

**Chapter
12****Maintaining Mathematical Proficiency**

Write and solve a proportion to answer the question.

1. What percent of 260 is 65?
2. What number is 32% of 75?
3. 15.01 is what percent of 19?

Display the data in a histogram.

4.

	Number of Strikeouts in One Game		
Strikeouts	0–3	4–7	8–11
Frequency	34	20	8

5.

	Number of Days of Exercise in One Week			
Days of Exercise	0–1	2–3	4–5	6–7
Frequency	4	26	22	6

12.1

Sample Spaces and Probability

For use with Exploration 12.1

Essential Question How can you list the possible outcomes in the sample space of an experiment?

The **sample space** of an experiment is the set of all possible outcomes for that experiment.

1 EXPLORATION: Finding the Sample Space of an Experiment

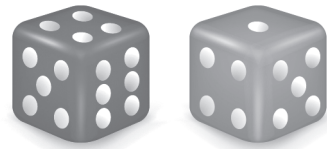
Work with a partner. In an experiment, three coins are flipped. List the possible outcomes in the sample space of the experiment.



2 EXPLORATION: Finding the Sample Space of an Experiment

Work with a partner. List the possible outcomes in the sample space of the experiment.

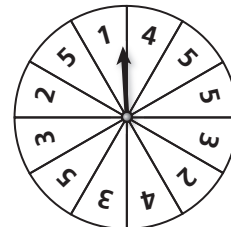
- a. One six-sided die is rolled.
- b. Two six-sided dice are rolled.



3 EXPLORATION: Finding the Sample Space of an Experiment

Work with a partner. In an experiment, a spinner is spun.

- a. How many ways can you spin a 1? 2? 3? 4? 5?

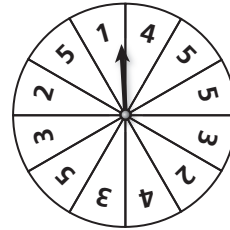


12.1 Sample Spaces and Probability (continued)

3 EXPLORATION: Finding the Sample Space of an Experiment (continued)

b. List the sample space.

c. What is the total number of outcomes?



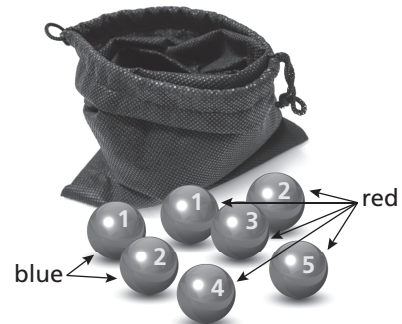
4 EXPLORATION: Finding the Sample Space of an Experiment

Work with a partner. In an experiment, a bag contains 2 blue marbles and 5 red marbles. Two marbles are drawn from the bag.

a. How many ways can you choose two blue? a red then blue? a blue then red? two red?

b. List the sample space.

c. What is the total number of outcomes?



Communicate Your Answer

5. How can you list the possible outcomes in the sample space of an experiment?

6. For Exploration 3, find the ratio of the number of each possible outcome to the total number of outcomes. Then find the sum of these ratios. Repeat for Exploration 4. What do you observe?

12.1**Notetaking with Vocabulary**

For use after Lesson 12.1

In your own words, write the meaning of each vocabulary term.

probability experiment

outcome

event

sample space

probability of an event

theoretical probability

geometric probability

experimental probability

Core Concepts**Probability of the Complement of an Event**

The probability of the complement of event A is

$$P(\overline{A}) = 1 - P(A).$$

Notes:

12.1 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1 and 2, find the number of possible outcomes in the sample space. Then list the possible outcomes.

1. A stack of cards contains the thirteen clubs from a standard deck of cards. You pick a card from the stack and flip two coins.
2. You spin a spinner with the numbers 1–5 on it and roll a die.
3. When two tiles with numbers between 1 and 10 are chosen from two different bags, there are 100 possible outcomes. Find the probability that (a) the sum of the two numbers is not 10 and (b) the product of the numbers is greater than 10.
4. At a school dance, the parents sell pizza slices. The table shows the number of pizza slices that are available. A student chooses a slice at random. What is the probability that the student chooses a thin crust slice with pepperoni?

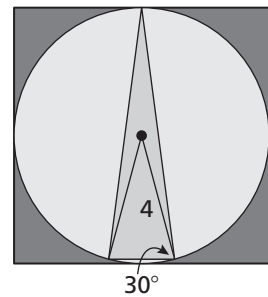
	Pepperoni	Plain Cheese
Thin Crust	34	36
Thick Crust	8	12

12.1 Notetaking with Vocabulary (continued)

5. Find the probability that the polynomial $x^2 - x - c$ can be factored if c is a randomly chosen integer from 1 to 12.

6. You throw a dart at the board shown. Your dart is equally likely to hit any point inside the square board.

a. What is the probability your dart lands in the smallest triangle?



b. What is the probability your dart does not land anywhere in the circle?

7. The sections of a spinner are numbered 1 through 12. Each section of the spinner has the same area. You spin the spinner 180 times. The table shows the results. For which number is the experimental probability of stopping on the number the same as the theoretical probability?

Spinner Results											
1	2	3	4	5	6	7	8	9	10	11	12
13	21	22	20	11	8	14	9	15	12	18	17

12.2**Independent and Dependent Events**

For use with Exploration 12.2

Essential Question How can you determine whether two events are independent or dependent?

Two events are **independent events** when the occurrence of one event does not affect the occurrence of the other event. Two events are **dependent events** when the occurrence of one event *does* affect the occurrence of the other event.

1 EXPLORATION: Identifying Independent and Dependent Events

Work with a partner. Determine whether the events are independent or dependent. Explain your reasoning.

- a. Two six-sided dice are rolled.

- b. Six pieces of paper, numbered 1 through 6, are in a bag. Two pieces of paper are selected one at a time without replacement.

**2 EXPLORATION: Finding Experimental Probabilities**

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

- a. In Exploration 1(a), experimentally estimate the probability that the sum of the two numbers rolled is 7. Describe your experiment.

- b. In Exploration 1(b), experimentally estimate the probability that the sum of the two numbers selected is 7. Describe your experiment.

12.2 Independent and Dependent Events (continued)**3 EXPLORATION: Finding Theoretical Probabilities**

Work with a partner.

- a. In Exploration 1(a), find the theoretical probability that the sum of the two numbers rolled is 7. Then compare your answer with the experimental probability you found in Exploration 2(a).

- b. In Exploration 1(b), find the theoretical probability that the sum of the two numbers selected is 7. Then compare your answer with the experimental probability you found in Exploration 2(b).

- c. Compare the probabilities you obtained in parts (a) and (b).

Communicate Your Answer

4. How can you determine whether two events are independent or dependent?

5. Determine whether the events are independent or dependent. Explain your reasoning.
 - a. You roll a 4 on a six-sided die and spin red on a spinner.

 - b. Your teacher chooses a student to lead a group, chooses another student to lead a second group, and chooses a third student to lead a third group.

12.2

Notetaking with Vocabulary

For use after Lesson 12.2

In your own words, write the meaning of each vocabulary term.

independent events

dependent events

conditional probability

Core Concepts

Probability of Independent Events

Words Two events A and B are independent events if and only if the probability that both events occur is the product of the probabilities of the events.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

Notes:

12.2 Notetaking with Vocabulary (continued)**Probability of Dependent Events**

Words If two events A and B are dependent events, then the probability that both events occur is the product of the probability of the first event and the conditional probability of the second event given the first event.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Example Using the information in Example 2:

$$\begin{aligned} P(\text{girl first and girl second}) &= P(\text{girl first}) \cdot P(\text{girl second}|\text{girl first}) \\ &= \frac{9}{12} \cdot \frac{6}{9} = \frac{1}{2} \end{aligned}$$

Notes:

Extra Practice

In Exercises 1 and 2, determine whether the events are independent. Explain your reasoning.

1. You have three white golf balls and two yellow golf balls in a bag. You randomly select one golf ball to hit now and another golf ball to place in your pocket. Use a sample space to determine whether randomly selecting a white golf ball first and then a white golf ball second are independent events.

2. Your friend writes a phone number down on a piece of paper but the last three numbers get smudged after being in your pocket all day long. You decide to randomly choose numbers for each of the three digits. Use a sample space to determine whether guessing the first digit correctly and the second digit correctly are independent events.

12.2 Notetaking with Vocabulary (continued)

3. You are trying to guess a three-letter password that uses only the letters A, E, I, O, U, and Y. Letters can be used more than once. Find the probability that you pick the correct password “YOU.”
4. You are trying to guess a three-letter password that uses only the letters A, E, I, O, U, and Y. Letters *cannot* be used more than once. Find the probability that you pick the correct password “AIE.”
5. The table shows the number of male and female college students who played collegiate basketball and collegiate soccer in the United States in a recent year.

	Collegiate Soccer	Collegiate Basketball
Male	37,240	31,863
Female	36,523	28,002

- a. Find the probability that a randomly selected collegiate soccer player is female.
- b. Find the probability that a randomly selected male student is a collegiate basketball player.

12.3

Two-Way Tables and Probability

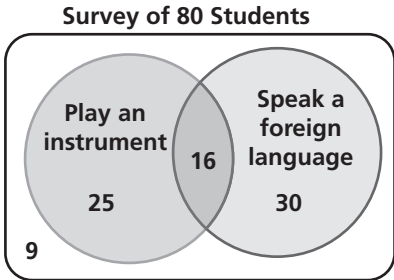
For use with Exploration 12.3

Essential Question How can you construct and interpret a two-way table?

1 EXPLORATION: Completing and Using a Two-Way Table

Work with a partner. A *two-way table* displays the same information as a Venn diagram. In a two-way table, one category is represented by the rows and the other category is represented by the columns.

The Venn diagram shows the results of a survey in which 80 students were asked whether they play a musical instrument and whether they speak a foreign language. Use the Venn diagram to complete the two-way table. Then use the two-way table to answer each question.



	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language			
Do Not Speak a Foreign Language			
Total			

- How many students play an instrument?
- How many students speak a foreign language?
- How many students play an instrument and speak a foreign language?
- How many students do not play an instrument and do not speak a foreign language?
- How many students play an instrument and do not speak a foreign language?

2 EXPLORATION: Two-Way Tables and Probability

Work with a partner. In Exploration 1, one student is selected at random from the 80 students who took the survey. Find the probability that the student

- plays an instrument.

12.3 Two-Way Tables and Probability (continued)**2 EXPLORATION: Two-Way Tables and Probability (continued)**

- b. speaks a foreign language.
- c. plays an instrument and speaks a foreign language.
- d. does not play an instrument and does not speak a foreign language.
- e. plays an instrument and does not speak a foreign language.

3 EXPLORATION: Conducting a Survey

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with your class. Conduct a survey of students in your class. Choose two categories that are different from those given in Explorations 1 and 2. Then summarize the results in both a Venn diagram and a two-way table. Discuss the results.

Communicate Your Answer

- 4. How can you construct and interpret a two-way table?
- 5. How can you use a two-way table to determine probabilities?

12.3**Notetaking with Vocabulary**

For use after Lesson 12.3

In your own words, write the meaning of each vocabulary term.

two-way table

joint frequency

marginal frequency

joint relative frequency

marginal relative frequency

conditional relative frequency

Core Concepts**Relative and Conditional Relative Frequencies**

A **joint relative frequency** is the ratio of a frequency that is not in the total row or the total column to the total number of values or observations.

A **marginal relative frequency** is the sum of the joint relative frequencies in a row or a column.

A **conditional relative frequency** is the ratio of a joint relative frequency to the marginal relative frequency. You can find a conditional relative frequency using a row total or a column total of a two-way table.

Notes:

12.3 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1 and 2, complete the two-way table.

1.

		Arrival		
		Tardy	On Time	Total
Method	Walk	22		
	City Bus			60
Total			58	130

2.

		Response		
		Yes	No	Total
Age	Under 21		24	25
	Over 21	29		
Total		30		75

3. A survey was taken of 100 families with one child and 86 families with two or more children to determine whether they were saving for college. Of those, 94 of the families with one child and 60 of the families with two or more children were saving for college. Organize these results in a two-way table. Then find and interpret the marginal frequencies.

12.3 Notetaking with Vocabulary (continued)

4. In a survey, 214 ninth graders played video games every day of the week and 22 ninth graders did not play video games every day of the week. Of those that played every day of the week, 36 had trouble sleeping at night. Of those that did not play every day of the week, 7 had trouble sleeping at night. Make a two-way table that shows the joint and marginal relative frequencies.
5. For financial reasons, a school district is debating about eliminating a Computer Programming class at the high school. The district surveyed parents, students, and teachers. The results, given as joint relative frequencies, are shown in the two-way table.

		Population		
		Parents	Students	Teachers
Response	Yes	0.58	0.08	0.10
	No	0.06	0.15	0.03

- a. What is the probability that a randomly selected parent voted to eliminate the class?
- b. What is the probability that a randomly selected student did not want to eliminate the class?
- c. Determine whether voting to eliminate the class and being a teacher are independent events.

12.4**Probability of Disjoint and Overlapping Events**

For use with Exploration 12.4

Essential Question How can you find probabilities of disjoint and overlapping events?

Two events are **disjoint**, or **mutually exclusive**, when they have no outcomes in common. Two events are **overlapping** when they have one or more outcomes in common.

1 EXPLORATION: Disjoint Events and Overlapping Events

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. A six-sided die is rolled. Draw a Venn diagram that relates the two events. Then decide whether the events are disjoint or overlapping.

- a. Event A : The result is an even number.
Event B : The result is a prime number.
- b. Event A : The result is 2 or 4.
Event B : The result is an odd number.

2 EXPLORATION: Finding the Probability that Two Events Occur

Work with a partner. A six-sided die is rolled. For each pair of events, find (a) $P(A)$, (b) $P(B)$, (c) $P(A \text{ and } B)$, and (d) $P(A \text{ or } B)$.

- a. Event A : The result is an even number.
Event B : The result is a prime number.
- b. Event A : The result is a 2 or 4.
Event B : The result is an odd number.

12.4 Probability of Disjoint and Overlapping Events (continued)**3** **EXPLORATION:** Discovering Probability Formulas

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- a. In general, if event A and event B are disjoint, then what is the probability that event A or event B will occur? Use a Venn diagram to justify your conclusion.

- b. In general, if event A and event B are overlapping, then what is the probability that event A or event B will occur? Use a Venn diagram to justify your conclusion.

- c. Conduct an experiment using a six-sided die. Roll the die 50 times and record the results. Then use the results to find the probabilities described in Exploration 2. How closely do your experimental probabilities compare to the theoretical probabilities you found in Exploration 2?

Communicate Your Answer

4. How can you find probabilities of disjoint and overlapping events?

5. Give examples of disjoint events and overlapping events that do not involve dice.

12.4**Notetaking with Vocabulary**

For use after Lesson 12.4

In your own words, write the meaning of each vocabulary term.

compound event

overlapping events

disjoint or mutually exclusive events

Core Concepts**Probability of Compound Events**

If A and B are any two events, then the probability of A or B is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

If A and B are disjoint events, then the probability of A or B is

$$P(A \text{ or } B) = P(A) + P(B).$$

Notes:

12.4 Notetaking with Vocabulary (continued)**Extra Practice**

1. Events A and B are disjoint. $P(A) = \frac{2}{3}$ and $P(B) = \frac{1}{6}$. Find $P(A \text{ or } B)$.
2. $P(A) = 0.8$, $P(B) = 0.05$, and $P(A \text{ or } B) = 0.6$. Find $P(A \text{ and } B)$.

In Exercises 3–6, a vehicle is randomly chosen from a parking lot. The parking lot contains three red minivans, two blue minivans, three blue convertibles, one black pickup truck, three black motorcycles, one red motorcycle and two blue scooters. Find the probability of selecting the type of vehicle.

3. A red vehicle or a minivan
4. A scooter or a black vehicle
5. A black vehicle or a motorcycle
6. A four-wheeled vehicle or a blue vehicle

12.4 Notetaking with Vocabulary (continued)

7. During a basketball game, the coach needs to select a player to make the free throw after a technical foul on the other team. There is a 68% chance that the coach will select you and a 26% chance that the coach will select your friend. What is the probability that you or your friend is selected to make the free throw?

8. Two six-sided dice are rolled. Find the probability of rolling the same number twice.

9. Out of 120 student parents, 90 of them can chaperone the Homecoming dance or the Prom. There are 40 parents who can chaperone the Homecoming dance and 65 parents who can chaperone the Prom. What is the probability that a randomly selected parent can chaperone both the Homecoming dance and the Prom?

10. A football team scores a touchdown first 75% of the time when they start with the ball. The team does not score first 51% of the time when their opponent starts with the ball. The team who gets the ball first is determined by a coin toss. What is the probability that the team scores a touchdown first?

12.5

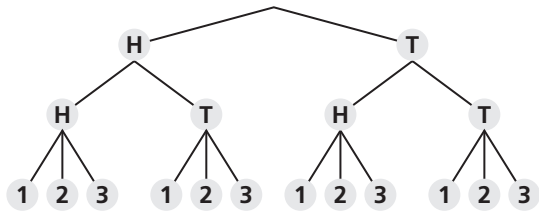
Permutations and Combinations

For use with Exploration 12.5

Essential Question How can a tree diagram help you visualize the number of ways in which two or more events can occur?

1 EXPLORATION: Reading a Tree Diagram

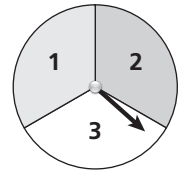
Work with a partner. Two coins are flipped and the spinner is spun. The tree diagram shows the possible outcomes.



Coin is flipped.

Coin is flipped.

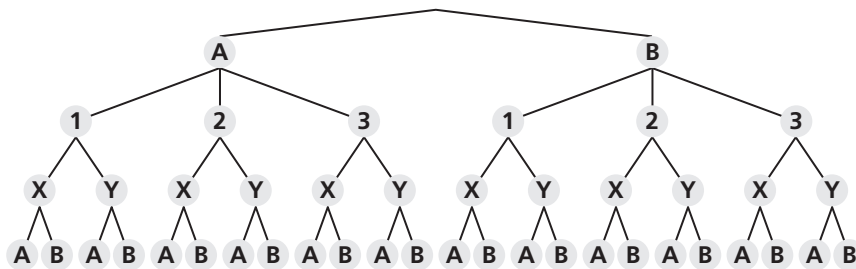
Spinner is spun.



- How many outcomes are possible?
- List the possible outcomes.

2 EXPLORATION: Reading a Tree Diagram

Work with a partner. Consider the tree diagram below.



- How many events are shown?
- What outcomes are possible for each event?
- How many outcomes are possible?
- List the possible outcomes.

12.5 Permutations and Combinations (continued)**3** **EXPLORATION:** Writing a Conjecture

Work with a partner.

- a. Consider the following general problem: Event 1 can occur in m ways and event 2 can occur in n ways. Write a conjecture about the number of ways the two events can occur. Explain your reasoning.

- b. Use the conjecture you wrote in part (a) to write a conjecture about the number of ways *more than* two events can occur. Explain your reasoning.

- c. Use the results of Explorations 1(a) and 2(c) to verify your conjectures.

Communicate Your Answer

4. How can a tree diagram help you visualize the number of ways in which two or more events can occur?

5. In Exploration 1, the spinner is spun a second time. How many outcomes are possible?

12.5**Notetaking with Vocabulary**

For use after Lesson 12.5

In your own words, write the meaning of each vocabulary term.

permutation

 n factorial

combination

Core Concepts**Permutations****Formulas**The number of permutations of n objects is given by

$${}_n P_n = n!$$

The number of permutations of n objects taken r at a time, where $r \leq n$, is given by

$${}_n P_r = \frac{n!}{(n-r)!}$$

Examples

The number of permutations of 4 objects is

$${}_4 P_4 = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24.$$

The number of permutations of 4 objects taken 2 at a time is

$${}_4 P_2 = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!}} = 12.$$

Notes:

12.5 Notetaking with Vocabulary (continued)**Combinations**

Formula The number of combinations of n objects taken r at a time, where $r \leq n$, is given by

$${}_n C_r = \frac{n!}{(n-r)! \cdot r!}$$

Example The number of combinations of 4 objects taken 2 at a time is

$$\begin{aligned} {}_4 C_2 &= \frac{4!}{(4-2)! \cdot 2!} \\ &= \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!} \cdot (2 \cdot 1)} \\ &= 6. \end{aligned}$$

Notes:

12.5 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1 and 2, find the number of ways you can arrange (a) all of the numbers and (b) 3 of the numbers in the given amount.

1. \$2,564,783
2. \$4,128,675,309

3. Your rock band has nine songs recorded but you only want to put five of them on your demo CD to hand out to local radio stations. How many possible ways could the five songs be ordered on your demo CD?

4. A witness at the scene of a hit-and-run accident saw that the car that caused the accident had a license plate with only the letters I, R, L, T, O, and A. Find the probability that the license plate starts with a T and ends with an R.

5. How many possible combinations of three colors can be chosen from the seven colors of the rainbow?

12.6

Binomial Distributions

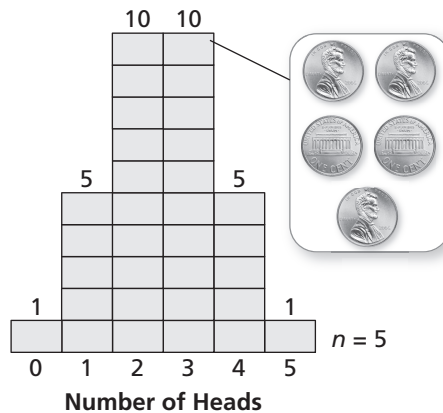
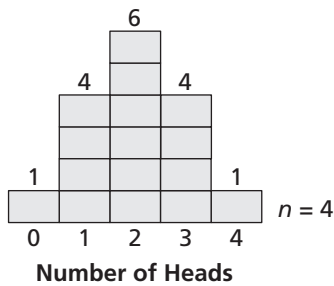
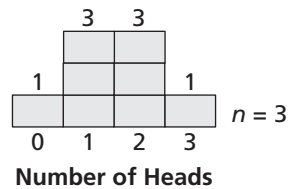
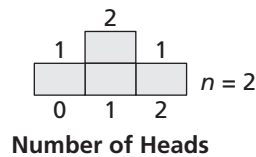
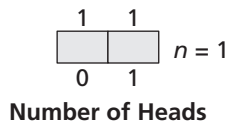
For use with Exploration 12.6

Essential Question How can you determine the frequency of each outcome of an event?

1 EXPLORATION: Analyzing Histograms

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. The histograms show the results when n coins are flipped.



- In how many ways can 3 heads occur when 5 coins are flipped?
- Draw a histogram that shows the numbers of heads that can occur when 6 coins are flipped.
- In how many ways can 3 heads occur when 6 coins are flipped?

12.6 Binomial Distributions (continued)**2** **EXPLORATION:** Determining the Number of Occurrences

Work with a partner.

- a. Complete the table showing the numbers of ways in which 2 heads can occur when n coins are flipped.

n	3	4	5	6	7
Occurrences of 2 heads					

- b. Determine the pattern shown in the table. Use your result to find the number of ways in which 2 heads can occur when 8 coins are flipped.

Communicate Your Answer

3. How can you determine the frequency of each outcome of an event?
4. How can you use a histogram to find the probability of an event?

12.6**Notetaking with Vocabulary**

For use after Lesson 12.6

In your own words, write the meaning of each vocabulary term.

random variable

probability distribution

binomial distribution

binomial experiment

Core Concepts**Probability Distributions**

A **probability distribution** is a function that gives the probability of each possible value of a random variable. The sum of all the probabilities in a probability distribution must equal 1.

Probability Distribution for Rolling a Six-Sided Die						
x	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Notes:

12.6 Notetaking with Vocabulary (continued)**Binomial Experiments**

A **binomial experiment** meets the following conditions.

- There are n independent trials.
- Each trial has only two possible outcomes: success and failure.
- The probability of success is the same for each trial. This probability is denoted by p . The probability of failure is $1 - p$.

For a binomial experiment, the probability of exactly k successes in n trials is

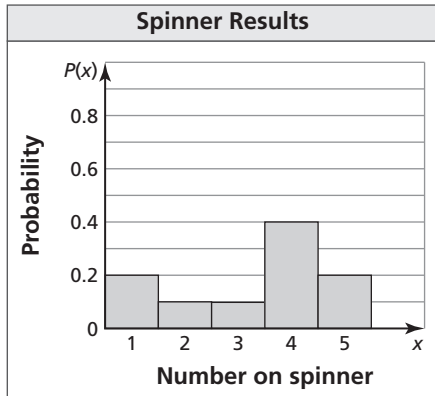
$$P(k \text{ successes}) = {}_n C_k p^k (1 - p)^{n-k}.$$

Notes:**Extra Practice**

1. Make a table and draw a histogram showing the probability distribution for the random variable P if $P =$ the product when two six-sided dice are rolled.

12.6 Notetaking with Vocabulary (continued)

2. Use the probability distribution to determine (a) the number that is most likely to be spun on a spinner, and (b) the probability of spinning a perfect square.



3. Calculate the probability of flipping a coin twenty times and getting nineteen heads.
4. According to a survey, 78% of women in a city watch professional football. You ask four randomly chosen women from the city whether they watch professional football.
- Draw a histogram of the binomial distribution for your survey.
 - What is the most likely outcome of your survey?
 - What is the probability that at most one woman watches professional football?

