

Inductive Reasoning

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

Example 1 Use inductive reasoning to make a conjecture about the result when the expression $n^2 - n + 11$ is evaluated at any natural number n .

Step 1 Find a pattern using the first few natural numbers.

$$\begin{array}{lll} 1^2 - 1 + 11 = 11 & 3^2 - 3 + 11 = 17 & 5^2 - 5 + 11 = 31 \\ 2^2 - 2 + 11 = 13 & 4^2 - 4 + 11 = 23 & 6^2 - 6 + 11 = 41 \end{array}$$

Step 2 Make a conjecture.

► **Conjecture** The expression $n^2 - n + 11$ gives a prime number when evaluated at any natural number n .

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

Example 2 Find a counterexample to show that the conjecture in Example 1 is false.

To find a counterexample, you need to find a natural number n for which $n^2 - n + 11$ gives a composite number.

$$11^2 - 11 + 11 = 121 \quad \text{Substitute 11 for } n.$$

Because the factors of 121 are 1, 11, and 121, it is composite.

► Because a counterexample exists, the conjecture is false.

Practice

Check your answers at BigIdeasMath.com.

Make a conjecture about the given quantity.

- the product of a nonzero number and its opposite
Sample answer: The product of a nonzero number and its opposite is negative.
- the sum of the first n positive odd integers
Sample answer: The sum of the first n positive odd integers is n^2 .
- the ones digit of 4^{200}
Sample answer: The ones digit of 4^{200} is 6.

Find a counterexample to show that the conjecture is false.

- If x is an integer, then $-x < x$.
any $x \leq 0$
- A nonzero number is always greater than its reciprocal.
any $n \leq -1$ or $0 < n \leq 1$
- If the product of two natural numbers is even, then both of the natural numbers are even.
any pair of natural numbers with one of them odd
- If the sum of two natural numbers is even, then the product of the two natural numbers is even.
any pair of natural numbers with both of them odd