

Solving Radical Equations

A **radical equation** is an equation that contains a radical expression with a variable in the radicand. To solve a radical equation involving a square root, first use properties of equality to isolate the radical on one side of the equation. Then use the following property to eliminate the radical and solve for the variable.

Squaring Each Side of an Equation

Words If two expressions are equal, then their squares are also equal.

Algebra If $a = b$, then $a^2 = b^2$.

Squaring each side of an equation can sometimes introduce an extraneous solution.

Example 1 Solve $x = \sqrt{x + 2}$. Check your solutions.

$x = \sqrt{x + 2}$	Write the equation.
$x^2 = (\sqrt{x + 2})^2$	Square each side of the equation.
$x^2 = x + 2$	Simplify.
$x^2 - x - 2 = 0$	Subtract x and 2 from each side.
$(x - 2)(x + 1) = 0$	Factor.
$x - 2 = 0$ or $x + 1 = 0$	Zero-Product Property
$x = 2$ or $x = -1$	Solve for x .

Check Check each solution in the original equation.

$2 \stackrel{?}{=} \sqrt{2 + 2}$	Substitute for x .	$-1 \stackrel{?}{=} \sqrt{-1 + 2}$
$2 \stackrel{?}{=} \sqrt{4}$	Simplify.	$-1 \stackrel{?}{=} \sqrt{1}$
$2 = 2$ ✓	Simplify.	$-1 \neq 1$ ✗

- Because $x = -1$ does not satisfy the original equation, it is an extraneous solution. The only solution is $x = 2$.

Practice

Check your answers at BigIdeasMath.com.

Solve the equation. Check your solution(s).

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|------------------------------------|--|---|
| 1. $4\sqrt{x} - 8 = 0$ $x = 4$ | 2. $2\sqrt{x} - 6 = 0$ $x = 9$ | 3. $2\sqrt{x - 6} - 3 = 5$ $x = 22$ |
| 4. $\sqrt{6x} + 6 = 0$ no solution | 5. $\sqrt{5x} + 1 = 0$ no solution | 6. $x = \sqrt{x + 12}$ $x = 4$ |
| 7. $\sqrt{2x + 8} = x$ $x = 4$ | 8. $\sqrt{2x + 3} = \sqrt{x + 2}$ $x = -1$ | 9. $\sqrt{-3x - 4} = \sqrt{2x + 11}$ $x = -3$ |

10. **PENDULUM** The period P (in seconds) of a pendulum is given by the function $P = 2\pi\sqrt{\frac{L}{32}}$, where L is the pendulum length (in feet). What is the length of a pendulum that has a period of 3 seconds? **about 7.3 feet**