The Pythagorean Theorem

In a right triangle, the **hypotenuse** is the side opposite the right angle. The **legs** are the two sides that form the right angle.

The **Pythagorean Theorem** states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

**Example 1**  Find the missing length of the triangle.

\[ a^2 + b^2 = c^2 \]

Write the Pythagorean Theorem.

\[ a^2 + 15^2 = 17^2 \]

Substitute 15 for \( b \) and 17 for \( c \).

\[ a^2 + 225 = 289 \]

Evaluate powers.

\[ a^2 = 64 \]

Subtract 225 from each side.

\[ a = 8 \]

Take positive square root of each side.

The missing length is 8 yards.

You can use the Pythagorean Theorem to develop the **Distance Formula**.

You can use the **Distance Formula** to find the distance \( d \) between any two points \((x_1, y_1)\) and \((x_2, y_2)\) in a coordinate plane.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

**Example 2**  Find the distance between the two points.

a. \((3, 6), (-2, 4)\)

Let \((x_1, y_1) = (3, 6)\) and \((x_2, y_2) = (-2, 4)\).

\[ d = \sqrt{(-2 - 3)^2 + (4 - 6)^2} \]

\[ = \sqrt{25 + 4} \]

\[ = \sqrt{29} \]

b. \((0, 5), (4, -1)\)

Let \((x_1, y_1) = (0, 5)\) and \((x_2, y_2) = (4, -1)\).

\[ d = \sqrt{(4 - 0)^2 + (-1 - 5)^2} \]

\[ = \sqrt{16 + 36} \]

\[ = 2\sqrt{13} \]

**Practice**

Find the missing length of the triangle.

1. \[
\begin{array}{c}
6 \text{ ft} \\
\hline
8 \text{ ft} \\
\hline
10 \text{ ft}
\end{array}
\]

2. \[
\begin{array}{c}
19.5 \text{ in.} \\
\hline
7.5 \text{ in.} \\
\hline
18 \text{ in.}
\end{array}
\]

3. \[
\begin{array}{c}
a \\
\hline
2.1 \text{ m} \\
\hline
2 \text{ m}
\end{array}
\]

Find the distance between the two points.

4. \((0, 0), (4, 3)\) 5

5. \((0, -7), (5, 5)\) 13

6. \((4, 2), (-1, 5)\) \(\sqrt{34}\)

7. \((-5, 6), (-7, -2)\) \(2\sqrt{17}\)

8. \((-1, -3), (9, 0)\) \(\sqrt{109}\)

9. \((-4, -4), (-1, -1)\) \(3\sqrt{2}\)

Check your answers at BigIdeasMath.com.