x$

In Exercises 25–28, solve the formula for the indicated variable. *Examples 3 and 5*

25. Profit: $P = R - C$; Solve for C.

26. Surface area of a cylinder: $S = 2\pi r^2 + 2\pi rh$; Solve for h.

27. Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$; Solve for b_2.

28. Average acceleration of an object: $a = \dfrac{v_1 - v_0}{t}$; Solve for v_1.

29. REWRITING A FORMULA A common statistic used in professional football is the quarterback rating. This rating is made up of four major factors. One factor is the completion rating given by the formula

$$R = 5\left(\frac{C}{A} - 0.3\right)$$

where C is the number of completed passes and A is the number of attempted passes. Solve the formula for C.

30. REWRITING A FORMULA Newton's law of gravitation is given by the formula

$$F = G\left(\frac{m_1 m_2}{d^2}\right)$$

where F is the force between two objects of masses m_1 and m_2, G is the gravitational constant, and d is the distance between the two objects. Solve the formula for m_1.

21. a. $x = \dfrac{C - 60}{85}$

b. 3 trips; 5 trips

22. a. $n = \dfrac{d + 2}{4}$

b. $1\frac{1}{4}$ in.; 2 in.; 3 in.

23. The equation is not solved for x because there is still a term with x on both sides; $x = y - x + 6$; $2x = y + 6$; $x = \dfrac{y + 6}{2}$

24. There is no x in $3b$ to factor out; $10 = ax - 3b$; $10 + 3b = ax$; $x = \dfrac{10 + 3b}{a}$

25. $C = R - P$

26. $h = \dfrac{S - 2\pi r^2}{2\pi r}$

27. $b_2 = \dfrac{2A}{h} - b_1$

28. $v_1 = at + v_0$

29. $C = A\left(\dfrac{R}{5} + 0.3\right)$

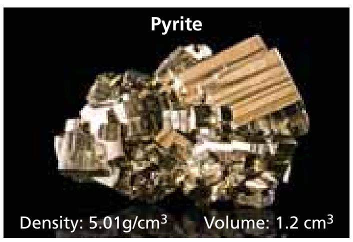
30. $m_1 = \dfrac{Fd^2}{m_2 G}$

31. MODELING REAL LIFE The sale price S (in dollars) of an item is given by the formula $S = L - rL$, where L is the list price (in dollars) and r is the percent of discount (in decimal form).
Examples 4 and 6

a. Solve the formula for r.

b. The list price of the shirt is \$21.50. What is the percent of discount?

32. MODELING REAL LIFE The density d of a substance is given by the formula $d = \frac{m}{V}$, where m is its mass and V is its volume.

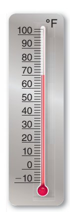


a. Solve the formula for each of the other two variables.

b. Find the mass of the pyrite sample. Explain how you found the mass.

33. MAKING AN ARGUMENT Your friend claims that Thermometer A displays a greater temperature than Thermometer B. Is your friend correct? Explain your reasoning.

Thermometer A



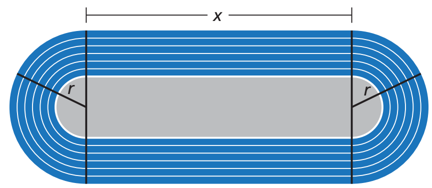
Thermometer B

34. MODELING REAL LIFE You deposit \$2000 in an account that earns simple interest at an annual rate of 4%. How long must you leave the money in the account to earn \$500 in interest? *Example 7*

35. MODELING REAL LIFE A flight averages 460 miles per hour. The return flight averages 500 miles per hour due to a tailwind. The total flying time is 4 hours and 48 minutes. How long is each flight? Explain.
Example 8

36. MODELING REAL LIFE An athletic facility is building an indoor track. The track is composed of a rectangle and two semicircles, as shown.

GO DIGITAL



a. Write a formula for the perimeter of the indoor track. Then solve the formula for x.

b. The perimeter of the track is 660 feet, and r is 50 feet. Find x. Round your answer to the nearest foot.

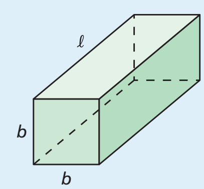
37. MODELING REAL LIFE A vehicle travels 55 miles per hour and 20 miles per gallon.

a. Write an equation that represents the distance d (in miles) that the vehicle travels in t hours. Then write an equation that represents the distance d (in miles) that the vehicle travels using g gallons of gasoline.

b. Write an equation that relates g and t. Then solve the equation for g.

c. The vehicle travels for 6 hours. How many gallons of gasoline does the vehicle use? How far does it travel? Explain.

38. HOW DO YOU SEE IT?
The rectangular prism shown has square bases.



a. Use the figure to write a formula for the surface area S of the prism.

b. Your teacher asks you to solve the formula for either b or ℓ. Which would you choose? Explain.

DIG DEEPER **In Exercises 39 and 40, solve the literal equation for *a*.**

39. $x = \frac{a + b + c}{ab}$

40. $y = x\left(\frac{ab}{a - b}\right)$

39. $a = \frac{b + c}{bx - 1}$

40. $a = \frac{yb}{y - xb}$

Slow Reveal

For Exercise 32, write only the equation $d = \frac{m}{V}$. Ask, "What do you think this formula is about?" Allow time for students to brainstorm ideas before asking, "What do you think the variables represent?" After discussing possible meanings, reveal the actual meanings if students have not identified them. Then ask, "How can you use this formula? Are you always finding the density of an object?" *Sample answer:* You may need to find the mass (or volume) when the density of an object is known.

ANSWERS

31. a. $r = \frac{L - S}{L}$

b. 20%

32. a. $m = dV$, $V = \frac{m}{d}$

b. 6.012 g; *Sample answer:* Substitute the values for density and volume into the equation $m = dV$.

33. no; 70°F is about 21.1°C, which is greater than 20°C.

34. 6.25 yr

35. 2.5 h; 2.3 h; $\frac{d}{460}$ represents the original trip time, and $\frac{d}{500}$ represents the return trip time. Add these expressions and solve for the one-way distance. Substituting the distance into each of the expressions gives the time for each flight.

36. a. $P = 2x + 2\pi r$; $x = \frac{1}{2}P - \pi r$

b. 173 ft

37. a. $d = 55t$; $d = 20g$

b. $55t = 20g$; $g = \frac{11t}{4}$

c. 16.5 gal; 330 mi; The amount of gasoline used can be found using the formula from part (b). Either of the original formulas can be used to find the distance.

38. a. $S = 4\ell b + 2b^2$

b. *Sample answer:* ℓ; The formula contains terms with both b and b^2, but only one term with ℓ.

Mini-Assessment

1. Solve $9 - y = 17x$ for y.
   $y = 9 - 17x$
2. Solve $nt + 6 - xt = s$ for t.
   $t = \dfrac{s - 6}{n - x}$
3. The formula for the volume of a rectangular prism is $V = \ell wh$. Solve the formula for the width w.
   $w = \dfrac{V}{\ell h}$
4. A circular trampoline has a padded rope that goes around the outside of the frame. The rope for this trampoline is sold for \$2.75 per foot and costs \$104.50 before tax. What is the diameter (to the nearest foot) of the trampoline? 12 ft
5. Your friend averages 56 miles per hour while driving to a college. On the return trip, your friend averages 48 miles per hour due to construction. The total driving time is 6 hours and 30 minutes. How long does each trip take?
   to college: 3 h; return trip: 3.5 h, or 3 h 30 min

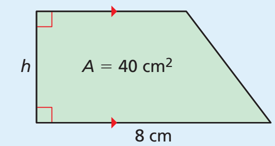
41. MODELING REAL LIFE One type of stone formation found in Carlsbad Caverns in New Mexico is called a column. This stone formation connects to the ceiling and the floor of a cave.

a. What is the radius (to the nearest tenth of a foot) of a cylindrical column that has a circumference of 7 feet? 8 feet? 9 feet?

b. Explain why you might rewrite the circumference formula to find the solutions in part (a).

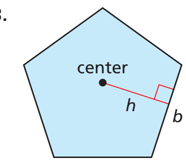
42. THOUGHT PROVOKING Give a possible value for h. Justify your answer. Draw and label the figure using your chosen value of h.

GO DIGITAL

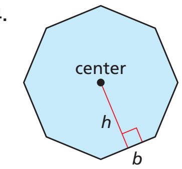


CONNECTING CONCEPTS In Exercises 43 and 44, write a formula for the area of the regular polygon. Solve the formula for the height h.

43.



44.



REVIEW & REFRESH

WATCH

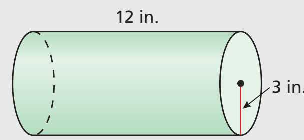
In Exercises 45–48, evaluate the expression when $a = 5$ and $b = 2$.

45. $a^2 + 12$ **46.** $9b - 4$

47. $\dfrac{a}{b} - \dfrac{3}{4}$ **48.** $3b\left(16 - \dfrac{1}{10}a\right)$

49. Solve the literal equation $6x - y = 12$ for y.

50. Find the surface area of the cylinder.



51. Tell whether the ratios 6 : 8 and 4 : 6 form a proportion.

52. MODELING REAL LIFE The volume G of an aquarium (in gallons) is given by the formula $G = \dfrac{\ell wh}{231}$, where ℓ is the length of the aquarium (in inches), w is the width (in inches), and h is the height (in inches).

a. Solve the formula for h.

b. Find the height of a 20-gallon aquarium with a length of 24 inches and a width of 12 inches.

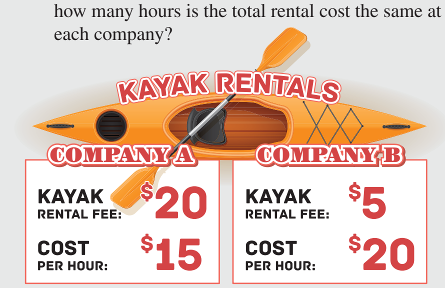
In Exercises 53–56, solve the equation. Graph the solutions, if possible.

53. $|x - 3| = 5$ **54.** $|3y - 12| - 7 = 2$

55. $2|2r + 4| = -16$ **56.** $-4|s + 9| = -24$

57. Tell whether the points $(-2, 3)$, $(2, 1)$, and $(3, 3)$ form a right triangle.

58. MODELING REAL LIFE You want to rent a kayak from one of the two rental companies shown. After how many hours is the total rental cost the same at each company?



In Exercises 59–61, solve the equation. Check your solution.

59. $\dfrac{z}{-5} + 2 = -4$ **60.** $1.9t = -5.7$

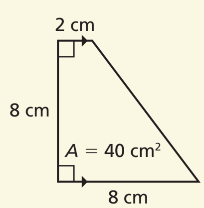
61. $27 = -3(8y - 7) + 20y$

ANSWERS

41. a. 1.1 ft; 1.3 ft; 1.4 ft

b. *Sample answer:* It is easier to find the radius when the circumference formula is rewritten for r.

42. *Sample answer:* $h = 8$; missing base $= 2$; $A = \frac{1}{2}h(b_1 + b_2)$, $40 = \frac{1}{2} \cdot 8 \cdot (2 + 8)$



43. $A = \dfrac{5}{2}bh$; $h = \dfrac{2A}{5b}$

44. $A = 4bh$; $h = \dfrac{A}{4b}$

45. 37

46. 14

47. $\frac{7}{4}$

48. 93

49. $y = 6x - 12$

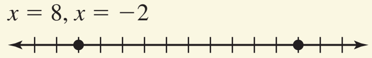
50. 90π in.^{2}

51. no

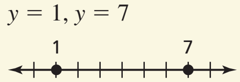
52. a. $h = \dfrac{231G}{\ell w}$

b. about 16 in.

53. $x = 8$, $x = -2$

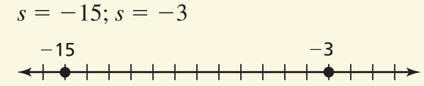


54. $y = 1$, $y = 7$



55. no solution

56. $s = -15$; $s = -3$



57. yes

58. 3 h

59. $z = 30$

60. $t = -3$

61. $y = -\frac{3}{2}$

1 Chapter Review WITH CalcChat®

GO DIGITAL

Chapter Learning Target Understand solving linear equations.

Chapter Success Criteria
- ◆ Solve simple and multi-step equations.
- ◆ Describe how to solve equations.
- ■ Analyze the measurements used to solve a problem and judge the level of accuracy appropriate for the solution.
- ■ Apply equation-solving techniques to solve real-life problems.

◆ Surface
■ Deep

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1.1 Solving Simple Equations *(pp. 3–10)* WATCH

Learning Target: Write and solve one-step linear equations.

Vocabulary AZ VOCAB
equation
linear equation in one variable
solution
equivalent equations

Solve the equation. Justify each step. Check your solution.

1. $z + 3 = -6$
2. $2.6 = -0.2t$
3. $-\frac{n}{5} = -2$
4. $5.9 = y - 2.7$
5. $\pi b = -\frac{1}{9}$
6. $3\frac{3}{10} = w + 1\frac{4}{5}$
7. Which property of equality should you use to solve the equation $\frac{a}{b} + x = c$, where a and b are negative numbers? Explain your reasoning.
8. You have \$175.29 in a savings account. You deposit an additional \$48.75. How much more do you need to save to buy the cell phone?

1.2 Solving Multi-Step Equations *(pp. 11–18)* WATCH

Learning Target: Write and solve multi-step linear equations.

Solve the equation. Check your solution.

9. $3y + 11 = -16$
10. $6 = 1 - b$
11. $n + 5n + 7 = 43$
12. $-4(2z + 6) - 12 = 4$
13. $\frac{3}{2}(x - 2) - 5 = 19$
14. $6 = \frac{1}{5}w + \frac{7}{5}w - 4$
15. What happens to the value of x in the equation $\frac{x}{b} + 8 = 15$ as the value of b decreases? Explain your reasoning.
16. Find three consecutive odd integers that have a sum of 75.

ELL SUPPORT

English language learners benefit from cooperative learning that has them practicing language as they learn and review. When reviewing, it is important to check that everyone understands the concepts—not just those who voluntarily answer questions in class. Most English language learners are too timid to volunteer an answer. There are a variety of ways to check comprehension. One way is to have a group of students discuss a problem and reach a consensus. Be sure to monitor these discussions and provide support as needed. Then have groups present their answers to the class for verification.

ANSWERS

1. $z = -9$; Subtract 3 from each side.
2. $t = -13$; Divide each side by -0.2.
3. $n = 10$; Multiply each side by -5.
4. $y = 8.6$; Add 2.7 to each side.
5. $b = -\frac{1}{9\pi}$; Divide each side by π.
6. $w = 1\frac{1}{2}$; Subtract each side by $1\frac{4}{5}$.
7. Subtraction Property of Equality; Because a and b are negative numbers, $\frac{a}{b}$ is positive and the inverse operation is subtraction.
8. \$25.95
9. $y = -9$
10. $b = -5$
11. $n = 6$
12. $z = -5$
13. $x = 18$
14. $w = \frac{25}{4}$
15. decreases; The equation can be rewritten as $x = 7b$, so when the value of b decreases, the value of $7b$ also decreases.
16. 23, 25, 27

ANSWERS

17. $d = 4$

18. $m = 4$

19. $b = 32$

20. 7647.5 m, or 7661.29 m

21. *Sample answer:* Concert A had more merchandise sales per person that attended. Concert B had more social media posts per ticket sold.

22. 24 in.^{2}

23. 150 people

24. $n = -4$

25. $y = 9$

26. all real numbers

27. no solution

28. 2475 mi

29. **a.** sometimes

b. sometimes

c. always

GO DIGITAL

1.3 Modeling Quantities (pp. 19–24)

Learning Target: Use proportional reasoning and analyze units when solving problems.

Vocabulary
ratio
proportion
rate

Solve the proportion.

17. $\frac{12}{d} = \frac{9}{3}$ **18.** $\frac{6}{15} = \frac{m}{10}$ **19.** $\frac{3}{8} = \frac{12}{b}$

20. Convert 4.75 miles to meters. Round your answer to the nearest hundredth, if necessary.

21. The table shows data collected by a musician after two concerts. Use rates to compare the data for each concert.

Concert	Tickets sold	Attendance	Merchandise sales	Social media posts
A	275	264	\$1760	66
B	325	299	\$1794	104

1.4 Accuracy with Measurements (pp. 25–30)

Learning Target: Choose an appropriate level of accuracy when calculating with measurements.

Vocabulary
precision
accuracy

22. You use an inch ruler to measure the dimensions of the photograph. Estimate the area of the photograph.

23. The land area of China is 9,326,410 square kilometers. According to United Nations estimates, the population of China in 2015 was 1,397,029,000. Estimate the population per square kilometer.

1.5 Solving Equations with Variables on Both Sides (pp. 31–36)

Learning Target: Write and solve equations with variables on both sides.

Vocabulary
identity

Solve the equation. Check your solution.

24. $3n - 3 = 4n + 1$ **25.** $2y - 16 = \frac{1}{3}(y - 3)$

26. $5(1 + x) = 5x + 5$ **27.** $3(n + 4) = \frac{1}{2}(6n + 4)$

28. An airplane leaves Los Angeles and travels 5 hours to New York City. The return trip travels along the same route and takes 6 hours and 15 minutes because the plane travels 99 miles per hour slower due to a headwind. Find the distance that the plane flies between the two cities.

29. Determine whether the equation $mx + 4 = \frac{1}{2}nx + 8$ *always*, *sometimes*, or *never* has a solution when (a) $m < n$, (b) $m = n$, and (c) $m > n$.

1.6 Solving Absolute Value Equations (pp. 37–44)

WATCH

GO DIGITAL

Learning Target: Write and solve equations involving absolute value.

Vocabulary (VOCAB)

absolute value equation
extraneous solution

Solve the equation. Check your solutions.

30. $|y + 3| = 17$

31. $2|k - 3| = 18$

32. $|16g - 40| = -4|4g - 10|$

33. $-|b + 6| = |3b - 2|$

34. $-2|5w - 7| + 9 = -7$

35. $|x - 2| = |4 + x|$

36. The minimum sustained wind speed of a Category 1 hurricane is 74 miles per hour. The maximum sustained wind speed is 95 miles per hour. Write an absolute value equation that represents the minimum and maximum speeds.

1.7 Rewriting Equations and Formulas (pp. 45–52)

WATCH

Learning Target: Solve literal equations for given variables.

Vocabulary (VOCAB)

literal equation
formula

Solve the literal equation for *y*.

37. $2x - 4y = 20$

38. $8x - 3 = 5 + 4y$

39. $3(2x + y) = -4x - y$

40. $a = 9y + 3yx$

41. The formula $F = \frac{9}{5}(K - 273.15) + 32$ converts a temperature from kelvin K to degrees Fahrenheit F.

a. Solve the formula for K.

b. Convert 180°F to kelvin. Round your answer to the nearest hundredth.

Mathematical Practices

Construct Viable Arguments and Critique the Reasoning of Others

Mathematically proficient students justify conclusions, communicate them to others, and respond to the arguments of others.

1. In Exercise 37 on page 51, explain why you are able to write an equation that relates the number of gallons g of gasoline and the time t.

2. Your friend says that Equation 1 and Equation 2 are equivalent. Is your friend correct? If so, explain why the equations are equivalent. If not, correct your friend's reasoning.

   Equation 1: $8x + 18 = 4x - 32$

   Equation 2: $2x + 18 = x - 32$

3. Write instructions to teach someone how to use properties of equality to solve a literal equation for a given variable.

ANSWERS

30. $y = 14, y = -20$

31. $k = 12, k = -6$

32. $g = \frac{5}{2}$

33. no solution

34. $w = 3, w = -\frac{1}{5}$

35. $x = -1$

36. $|v - 84.5| = 10.5$

37. $y = \frac{1}{2}x - 5$

38. $y = 2x - 2$

39. $y = -\frac{5}{2}x$

40. $y = \dfrac{a}{9 + 3x}$

41. a. $K = \frac{5}{9}(F - 32) + 273.15$

b. 355.37 K

MATHEMATICAL PRACTICES ANSWERS

1. The two equations were written in terms of the distance d, so you can use substitution to equate the equations.
2. no; By the Multiplication Property of Equality, each term of Equation 2 should be multiplied by 4, not just the variable terms.
3. *Sample answer:* Use the Addition and Subtraction Properties of Equality to rewrite the equation with the variable terms and constant terms on different sides of the equation. Then use the Multiplication and Division Properties of Equality to isolate the given variable.

Item Correlation to Sections

Practice Test Exercises	Section to Review
1	1.1
2, 6	1.2
10	1.3
13	1.4
5, 7, 12	1.5
3, 4, 8, 9, 14	1.6
11	1.7

Item Leveling

DOK	Exercises
1	1–6
2	7–13
3	14

ANSWERS

1. $x = 22$
2. $x = -3$
3. $x = -3, x = 9$
4. $x = 3$
5. all real numbers
6. no solution
7. $c \neq 5$; If c is 5, then the equation is an identity. For all other values of c, subtracting $3x$ from each side will give a statement that is always false.
8. $c < 0$; An absolute value cannot be negative.
9. $|h - 34| = 4$
10. skateboard
11. **a.** $w = \dfrac{P - 2\ell}{2}$
    b. 65 yd
    c. 4.8%
12. **a.** 2.1 h; The cost at the dealership is $24 + 99t$ and the cost at the local mechanic is $45 + 89t$. Set these two expressions equal and solve for the time.
    b. time is less than 2.1 h; time is greater than 2.1 h; Because the expressions are equal for 2.1 hours, that is the cutoff point from the dealership being less expensive to the local mechanic being less expensive.
13. 8.3 min
14. It will give a negative value on the right and absolute value cannot be negative.

1 Practice Test WITH CalcChat®

GO DIGITAL

Solve the equation.

1. $x - 7 = 15$

2. $\frac{2}{3}x + 5 = 3$

3. $2|x - 3| - 5 = 7$

4. $|2x - 19| = 4x + 1$

5. $-2 + 5x - 7 = 3x - 9 + 2x$

6. $\frac{1}{3}(6x + 12) - 2(x - 7) = 19$

Describe the values of c for which the equation has no solution. Explain your reasoning.

7. $3x - 5 = 3x - c$

8. $|x - 7| = c$

9. A safety regulation states that the minimum height of a handrail is 30 inches. The maximum height is 38 inches. Write an absolute value equation that represents the minimum and maximum heights.

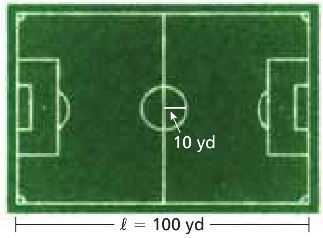
10. The fastest recorded speed in a standing position on a skateboard is about 146.7 kilometers per hour. The fastest recorded speed on inline skates is about 113.6 feet per second. Which speed is faster?

11. The perimeter P (in yards) of a soccer field is represented by the formula $P = 2\ell + 2w$, where ℓ is the length (in yards) and w is the width (in yards).

a. Solve the formula for w.

b. The perimeter of the soccer field is 330 yards. Find the width of the field.

c. About what percent of the field is inside the circle?



12. Your car needs new brakes. You call a dealership and a local mechanic for prices.

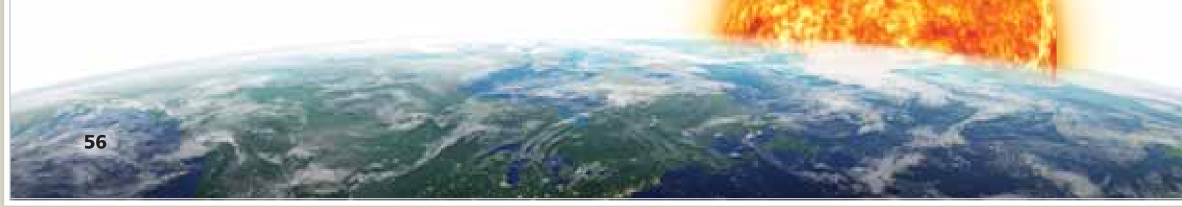
	Cost of parts	Labor cost per hour
Dealership	$24	$99
Local mechanic	$45	$89

a. After how many hours are the total costs the same at both places? Justify your answer.

b. When do the repairs cost less at the dealership? at the local mechanic? Explain.

13. The speed of light is 11,176,920 miles per minute and the distance from Earth to the Sun at a specific time is 92,955,807 miles. About how long does it take for sunlight to reach Earth?

14. Consider the equation $|4x + 20| = 6x$. Without solving, how do you know that $x = -2$ is an extraneous solution?

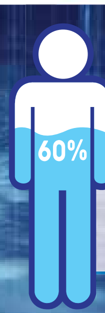



56

1 Performance Task
Every Drop Counts

GO DIGITAL

The average American family uses more than 300 gallons of water per day!



About 60% of an adult's body is water.

WATER LOG

INFO

Record your water activities and water consumption for one week. Then make a plan to reduce the amount of water you use each week. Determine the number of gallons you want to use for each activity and solve equations to show how you will achieve your goal.

57

Rubric

Every Drop Counts	
Component	**Points**
Record of water activities and water consumption for one week	**3** Complete record, including water activities and water consumption for one week **2** Mostly complete record **1** Partially complete record
Plan to reduce the amount of water used each week	**2** Thoughtful response that justifies an acceptable plan **1** Plan chosen is acceptable but not justified
Calculations, including equations, showing how to find the number of gallons you want to use for each activity	**3** Correct calculations and equations **2** Mostly correct calculations and equations **1** Partially correct calculations and equations
Total Points	**8 Points**

Laurie's Notes

- Students may want to research additional water use statistics and successful methods for reducing water consumption.
- Brainstorm other activities that use water, such as washing a car, watering a lawn, watering house or outdoor plants, and cleaning a house.
- Caution students not to be overly zealous. People who live in communities where water shortages have occurred know that reducing water use by more than 20% is very challenging.
- Note that students are asked to determine the amount of water they will use for each activity, including some that may not be listed. They may set goals that involve fixed amounts or percent decreases.

ANSWERS WITH ITEM ANALYSIS

1. **A.** The student divides A by 2 instead of multiplying by 2.
   B. The student subtracts $(b_1 + b_2)$ from $2A$ instead of dividing $2A$ by $(b_1 + b_2)$.
   C. The student incorrectly distributes $h(b_1 + b_2)$ by not distributing h to b_2.
   D. Correct answer
2. **A.** The student incorrectly thinks the equations are not equivalent.
   B. Correct answer
   C. The student incorrectly thinks the equations are not equivalent.
   D. The student incorrectly thinks the equations are not equivalent.
3. 18 beginner, 15 intermediate, 15 expert
4. **a.** <
   b. <
   c. >
   d. <
   e. =
   f. =
5. $8x + 6 = -2x - 14$ and $5x + 3 = -7$

1 College and Career Readiness WITH CalcChat®

GO DIGITAL

WATCH Tutorial videos are available for each exercise.

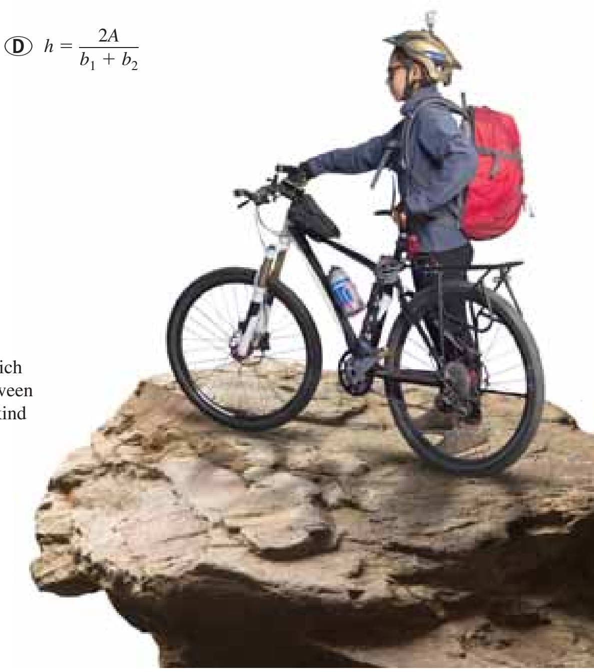
1. Which equation is equivalent to the formula for the area A of a trapezoid, $A = \frac{1}{2}h(b_1 + b_2)$?

Ⓐ $h = \dfrac{A}{2(b_1 + b_2)}$

Ⓑ $h = 2A - (b_1 + b_2)$

Ⓒ $h = \dfrac{2A - b_2}{b_1}$

Ⓓ $h = \dfrac{2A}{b_1 + b_2}$



2. Which equation is *not* equivalent to $cx - a = b$?

Ⓐ $cx - a + b = 2b$

Ⓑ $cx - a + b = 0$

Ⓒ $2cx - 2a = 2b$

Ⓓ $b + a = cx$

3. A mountain biking park has 48 trails, 37.5% of which are beginner trails. The rest are divided evenly between intermediate and expert trails. How many of each kind of trail are in the park?

4. Let N represent the number of solutions of the equation $3(x - a) = 3x - 6$. Complete each statement with the symbol <, >, or =.

a. When $a = 3$, N ____ 1.

b. When $a = -3$, N ____ 1.

c. When $a = 2$, N ____ 1.

d. When $a = -2$, N ____ 1.

e. When $a = x$, N ____ 1.

f. When $a = -x$, N ____ 1.

5. Which of the equations are equivalent?

$6x + 6 = -14$	$8x + 6 = -2x - 14$
$5x + 3 = -7$	$7x + 3 = 2x - 13$

GO DIGITAL

6. You are painting your dining room white and your living room blue. You spend \$132 on 5 cans of paint. The white paint costs \$24 per can, and the blue paint costs \$28 per can.

a. Write an equation that you can use to find the number of cans of each color that you buy.

b. How much would you save by switching the colors of the dining room and living room? Explain.

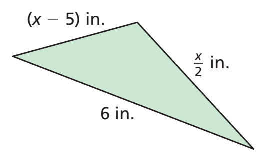
7. The perimeter of the triangle is 13 inches. What is the length of the shortest side?

Ⓐ 3 in.

Ⓑ 4 in.

Ⓒ 6 in.

Ⓓ 8 in.



8. You pay \$45 per month for cable TV. Your friend buys a satellite TV receiver for \$99 and pays \$36 per month for satellite TV. Your friend claims that the expenses for a year of satellite TV are less than the expenses for one year of cable. Is your friend correct? Explain.

9. Which table represents a proportional relationship?

Ⓐ

x	1	2	3	4
y	18	15	12	9

Ⓑ

x	2	4	5	8
y	7	14	17.5	28

Ⓒ

x	1	2	3	4
y	5	5	5	5

Ⓓ

x	40	20	10	5
y	1	2	4	8

10. Place each equation into one of the four categories.

No solution	One solution	Two solutions	Infinitely many solutions

$\lvert 8x + 3\rvert = 0$	$-6 = 5x - 9$	$3x - 12 = 3(x - 4) + 1$
$-2x + 4 = 2x + 4$	$0 = \lvert x + 13\rvert + 2$	$-4(x + 4) = -4x - 16$
$12x - 2x = 10x - 8$	$9 = 3\lvert 2x - 11\rvert$	$7 - 2x = 3 - 2(x - 2)$

11. A car travels 1100 feet in 12.5 seconds. How fast does the car travel in miles per hour?

Ⓐ $\frac{1}{60}$ mi/h

Ⓑ 1 mi/h

Ⓒ 60 mi/h

Ⓓ 88 mi/h

ANSWERS WITH ITEM ANALYSIS

6. **a.** $24x + 28(5 - x) = 132$ or $24(5 - x) + 28x = 132$

b. \$4; *Sample answer:* Switching gives a total cost of \$128, which is \$4 less than \$132.

7. **A.** Correct answer

B. The student incorrectly thinks $\frac{x}{2}$ must be less than $(x - 5)$.

C. The student finds the length of the longest side instead of the shortest side.

D. The student finds the value of x instead of the length of the shortest side.

8. yes; Because the expenses are equal for month 11, any time after that satellite TV will be less expensive.

9. **A.** The student incorrectly thinks a linear relationship must be proportional.

B. Correct answer

C. The student incorrectly thinks a linear relationship must be proportional.

D. The student compares the values of the products xy instead of the values of the ratios x to y.

10. no solution: $12x - 2x = 10x - 8$, $0 = \lvert x + 13\rvert + 2$, $3x - 12 = 3(x - 4) + 1$

one solution: $\lvert 8x + 3\rvert = 0$, $-2x + 4 = 2x + 4$, $-6 = 5x - 9$

two solutions: $9 = 3\lvert 2x - 11\rvert$

infinitely many solutions: $-4(x + 4) = -4x - 16$, $7 - 2x = 3 - 2(x - 2)$

11. **A.** The student finds the speed in miles per second instead of miles per hour.

B. The student finds the speed in miles per minute instead of miles per hour.

C. Correct answer

D. The student finds the speed in feet per second instead of miles per hour.