Prime Factorization

Because 3 is a factor of 24 and \(3 \cdot 8 = 24\), 8 is also a factor of 24. The pair 3, 8 is called a \textbf{factor pair} of 24.

The \textbf{prime factorization} of a composite number is the number written as a product of its prime factors. You can use factor pairs and a \textbf{factor tree} to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product.

**Example 1** A classroom has 42 students. The teacher arranges the students in rows. Each row has the same number of students. How many possible arrangements are there?

Use the factor pairs of 42 to find the number of arrangements.

\[
42 = 1 \cdot 42 \quad \text{1 row of 42 or 42 rows of 1} \\
42 = 2 \cdot 21 \quad \text{2 rows of 21 or 21 rows of 2} \\
42 = 3 \cdot 14 \quad \text{3 rows of 14 or 14 rows of 3} \\
42 = 6 \cdot 7 \quad \text{6 rows of 7 or 7 rows of 6}
\]

There are 8 possible arrangements: 1 row of 42, 42 rows of 1, 2 rows of 21, 21 rows of 2, 3 rows of 14, 14 rows of 3, 6 rows of 7, or 7 rows of 6.

**Example 2** Write the prime factorization of 54.

Choose any factor pair of 54 to begin the factor tree.

\[
\begin{array}{c}
\text{Tree 1} \\
54 \\
\downarrow \\
3 \cdot 18 \\
\downarrow \\
2 \cdot 9 \\
\downarrow \\
3 \cdot 3 \\
\end{array} \quad \begin{array}{c}
\text{Tree 2} \\
54 \\
\downarrow \\
2 \cdot 27 \\
\downarrow \\
3 \cdot 9 \\
\downarrow \\
3 \cdot 3 \\
\end{array}
\]

\[54 = 3 \cdot 2 \cdot 3 \cdot 3 \quad \quad 54 = 2 \cdot 3 \cdot 3 \cdot 3\]

The prime factorization of 54 is \(2 \cdot 3^3\).

**Practice**

List the factor pairs of the number.

1. 16 \(1, 16; 2, 8; 4, 4\)
2. 30 \(1, 30; 2, 15; 3, 10; 5, 6\)
3. 63 \(1, 63; 3, 21; 7, 9\)
4. 100 \(1, 100; 2, 50; 4, 25; 5, 20; 10, 10\)
5. 135
6. 275 \(1, 275; 5, 55; 11, 25\)

Write the prime factorization of the number.

7. 24 \(2^3 \cdot 3\)
8. 66 \(2 \cdot 3 \cdot 11\)
9. 50 \(2 \cdot 5^2\)
10. 80 \(2^4 \cdot 5\)
11. 98 \(2 \cdot 7^2\)
12. 126 \(2 \cdot 3^2 \cdot 7\)
13. 154 \(2 \cdot 7 \cdot 11\)
14. 310 \(2 \cdot 5 \cdot 31\)

Find the greatest perfect square that is a factor of the number.

15. 117 \(9\)
16. 150 \(25\)
17. 539 \(49\)
18. 936 \(36\)

19. **EXERCISE** An exercise class has 28 participants. The instructor arranges the participants in rows. Each row has the same number of participants. How many possible arrangements are there? \(6\) possible arrangements