Name _____ Date _____

Solving Absolute Value Equations

An **absolute value equation** is an equation that contains an absolute value expression. You can solve these types of equations by solving two related linear equations.

To solve $|ax + b| = c$ when $c \geq 0$, solve the related linear equations

$$ax + b = c \quad or \quad ax + b = -c.$$

When $c < 0$, the absolute value equation $|ax + b| = c$ has no solution because absolute value always indicates a number that is not negative.

To solve $|ax + b| = |cx + d|$, solve the related linear equations

$$ax + b = cx + d \quad or \quad ax + b = -(cx + d).$$

When you solve an absolute value equation, it is possible for a solution to be *extraneous*. An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

Example 1 **Solve $|x - 7| = 8$.**

Write the two related linear equations for $|x - 7| = 8$. Then solve.

$$\begin{array}{rcl} x - 7 = 8 & \quad or \quad & x - 7 = -8 \\ +7 \quad +7 & & +7 \quad +7 \\ x = 15 & & x = -1 \end{array}$$

▶ The solutions are $x = 15$ and $x = -1$.

Check

$$|x - 7| = 8$$
$$|15 - 7| \stackrel{?}{=} 8$$
$$|8| \stackrel{?}{=} |8|$$
$$8 = 8 \checkmark$$

$$|x - 7| = 8$$
$$|-1 - 7| \stackrel{?}{=} 8$$
$$|-8| \stackrel{?}{=} |-8|$$
$$8 = 8 \checkmark$$

Example 2 **Solve $|x + 3| = |x + 9|$.**

By equating the expression $x + 3$ and the opposite of $x + 9$, you obtain

$x + 3 = -(x + 9)$	Write related linear equation.
$x + 3 = -x - 9$	Distributive Property
$2x + 3 = -9$	Add x to each side.
$2x = -12$	Subtract 3 from each side.
$x = -6.$	Divide each side by 2.

However, by equating the expressions $x + 3$ and $x + 9$, you obtain

$x + 3 = x + 9$	Write related linear equation.
$x = x + 6$	Subtract 3 from each side.
$0 = 6$ ✗	Subtract x from each side.

which is a false statement. So, the original equation has only one solution.

▶ The solution is $x = -6$.

Practice

Check your answers at BigIdeasMath.com.

Solve the equation. Check your solutions.

1. $|x - 3| = 6$ $x = -3, x = 9$
2. $|2x - 1| = 9$ $x = -4, x = 5$
3. $|x - 5| = |x + 7|$ $x = -1$
4. $|x + 2| = |x + 8|$ $x = -5$
5. $|x - 3| = |x - 5|$ $x = -4$
6. $|x + 2| = |2x + 1|$ $x = -1, x = 1$