Chapter 11 **Maintaining Mathematical Proficiency**

Find the surface area of the prism.

1.

2 in. | 4 in.
12 in.

2.

13 cm

12 cm

13 cm

15 cm

10 cm
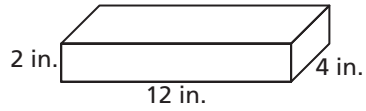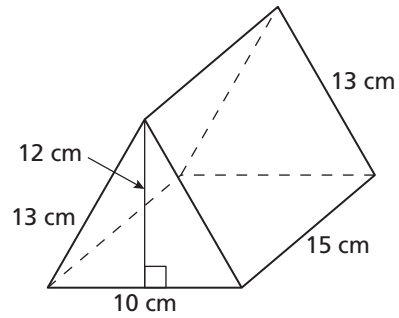
Find the missing dimension.

3. A rectangle has an area of 25 square inches and a length of 10 inches. What is the width of the rectangle?

4. A triangle has an area of 32 square centimeters and a base of 8 centimeters. What is the height of the triangle?
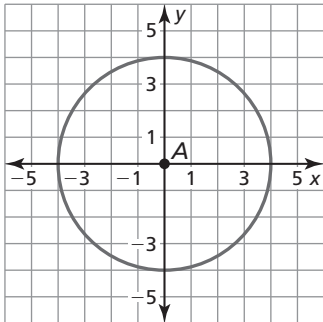
11.1 Circumference and Arc Length

For use with Exploration 11.1

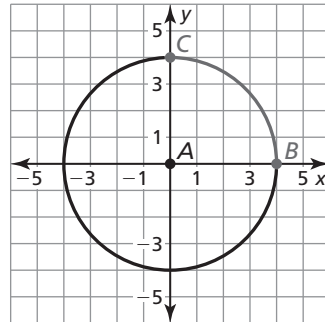**Essential Question** How can you find the length of a circular arc?

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1 EXPLORATION: Finding the Length of a Circular Arc
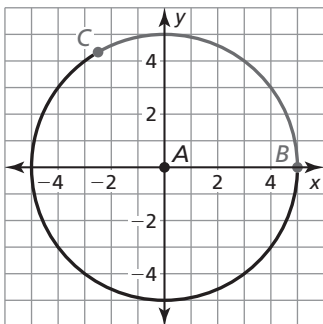
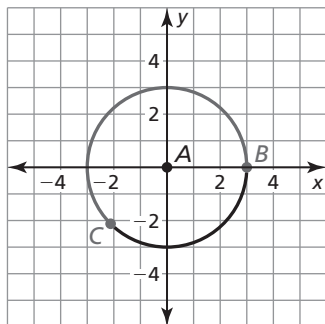**Work with a partner.** Find the length of each gray circular arc.

a. entire circle

b. one-fourth of a circle

c. one-third of a circle

d. five-eighths of a circle

11.1 **Circumference and Arc Length** (continued)

2 **EXPLORATION:** Writing a Conjecture

Work with a partner. The rider is attempting to stop with the front tire of the motorcycle in the painted rectangular box for a skills test. The front tire makes exactly one-half additional revolution before stopping. The diameter of the tire is 25 inches. Is the front tire still in contact with the painted box? Explain.

|——3 ft——|
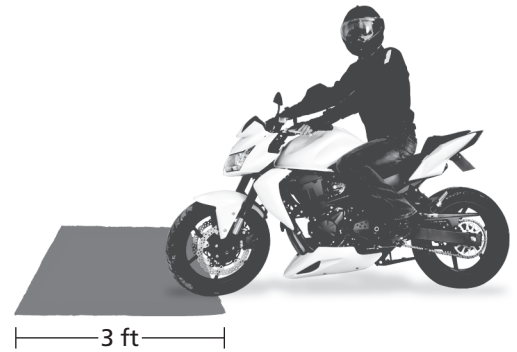
Communicate Your Answer

3. How can you find the length of a circular arc?

4. A motorcycle tire has a diameter of 24 inches. Approximately how many inches does the motorcycle travel when its front tire makes three-fourths of a revolution?

Name _____ Date _____

Notetaking with Vocabulary
For use after Lesson 11.1

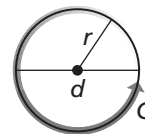**In your own words, write the meaning of each vocabulary term.**

circumference

arc length

radian

Core Concepts
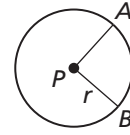
Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.

Notes:

$C = \pi d = 2\pi r$

11.1 **Notetaking with Vocabulary (continued)**

Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

$$\frac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360°}, \text{ or}$$

$$\text{Arc length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$$

Notes:

Converting Between Degrees and Radians

Degrees to radians

Multiply degree measure by

$$\frac{2\pi \text{ radians}}{360°}, \text{ or } \frac{\pi \text{ radians}}{180°}.$$
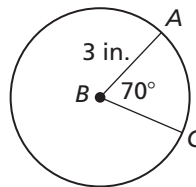
Radians to degrees

Multiply radian measure by

$$\frac{360°}{2\pi \text{ radians}}, \text{ or } \frac{180°}{\pi \text{ radians}}.$$

Notes:

11.1 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–5, find the indicated measure.

1. diameter of a circle with a circumference of 10 inches

2. circumference of a circle with a radius of 3 centimeters

3. radius of a circle with a circumference of 8 feet

4. circumference of a circle with a diameter of 2.4 meters

5. arc length of $\overset{\frown}{AC}$

In Exercises 6 and 7, convert the angle measure.

6. Convert 60° to radians.

7. Convert $\dfrac{5\pi}{6}$ radians to degrees.

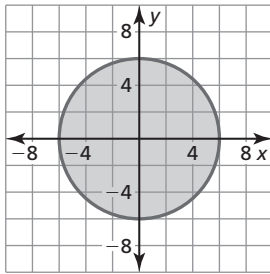Name_____ Date_____

11.2 Areas of Circles and Sectors
For use with Exploration 11.2

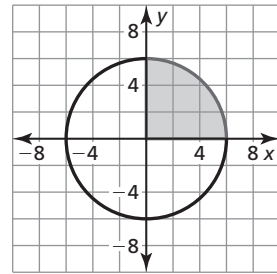**Essential Question** How can you find the area of a sector of a circle?

1 EXPLORATION: Finding the Area of a Sector of a Circle

Work with a partner. A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. Find the area of each shaded circle or sector of a circle.
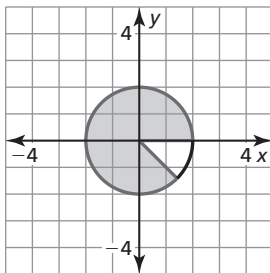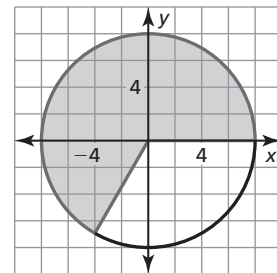
a. entire circle

b. one-fourth of a circle

**c.** seven-eighths of a circle

d. two-thirds of a circle

11.2 **Areas of Circles and Sectors** (continued)

2 **EXPLORATION:** Finding the Area of a Circular Sector

Work with a partner. A center pivot irrigation system consists of 400 meters of sprinkler equipment that rotates around a central pivot point at a rate of once every 3 days to irrigate a circular region with a diameter of 800 meters. Find the area of the sector that is irrigated by this system in one day.
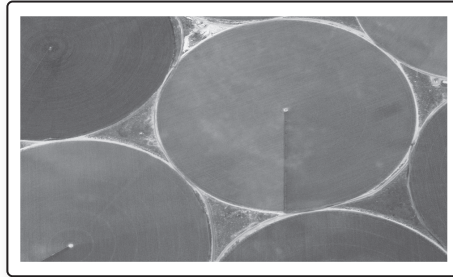
Communicate Your Answer

3. How can you find the area of a sector of a circle?

4. In Exploration 2, find the area of the sector that is irrigated in 2 hours.

11.2 Notetaking with Vocabulary

For use after Lesson 11.2

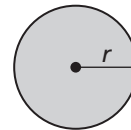**In your own words, write the meaning of each vocabulary term.**

population density

sector of a circle
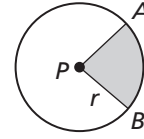
Core Concepts

Area of a Circle

The area of a circle is

$$A = \pi r^2$$

where *r* is the radius of the circle.

Notes:

11.2 Notetaking with Vocabulary **(continued)**

Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle (πr^2) is equal to the ratio of the measure of the intercepted arc to 360°.
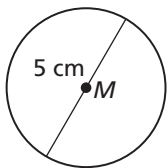
$$\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m\widehat{AB}}{360°}, \text{ or}$$

$$\text{Area of sector } APB = \frac{m\widehat{AB}}{360°} \cdot \pi r^2$$

Notes:
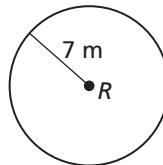
Extra Practice

In Exercises 1–2, find the indicated measure.

   1. area of $\odot M$

     5 cm

   2. area of $\odot R$

     7 m

11.2 **Notetaking with Vocabulary** (continued)

In Exercises 3–8, find the indicated measure.
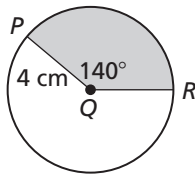
3. area of a circle with a diameter of 1.8 inches

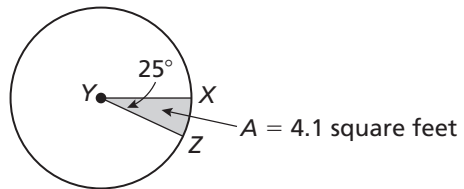**4.** diameter of a circle with an area of 10 square feet

5. radius of a circle with an area of 65 square centimeters

6. area of a circle with a radius of 6.1 yards

7. areas of the sectors formed by $\angle PQR$

8. area of $\odot Y$

9. About 70,000 people live in a region with a 30-mile radius. Find the population density in people per square mile.

Name _____ Date _____
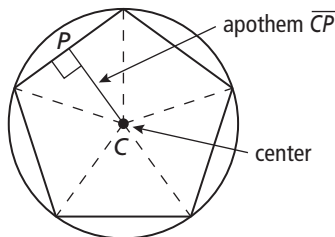
11.3 Areas of Polygons
For use with Exploration 11.3

Essential Question How can you find the area of a regular polygon?

The **center of a regular polygon** is the center of its circumscribed circle.

The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**.

apothem $\overline{CP}$

center

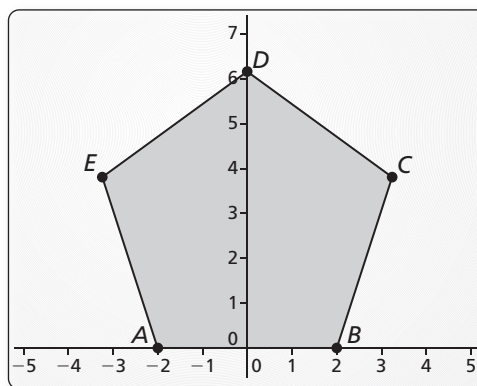**1   EXPLORATION:** Finding the Area of a Regular Polygon

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to construct each regular polygon with side lengths of 4, as shown. Find the apothem and use it to find the area of the polygon. Describe the steps that you used.
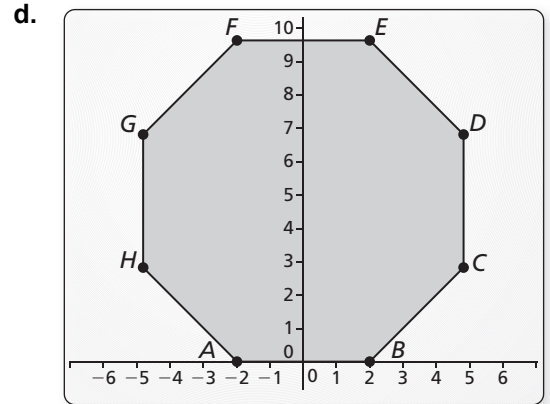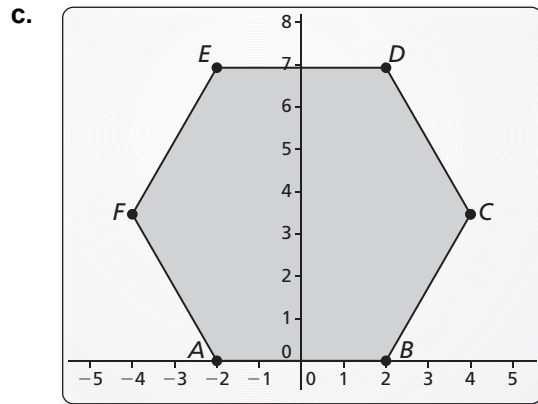
a.
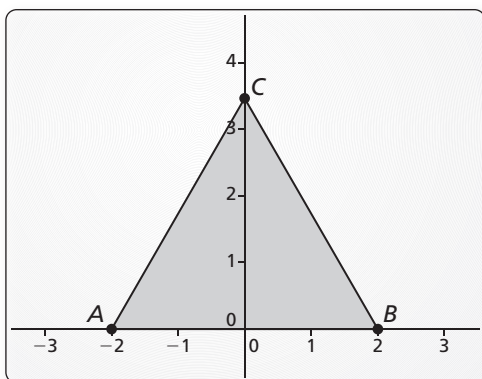
b.

11.3 **Areas of Polygons** (continued)

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1 **EXPLORATION:** Finding the Area of a Regular Polygon (continued)

c.

d.

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2 **EXPLORATION:** Writing a Formula for Area

Work with a partner. Generalize the steps you used in Exploration 1 to develop a formula for the area of a regular polygon.

Communicate Your Answer

3. How can you find the area of a regular polygon?

4. Regular pentagon *ABCDE* has side lengths of 6 meters and an apothem of approximately 4.13 meters. Find the area of *ABCDE*.
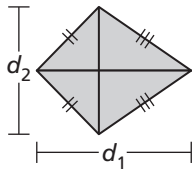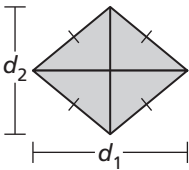
Name _____ Date _____

In your own words, write the meaning of each vocabulary term.

center of a regular polygon

radius of a regular polygon

apothem of a regular polygon
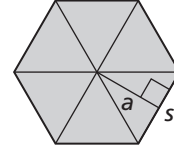
central angle of a regular polygon

Core Concepts

Area of a Rhombus or Kite

The area of a rhombus or kite with diagonals d_1 and d_2 is $\frac{1}{2}d_1d_2$.

Notes:

11.3 **Notetaking with Vocabulary** (continued)

Area of a Regular Polygon

The area of a regular *n*-gon with side length *s* is one-half the product of the apothem *a* and the perimeter *P*.

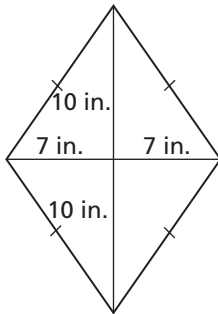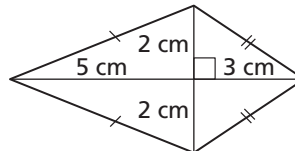$$A = \frac{1}{2}aP, \text{ or } A = \frac{1}{2}a \cdot ns$$

Notes:

Extra Practice

In Exercises 1 and 2, find the area of the kite or rhombus.

1.

10 in.

7 in.   7 in.

10 in.

2.

2 cm

5 cm   3 cm

2 cm

11.3 **Notetaking with Vocabulary (continued)**

3. Find the measure of a central angle of a regular polygon with 8 sides.

4. The central angles of a regular polygon are 40°. How many sides does the polygon have?

5. A regular pentagon has a radius of 4 inches and a side length of 3 inches.

   a. Find the apothem of the pentagon.

   b. Find the area of the pentagon.

6. A regular hexagon has an apothem of 10 units.

   a. Find the radius of the hexagon and the length of one side.

   **b.** Find the area of the hexagon.

Name_____ Date _____

Essential Question What is the relationship between the numbers of vertices *V*, edges *E*, and faces *F* of a polyhedron?

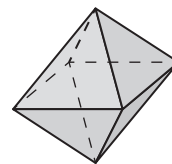**1 EXPLORATION:** Analyzing a Property of Polyhedra

Work with a partner. The five *Platonic solids* are shown below. Each of these solids has congruent regular polygons as faces. Complete the table by listing the numbers of vertices, edges, and faces of each Platonic solid.
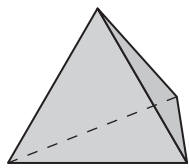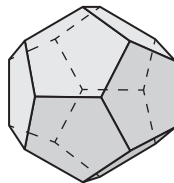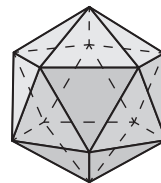
tetrahedron   cube   octahedron
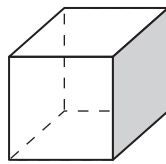
dodecahedron   icosahedron

Solid	Vertices, *V*	Edges, *E*	Faces, *F*
tetrahedron			
cube			
octahedron			
dodecahedron			
icosahedron			

11.4 **Three-Dimensional Figures** (continued)

Communicate Your Answer

2. What is the relationship between the numbers of vertices V, edges E, and faces F of a polyhedron? (*Note*: Swiss mathematician Leonhard Euler (1707–1783) discovered a formula that relates these quantities.)

3. Draw three polyhedra that are different from the Platonic solids given in Exploration 1. Count the number of vertices, edges, and faces of each polyhedron. Then verify that the relationship you found in Question 2 is valid for each polyhedron.

Name_____ Date _____

11.4 Notetaking with Vocabulary
For use after Lesson 11.4

In your own words, write the meaning of each vocabulary term.

polyhedron

face

edge

vertex
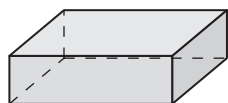

cross section

solid of revolution

axis of revolution

Notes:

Name _____ Date _____

Core Concepts

Types of Solids

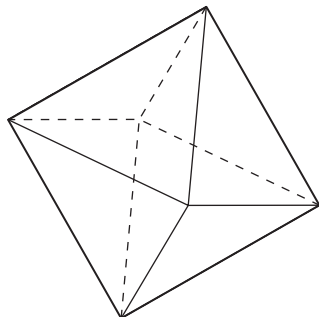| Polyhedra	Not Polyhedra

prism
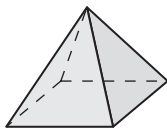
cylinder   cone

pyramid

sphere

Notes:

Extra Practice

In Exercises 1 and 2, tell whether the solid is a polyhedron. If it is, name the polyhedron.
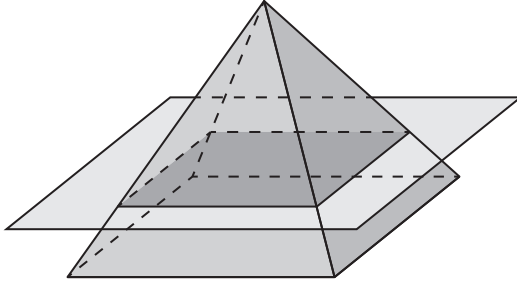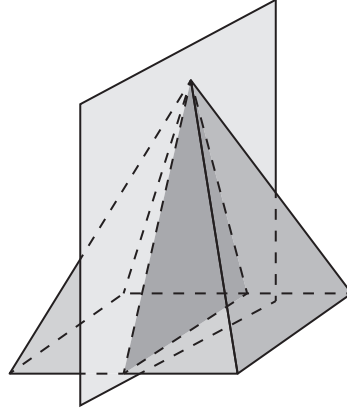
1.

2.

Name_____ Date _____
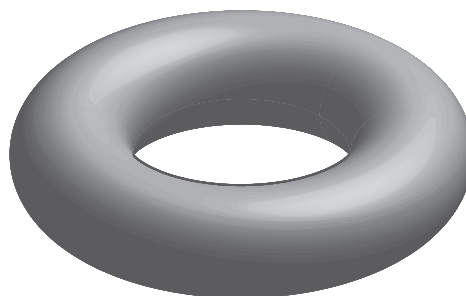
11.4 **Notetaking with Vocabulary** (continued)

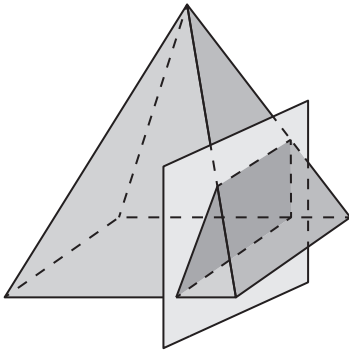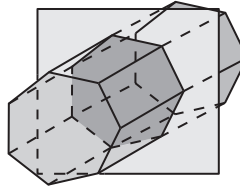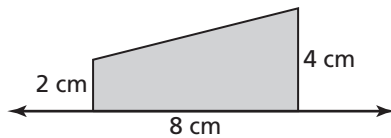**In Exercises 3–6, describe the cross section formed by the intersection of the plane and the solid.**
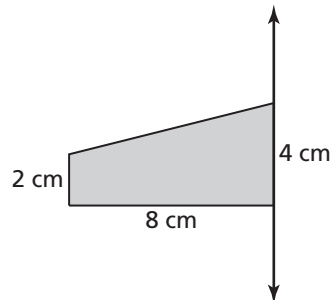
3.

4.

5.

6.

In Exercises 7 and 8, sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

7.

4 cm

2 cm

8 cm

8.

4 cm

2 cm
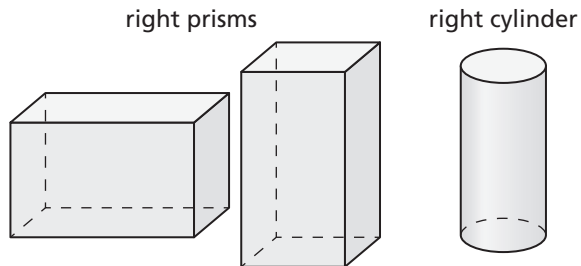
8 cm

Name _____ Date _____

11.5 Volumes of Prisms and Cylinders
For use with Exploration 11.5

Essential Question How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

Recall that the volume V of a right prism or a right cylinder is equal to the product of the area of a base B and the height h.
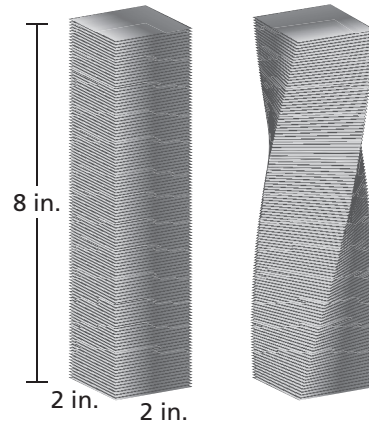
$$V = Bh$$

right prisms   right cylinder

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1   **EXPLORATION:** Finding Volume

Work with a partner. Consider a stack of square papers that is in the form of a right prism.

    a. What is the volume of the prism?

    b. When you twist the stack of papers, as shown at the right, do you change the volume? Explain your reasoning.
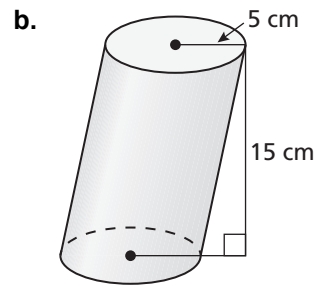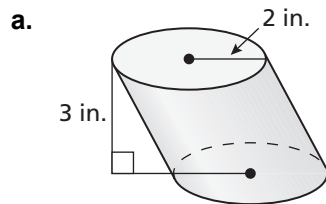
8 in.

2 in.   2 in.

    c. Write a carefully worded conjecture that describes the conclusion you reached in part (b).

    d. Use your conjecture to find the volume of the twisted stack of papers.

11.5 **Volumes of Prisms and Cylinders** (continued)

2 **EXPLORATION:** Finding Volume

Work with a partner. Use the conjecture you wrote in Exploration 1 to find the volume of the cylinder.

a.

2 in.

3 in.

b.

5 cm

15 cm

Communicate Your Answer

3. How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

4. In Exploration 1, would the conjecture you wrote change if the papers in each stack were not squares? Explain your reasoning.

Name _____ Date _____

Notetaking with Vocabulary
For use after Lesson 11.5

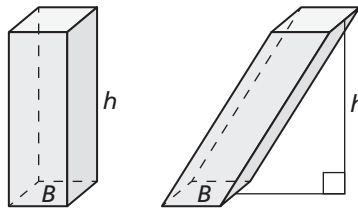**In your own words, write the meaning of each vocabulary term.**

volume

Cavalieri's Principle

density

similar solids

Core Concepts

Volume of a Prism

The volume V of a prism is

$$V = Bh$$

where B is the area of a base and h is the height.
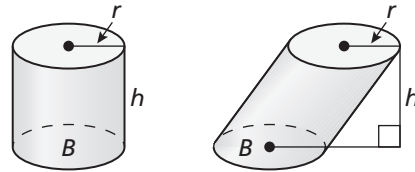

Notes:

11.5 **Notetaking with Vocabulary** (continued)

Volume of a Cylinder

The volume V of a cylinder is

$$V = Bh = \pi r^2 h$$

where B is the area of a base, h is the
height, and r is the radius of a base.

Notes:

Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights
or radii, are called **similar solids**. The ratio of the corresponding linear measures of two similar
solids is called the *scale factor*. If two similar solids have a scale factor of k, then the ratio of
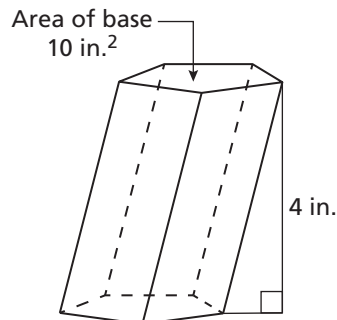their volumes is equal to k^3.

Notes:

Extra Practice

In Exercises 1 and 2, find the volume of the prism.

1.
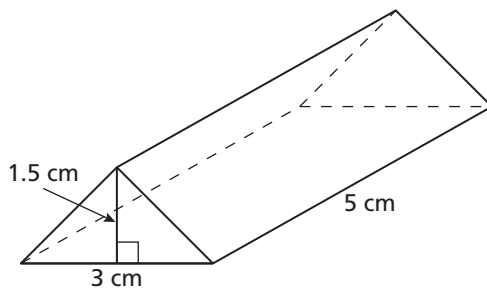
1.5 cm
5 cm
3 cm

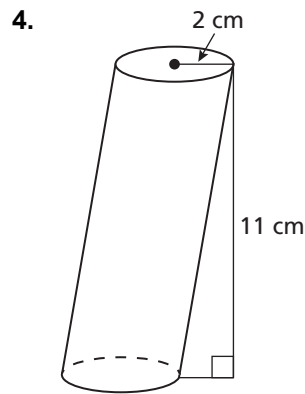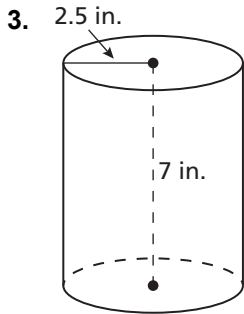**2.** Area of base
10 in.^{2}

4 in.

11.5 Notetaking with Vocabulary (continued)

In Exercises 3 and 4, find the volume of the cylinder.

**3.** 2.5 in.

7 in.

4. 2 cm

11 cm

In Exercises 5 and 6, find the indicated measure.

5. height of a cylinder with a base radius of 8 inches and a volume of 2010 cubic inches

6. area of the base of a pentagonal prism with a volume of 50 cubic centimeters and a height of 7.5 centimeters

**In Exercises 7 and 8, find the missing dimension of the prism or cylinder.**

7. Volume = 661.5 cm³

3.5 cm

y

18 cm

8. Volume = 75.36 in.³

p

6 in.

Name_____   Date_____
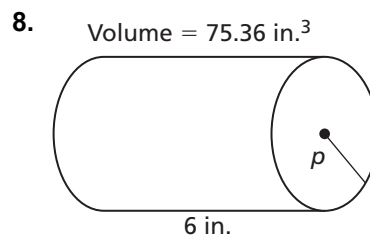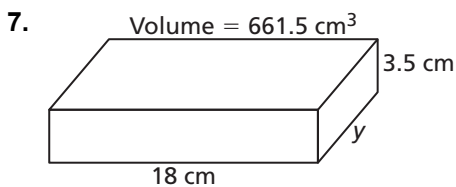
11.6 Volumes of Pyramids
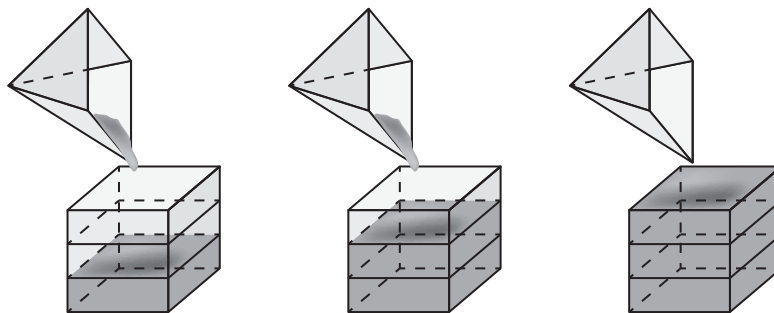**For use with Exploration 11.6**

Essential Question   How can you find the volume of a pyramid?

1   EXPLORATION: Finding the Volume of a Pyramid

Work with a partner. The pyramid and the prism have the same height and the same square base.
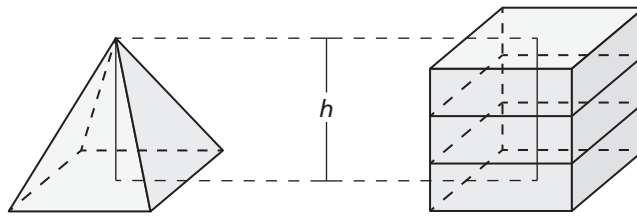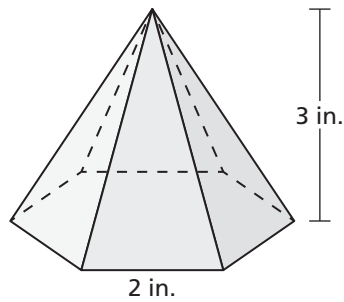
When the pyramid is filled with sand and poured into the prism, it takes three pyramids to fill the prism.

Use this information to write a formula for the volume V of a pyramid.

Name _____ Date _____

2 **EXPLORATION:** Finding the Volume of a Pyramid

Work with a partner. Use the formula you wrote in Exploration 1 to find the volume of the hexagonal pyramid.

3 in.

2 in.

Communicate Your Answer

3. How can you find the volume of a pyramid?

4. In Section 11.7, you will study volumes of cones. How do you think you could use a method similar to the one presented in Exploration 1 to write a formula for the volume of a cone? Explain your reasoning.

11.6 Notetaking with Vocabulary
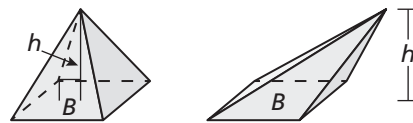**For use after Lesson 11.6**

In your own words, write the meaning of each vocabulary term.

pyramid

composite solid

Core Concepts

Volume of a Pyramid
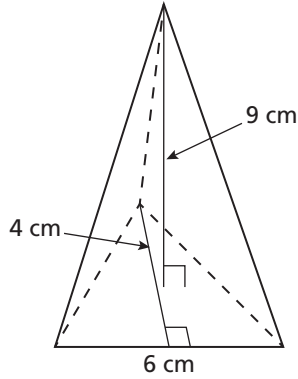
The volume V of a pyramid is

$$V = \frac{1}{3}Bh$$

where B is the area of a base and h is the height.

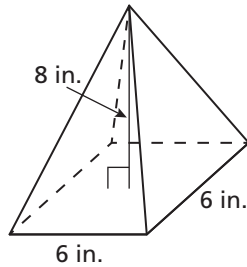**Notes:**

11.6 **Notetaking with Vocabulary** (continued)

Extra Practice

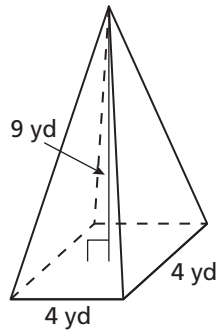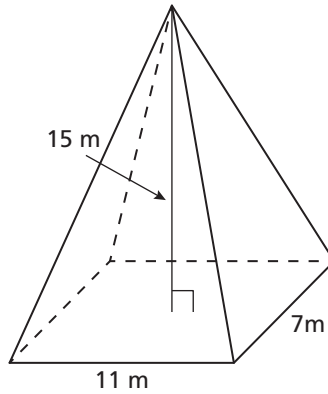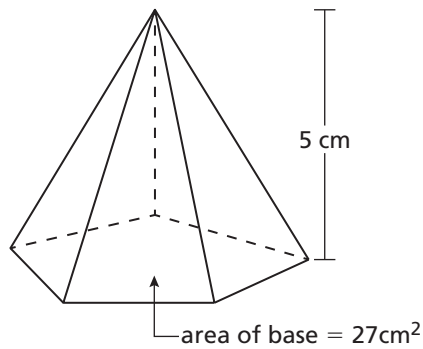**In Exercises 1–6, find the volume of the pyramid.**

1.

9 cm

4 cm

6 cm

2.

8 in.

6 in.

6 in.

3.

9 yd

4 yd

4 yd

4.

15 m

7m

11 m

5.

5 cm

area of base = 27cm²
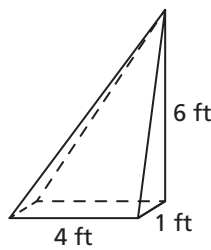
6.

6 ft

4 ft

1 ft

**11.6** **Notetaking with Vocabulary** (continued)

In Exercises 7–9, find the indicated measure.

7. A pyramid with a square base has a volume of 128 cubic inches and a height of 6 inches. Find the side length of the square base.

8. A pyramid with a rectangular base has a volume of 6 cubic feet. The length of the rectangular base is 3 feet and the width of the base is 1.5 feet. Find the height of the pyramid.

9. A pyramid with a triangular base has a volume of 18 cubic centimeters. The height of the pyramid is 9 centimeters and the height of the triangular base is 3 centimeters. Find the width of the base.

10. The pyramids are similar. Find the volume of pyramid B.

Pyramid A

15 in.

$V = 125$ in.^{3}

Pyramid B

10 in.

Name_____ Date _____
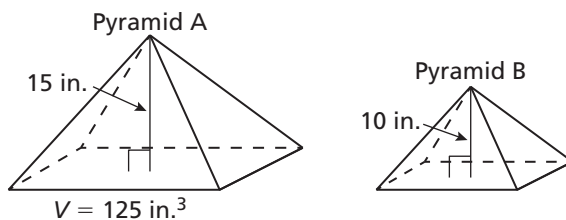
Surface Areas and Volumes of Cones
For use with Exploration 11.7

Essential Question   How can you find the surface area and the volume of a cone?

1   EXPLORATION: Finding the Surface Area of a Cone

Work with a partner. Construct a circle with a radius of 3 inches. Mark the circumference of the circle into six equal parts, and label the length of each part. Then cut out one sector of the circle and make a cone.
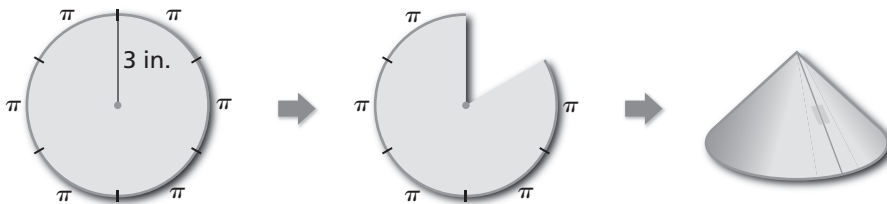

a. Explain why the base of the cone is a circle. What are the circumference and radius of the base?

b. What is the area of the original circle? What is the area with one sector missing?

c. Describe the surface area of the cone, including the base. Use your description to find the surface area.
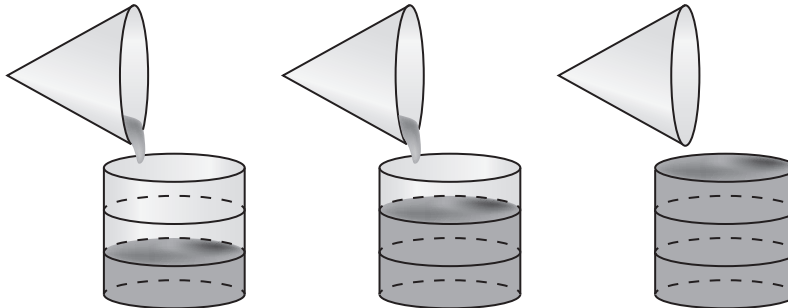
11.7 **Surface Areas and Volumes of Cones** (continued)

2 **EXPLORATION:** Finding the Volume of a Cone

Work with a partner. The cone and the cylinder have the same height and the same circular base.
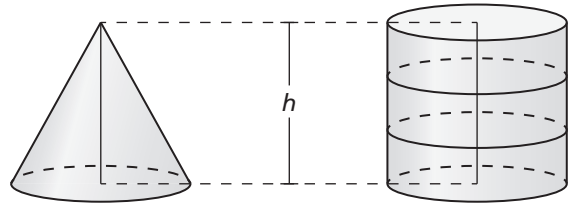
When the cone is filled with sand and poured into the cylinder, it takes three cones to fill the cylinder.

Use this information to write a formula for the volume V of a cone.

Communicate Your Answer

3. How can you find the surface area and the volume of a cone?

4. In Exploration 1, cut another sector from the circle and make a cone. Find the radius of the base and the surface area of the cone. Repeat this three times, recording your results in a table. Describe the pattern.

Radius of Base	Surface Area of Cone

11.7 Notetaking with Vocabulary
For use after Lesson 11.7

In your own words, write the meaning of each vocabulary term.

lateral surface of a cone

Notes:

Core Concepts

Surface Area of a Right Cone

The surface area S of a right cone is

$$S = \pi r^2 + \pi r \ell$$

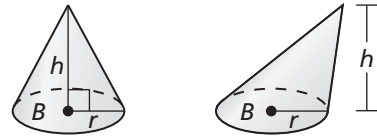where r is the radius of the base and ℓ is the slant height.

Notes:

11.7 **Notetaking with Vocabulary** (continued)

Volume of a Cone

The volume V of a cone is

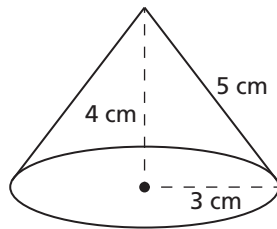$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$

where B is the area of a base, h is the height, and r is the radius of the base.

Notes:

Extra Practice

In Exercises 1 and 2, find the surface area of the right cone.
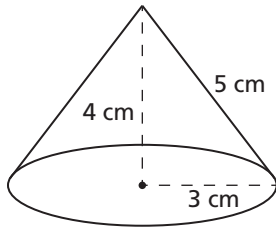
1.

5 cm

4 cm

3 cm

2. A right cone has a diameter of 1.8 inches and a height of 3 inches.

11.7   **Notetaking with Vocabulary** (continued)

In Exercises 3 and 4, find the volume of the cone.

3.

**4.** A right cone has a radius of 5 feet and a slant height of 13 feet.

In Exercises 5–7, find the indicated measure.

5. A right cone has a surface area of 440 square inches and a radius of 7 inches. Find its slant height.

6. A right cone has a volume of 528 cubic meters and a diameter of 12 meters. Find its height.

7. Cone A and cone B are similar. The radius of cone A is 4 cm and the radius of cone B is 10 cm. The volume of cone A is 134 cm^3, find the volume of cone B.
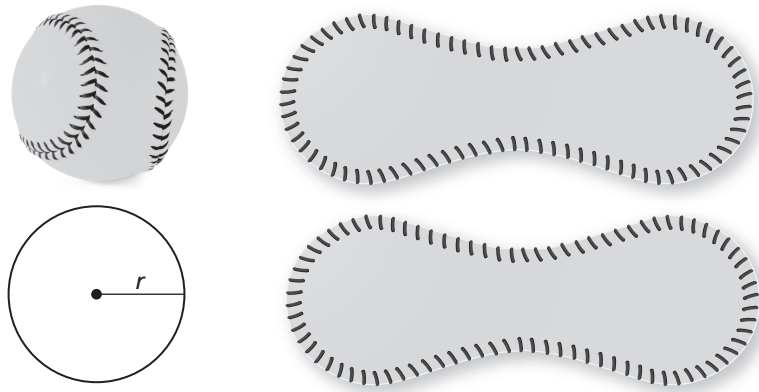
8. Find the volume of the composite solid.

Name_____ Date _____
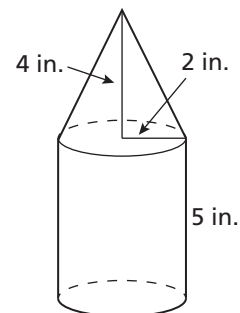
11.8 Surface Areas and Volumes of Spheres
For use with Exploration 11.8

Essential Question How can you find the surface area and the volume of a sphere?

1 EXPLORATION: Finding the Surface Area of a Sphere

Work with a partner. Remove the covering from a baseball or softball.


You will end up with two "figure 8" pieces of material, as shown above. From the amount of material it takes to cover the ball, what would you estimate the surface area S of the ball to be? Express your answer in terms of the radius r of the ball.

S = _____   Surface area of a sphere

Use the Internet or some other resource to confirm that the formula you wrote for the surface area of a sphere is correct.

11.8 **Surface Areas and Volumes of Spheres** (continued)

2 **EXPLORATION:** Finding the Volume of a Sphere

Work with a partner. A cylinder is circumscribed about a sphere, as shown. Write a formula for the volume V of the cylinder in terms of the radius r.
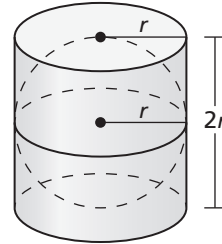
V = _____   Volume of cylinder

When half of the sphere (a *hemisphere*) is filled with sand and poured into the cylinder, it takes three hemispheres to fill the cylinder. Use this information to write a formula for the volume V of a sphere in terms of the radius r

V = _____   Volume of a sphere

Communicate Your Answer

3. How can you find the surface area and the volume of a sphere?

4. Use the results of Explorations 1 and 2 to find the surface area and the volume of a sphere with a radius of (a) 3 inches and (b) 2 centimeters.

11.8 Notetaking with Vocabulary

For use after Lesson 11.8

In your own words, write the meaning of each vocabulary term.

chord of a sphere

great circle

Core Concepts

Surface Area of a Sphere

The surface area S of a sphere is

$$S = 4\pi r^2$$

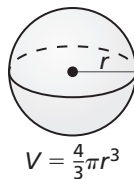where r is the radius of the sphere.

$S = 4\pi r^2$

Notes:

Name _____ Date _____

Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.

$$V = \frac{4}{3}\pi r^3$$
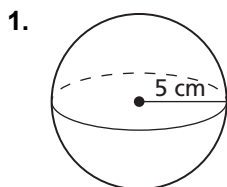
Notes:

Extra Practice

In Exercises 1–4, find the surface area of the solid.

1.

5 cm

2.

9 in.

3.

2 ft

4.

11.2 m

Name_____ Date _____
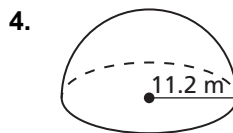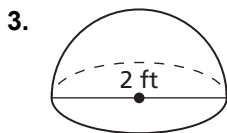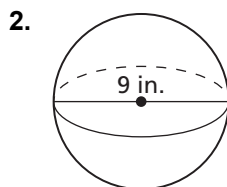
In Exercises 5–8, find the volume of the sphere.
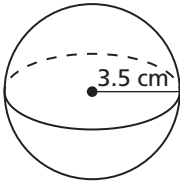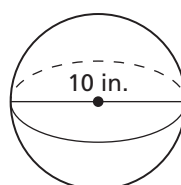
5.

15 in.

6.

13 cm

7.

3.5 cm

8.

10 in.
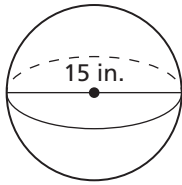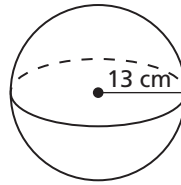
In Exercises 9–11, find the indicated measure.

9. Find the diameter of a sphere with a surface area of 144π square centimeters.

10. Find the volume of a sphere with a surface area of 256π square inches.

11. Find the volume of a sphere with a surface area of 400π square feet.