Name ________________________________ Date ______________

Inductive Reasoning

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

Example 1 **Use inductive reasoning to make a conjecture about the result when the expression $n^2 - n + 11$ is evaluated at any natural number n.**

Step 1 Find a pattern using the first few natural numbers.

$1^2 - 1 + 11 = 11$	$3^2 - 3 + 11 = 17$	$5^2 - 5 + 11 = 31$
$2^2 - 2 + 11 = 13$	$4^2 - 4 + 11 = 23$	$6^2 - 6 + 11 = 41$

Step 2 Make a conjecture.

▶ **Conjecture** The expression $n^2 - n + 11$ gives a prime number when evaluated at any natural number n.

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

Example 2 **Find a counterexample to show that the conjecture in Example 1 is false.**

To find a counterexample, you need to find a natural number n for which $n^2 - n + 11$ gives a composite number.

$11^2 - 11 + 11 = 121$ Substitute 11 for n.

Because the factors of 121 are 1, 11, and 121, it is composite.

▶ Because a counterexample exists, the conjecture is false.

Practice

Check your answers at BigIdeasMath.com.

Make a conjecture about the given quantity.

1. the product of a nonzero number and its opposite
2. the sum of the first n positive odd integers
3. the ones digit of 4^{200}

Find a counterexample to show that the conjecture is false.

4. If x is an integer, then $-x < x$.
5. A nonzero number is always greater than its reciprocal.
6. If the product of two natural numbers is even, then both of the natural numbers are even.
7. If the sum of two natural numbers is even, then the product of the two natural numbers is even.