

# Solving Absolute Value Equations

An **absolute value equation** is an equation that contains an absolute value expression. You can solve these types of equations by solving two related linear equations.

To solve  $|ax + b| = c$  when  $c \geq 0$ , solve the related linear equations

$$ax + b = c \quad \text{or} \quad ax + b = -c.$$

When  $c < 0$ , the absolute value equation  $|ax + b| = c$  has no solution because absolute value always indicates a number that is not negative.

To solve  $|ax + b| = |cx + d|$ , solve the related linear equations

$$ax + b = cx + d \quad \text{or} \quad ax + b = -(cx + d).$$

When you solve an absolute value equation, it is possible for a solution to be *extraneous*. An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

**Example 1** Solve  $|x - 7| = 8$ .

Write the two related linear equations for  $|x - 7| = 8$ . Then solve.

$$\begin{array}{rcl} x - 7 = 8 & \text{or} & x - 7 = -8 \\ \underline{+7} \quad \underline{+7} & & \underline{+7} \quad \underline{+7} \\ x = 15 & & x = -1 \end{array}$$

► The solutions are  $x = 15$  and  $x = -1$ .

**Example 2** Solve  $|x + 3| = |x + 9|$ .

By equating the expression  $x + 3$  and the opposite of  $x + 9$ , you obtain

$$\begin{array}{ll} x + 3 = -(x + 9) & \text{Write related linear equation.} \\ x + 3 = -x - 9 & \text{Distributive Property} \\ 2x + 3 = -9 & \text{Add } x \text{ to each side.} \\ 2x = -12 & \text{Subtract 3 from each side.} \\ x = -6. & \text{Divide each side by 2.} \end{array}$$

However, by equating the expressions  $x + 3$  and  $x + 9$ , you obtain

$$\begin{array}{ll} x + 3 = x + 9 & \text{Write related linear equation.} \\ x = x + 6 & \text{Subtract 3 from each side.} \\ 0 = 6 \quad \times & \text{Subtract } x \text{ from each side.} \end{array}$$

which is a false statement. So, the original equation has only one solution.

► The solution is  $x = -6$ .

## Check

$$\begin{array}{l} |x - 7| = 8 \\ |15 - 7| \stackrel{?}{=} 8 \\ |8| \stackrel{?}{=} |8| \\ 8 = 8 \quad \checkmark \end{array}$$

$$\begin{array}{l} |x - 7| = 8 \\ |-1 - 7| \stackrel{?}{=} 8 \\ |-8| \stackrel{?}{=} |-8| \\ 8 = 8 \quad \checkmark \end{array}$$

## Practice

Check your answers at [BigIdeasMath.com](http://BigIdeasMath.com).

Solve the equation. Check your solutions.

1.  $|x - 3| = 6$

2.  $|2x - 1| = 9$

3.  $|x - 5| = |x + 7|$

4.  $|x + 2| = |x + 8|$

5.  $|x - 3| = |x - 5|$

6.  $|x + 2| = |2x + 1|$