Find the surface area of the prism.

1. 
   \[ \text{Surface Area} = 2 \times (2 \times 12 + 2 \times 4 + 12 \times 4) \]
   \[ = 2 \times (24 + 8 + 48) \]
   \[ = 2 \times 80 \]
   \[ = 160 \text{ square inches} \]

2. 
   \[ \text{Surface Area} = 1 \times (13 \times 15 + 13 \times 12 + 15 \times 12) \]
   \[ = 1 \times (195 + 156 + 180) \]
   \[ = 1 \times 531 \]
   \[ = 531 \text{ square centimeters} \]

Find the missing dimension.

3. A rectangle has an area of 25 square inches and a length of 10 inches. What is the width of the rectangle?
   \[ \text{Area} = \text{Length} \times \text{Width} \]
   \[ 25 = 10 \times \text{Width} \]
   \[ \text{Width} = \frac{25}{10} \]
   \[ \text{Width} = 2.5 \text{ inches} \]

4. A triangle has an area of 32 square centimeters and a base of 8 centimeters. What is the height of the triangle?
   \[ \text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} \]
   \[ 32 = \frac{1}{2} \times 8 \times \text{Height} \]
   \[ \text{Height} = \frac{32 \times 2}{8} \]
   \[ \text{Height} = 8 \text{ centimeters} \]
11.1 Circumference and Arc Length
For use with Exploration 11.1

Essential Question  How can you find the length of a circular arc?

1 EXPLORATION: Finding the Length of a Circular Arc

Work with a partner. Find the length of each gray circular arc.

a. entire circle

b. one-fourth of a circle

c. one-third of a circle

d. five-eighths of a circle
Work with a partner. The rider is attempting to stop with the front tire of the motorcycle in the painted rectangular box for a skills test. The front tire makes exactly one-half additional revolution before stopping. The diameter of the tire is 25 inches. Is the front tire still in contact with the painted box? Explain.

Communicate Your Answer

3. How can you find the length of a circular arc?

4. A motorcycle tire has a diameter of 24 inches. Approximately how many inches does the motorcycle travel when its front tire makes three-fourths of a revolution?
In your own words, write the meaning of each vocabulary term.

**circumference**

**arc length**

**radian**

---

**Core Concepts**

**Circumference of a Circle**

The circumference $C$ of a circle is $C = \pi d$ or $C = 2\pi r$, where $d$ is the diameter of the circle and $r$ is the radius of the circle.

**Notes:**
11.1 Notetaking with Vocabulary (continued)

Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

\[
\frac{\text{Arc length of } \overarc{AB}}{2\pi r} = \frac{m\overarc{AB}}{360^\circ}, \text{ or }
\]

\[
\text{Arc length of } \overarc{AB} = \frac{m\overarc{AB}}{360^\circ} \cdot 2\pi r
\]

Notes:

Converting Between Degrees and Radians

Degrees to radians

Multiply degree measure by

\[
\frac{2\pi \text{ radians}}{360^\circ}, \text{ or } \frac{\pi \text{ radians}}{180^\circ}
\]

Notes:
11.1 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–5, find the indicated measure.

1. diameter of a circle with a circumference of 10 inches

2. circumference of a circle with a radius of 3 centimeters

3. radius of a circle with a circumference of 8 feet

4. circumference of a circle with a diameter of 2.4 meters

5. arc length of $\overline{AC}$

In Exercises 6 and 7, convert the angle measure.

6. Convert $60^\circ$ to radians.

7. Convert $\frac{5\pi}{6}$ radians to degrees.
11.2 Areas of Circles and Sectors
For use with Exploration 11.2

**Essential Question** How can you find the area of a sector of a circle?

1 **EXPLORATION:** Finding the Area of a Sector of a Circle

Work with a partner. A sector of a circle is the region bounded by two radii of the circle and their intercepted arc. Find the area of each shaded circle or sector of a circle.

- **a.** entire circle
- **b.** one-fourth of a circle
- **c.** seven-eighths of a circle
- **d.** two-thirds of a circle
Work with a partner. A center pivot irrigation system consists of 400 meters of sprinkler equipment that rotates around a central pivot point at a rate of once every 3 days to irrigate a circular region with a diameter of 800 meters. Find the area of the sector that is irrigated by this system in one day.

Communicate Your Answer

3. How can you find the area of a sector of a circle?

4. In Exploration 2, find the area of the sector that is irrigated in 2 hours.
11.2 Notetaking with Vocabulary
For use after Lesson 11.2

In your own words, write the meaning of each vocabulary term.

population density

sector of a circle

Core Concepts

Area of a Circle

The area of a circle is

\[ A = \pi r^2 \]

where \( r \) is the radius of the circle.

Notes:
Area of a Sector

The ratio of the area of a sector of a circle to the area of the whole circle \( \pi r^2 \) is equal to the ratio of the measure of the intercepted arc to 360°.

\[
\frac{\text{Area of sector } APB}{\pi r^2} = \frac{m \overarc{AB}}{360^\circ}, \text{ or }
\]

\[
\text{Area of sector } APB = \frac{m \overarc{AB}}{360^\circ} \cdot \pi r^2
\]

Notes:

Extra Practice

In Exercises 1–2, find the indicated measure.

1. area of \( \odot M \)

2. area of \( \odot R \)
11.2 Notetaking with Vocabulary (continued)

In Exercises 3–8, find the indicated measure.

3. area of a circle with a diameter of 1.8 inches

4. diameter of a circle with an area of 10 square feet

5. radius of a circle with an area of 65 square centimeters

6. area of a circle with a radius of 6.1 yards

7. areas of the sectors formed by $\angle PQR$

8. area of $\odot Y$

9. About 70,000 people live in a region with a 30-mile radius. Find the population density in people per square mile.
11.3 Areas of Polygons
For use with Exploration 11.3

Essential Question  How can you find the area of a regular polygon?

The center of a regular polygon is the center of its circumscribed circle.

The distance from the center to any side of a regular polygon is called the apothem of a regular polygon.

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to construct each regular polygon with side lengths of 4, as shown. Find the apothem and use it to find the area of the polygon. Describe the steps that you used.

a. 

b.
11.3 Areas of Polygons (continued)

1. EXPLORATION: Finding the Area of a Regular Polygon (continued)

2. EXPLORATION: Writing a Formula for Area

   Work with a partner. Generalize the steps you used in Exploration 1 to develop a formula for the area of a regular polygon.

3. Communicate Your Answer

   3. How can you find the area of a regular polygon?

   4. Regular pentagon $ABCDE$ has side lengths of 6 meters and an apothem of approximately 4.13 meters. Find the area of $ABCDE$. 
In your own words, write the meaning of each vocabulary term.

center of a regular polygon

radius of a regular polygon

apothem of a regular polygon

central angle of a regular polygon

**Core Concepts**

**Area of a Rhombus or Kite**

The area of a rhombus or kite with diagonals $d_1$ and $d_2$ is $\frac{1}{2}d_1d_2$. 

![Diagram of a rhombus and kite with diagonals](image)
11.3 Notetaking with Vocabulary (continued)

Area of a Regular Polygon

The area of a regular \( n \)-gon with side length \( s \) is one-half the product of the apothem \( a \) and the perimeter \( P \).

\[
A = \frac{1}{2} aP, \text{ or } A = \frac{1}{2} a \cdot ns
\]

Notes:

Extra Practice

In Exercises 1 and 2, find the area of the kite or rhombus.

1. \[ \text{10 in. to 7 in. to 10 in.} \]

2. \[ \text{5 cm to 2 cm to 3 cm} \]
3. Find the measure of a central angle of a regular polygon with 8 sides.

4. The central angles of a regular polygon are $40^\circ$. How many sides does the polygon have?

5. A regular pentagon has a radius of 4 inches and a side length of 3 inches.
   a. Find the apothem of the pentagon.
   b. Find the area of the pentagon.

6. A regular hexagon has an apothem of 10 units.
   a. Find the radius of the hexagon and the length of one side.
   b. Find the area of the hexagon.
11.4 Three-Dimensional Figures
For use with Exploration 11.4

Essential Question  What is the relationship between the numbers of vertices $V$, edges $E$, and faces $F$ of a polyhedron?

1 EXPLORATION: Analyzing a Property of Polyhedra

Work with a partner. The five Platonic solids are shown below. Each of these solids has congruent regular polygons as faces. Complete the table by listing the numbers of vertices, edges, and faces of each Platonic solid.

<table>
<thead>
<tr>
<th>Solid</th>
<th>Vertices, $V$</th>
<th>Edges, $E$</th>
<th>Faces, $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>tetrahedron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>octahedron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dodecahedron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>icosahedron</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Communicate Your Answer

2. What is the relationship between the numbers of vertices $V$, edges $E$, and faces $F$ of a polyhedron? (*Note:* Swiss mathematician Leonhard Euler (1707–1783) discovered a formula that relates these quantities.)

3. Draw three polyhedra that are different from the Platonic solids given in Exploration 1. Count the number of vertices, edges, and faces of each polyhedron. Then verify that the relationship you found in Question 2 is valid for each polyhedron.
11.4 Notetaking with Vocabulary
For use after Lesson 11.4

In your own words, write the meaning of each vocabulary term.

polyhedron

face

edge

vertex

cross section

solid of revolution

axis of revolution

Notes:
**Core Concepts**

**Types of Solids**

- **Polyhedra**
  - prism
  - pyramid

- **Not Polyhedra**
  - cylinder
  - cone
  - sphere

**Extra Practice**

In Exercises 1 and 2, tell whether the solid is a polyhedron. If it is, name the polyhedron.

1. [Image of a polyhedron]
2. [Image of a not polyhedron]
In Exercises 3–6, describe the cross section formed by the intersection of the plane and the solid.

3.

4.

5.

6.

In Exercises 7 and 8, sketch the solid produced by rotating the figure around the given axis. Then identify and describe the solid.

7.

8.
11.5 Volumes of Prisms and Cylinders
For use with Exploration 11.5

Essential Question  How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

Recall that the volume $V$ of a right prism or a right cylinder is equal to the product of the area of a base $B$ and the height $h$.

$$V = Bh$$

1 EXPLORATION: Finding Volume

Work with a partner. Consider a stack of square papers that is in the form of a right prism.

a. What is the volume of the prism?

b. When you twist the stack of papers, as shown at the right, do you change the volume? Explain your reasoning.

c. Write a carefully worded conjecture that describes the conclusion you reached in part (b).

d. Use your conjecture to find the volume of the twisted stack of papers.
2 EXPLORATION: Finding Volume

Work with a partner. Use the conjecture you wrote in Exploration 1 to find the volume of the cylinder.

a. 

b. 

Communicate Your Answer

3. How can you find the volume of a prism or cylinder that is not a right prism or right cylinder?

4. In Exploration 1, would the conjecture you wrote change if the papers in each stack were not squares? Explain your reasoning.
In your own words, write the meaning of each vocabulary term.

**volume**

Cavalieri’s Principle

**density**

similar solids

**Core Concepts**

**Volume of a Prism**

The volume $V$ of a prism is

$$V = Bh$$

where $B$ is the area of a base and $h$ is the height.

**Notes:**
11.5 Notetaking with Vocabulary (continued)

Volume of a Cylinder

The volume $V$ of a cylinder is

$$V = Bh = \pi r^2 h$$

where $B$ is the area of a base, $h$ is the height, and $r$ is the radius of a base.

Notes:

Similar Solids

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The ratio of the corresponding linear measures of two similar solids is called the **scale factor**. If two similar solids have a scale factor of $k$, then the ratio of their volumes is equal to $k^3$.

Notes:

Extra Practice

In Exercises 1 and 2, find the volume of the prism.

1. 

![Diagram of a prism with dimensions 1.5 cm, 5 cm, and 3 cm]

2. 

![Diagram of a prism with dimensions 4 in. and an area of base 10 in.$^2$]
In Exercises 3 and 4, find the volume of the cylinder.

3. Height = 7 in., radius = 2.5 in. (volume = \(661.5\) cm\(^3\))

4. Height = 11 cm, radius = 2 cm (volume = \(75.36\) in\(^3\))

In Exercises 5 and 6, find the indicated measure.

5. Height of a cylinder with a base radius of 8 inches and a volume of 2010 cubic inches

6. Area of the base of a pentagonal prism with a volume of 50 cubic centimeters and a height of 7.5 centimeters

In Exercises 7 and 8, find the missing dimension of the prism or cylinder.

7. Volume = 661.5 cm\(^3\), height = 3.5 cm, base = 18 cm

8. Volume = 75.36 in\(^3\), radius = 6 in.
11.6 Volumes of Pyramids
For use with Exploration 11.6

Essential Question  How can you find the volume of a pyramid?

1 EXPLORATION: Finding the Volume of a Pyramid

Work with a partner. The pyramid and the prism have the same height and the same square base.

When the pyramid is filled with sand and poured into the prism, it takes three pyramids to fill the prism.

Use this information to write a formula for the volume $V$ of a pyramid.
11.6 Volumes of Pyramids (continued)

2 EXPLORATION: Finding the Volume of a Pyramid

Work with a partner. Use the formula you wrote in Exploration 1 to find the volume of the hexagonal pyramid.

Communicate Your Answer

3. How can you find the volume of a pyramid?

4. In Section 11.7, you will study volumes of cones. How do you think you could use a method similar to the one presented in Exploration 1 to write a formula for the volume of a cone? Explain your reasoning.
In your own words, write the meaning of each vocabulary term.

**pyramid**

**composite solid**

**Core Concepts**

**Volume of a Pyramid**

The volume $V$ of a pyramid is

$$V = \frac{1}{3}Bh$$

where $B$ is the area of a base and $h$ is the height.

**Notes:**
Extra Practice

In Exercises 1–6, find the volume of the pyramid.

1. 
   ![Diagram of a pyramid with dimensions 9 cm, 4 cm, and 6 cm]
   
2. 
   ![Diagram of a pyramid with dimensions 8 in., 6 in., and 6 in.]

3. 
   ![Diagram of a pyramid with dimensions 9 yd, 4 yd, and 4 yd]

4. 
   ![Diagram of a pyramid with dimensions 15 m, 7 m, and 11 m]

5. 
   ![Diagram of a pyramid with dimensions 5 cm and area of base = 27cm²]

6. 
   ![Diagram of a pyramid with dimensions 6 ft, 4 ft, and 1 ft]
In Exercises 7–9, find the indicated measure.

7. A pyramid with a square base has a volume of 128 cubic inches and a height of 6 inches. Find the side length of the square base.

8. A pyramid with a rectangular base has a volume of 6 cubic feet. The length of the rectangular base is 3 feet and the width of the base is 1.5 feet. Find the height of the pyramid.

9. A pyramid with a triangular base has a volume of 18 cubic centimeters. The height of the pyramid is 9 centimeters and the height of the triangular base is 3 centimeters. Find the width of the base.

10. The pyramids are similar. Find the volume of pyramid B.
11.7 Surface Areas and Volumes of Cones
For use with Exploration 11.7

Essential Question  How can you find the surface area and the volume of a cone?

1 EXPLORATION: Finding the Surface Area of a Cone

Work with a partner. Construct a circle with a radius of 3 inches. Mark the circumference of the circle into six equal parts, and label the length of each part. Then cut out one sector of the circle and make a cone.

a. Explain why the base of the cone is a circle. What are the circumference and radius of the base?

b. What is the area of the original circle? What is the area with one sector missing?

c. Describe the surface area of the cone, including the base. Use your description to find the surface area.
11.7 Surface Areas and Volumes of Cones (continued)

2 EXPLORATION: Finding the Volume of a Cone

Work with a partner. The cone and the cylinder have the same height and the same circular base.

When the cone is filled with sand and poured into the cylinder, it takes three cones to fill the cylinder.

Use this information to write a formula for the volume $V$ of a cone.

Communicate Your Answer

3. How can you find the surface area and the volume of a cone?

4. In Exploration 1, cut another sector from the circle and make a cone. Find the radius of the base and the surface area of the cone. Repeat this three times, recording your results in a table. Describe the pattern.

<table>
<thead>
<tr>
<th>Radius of Base</th>
<th>Surface Area of Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
11.7 Notetaking with Vocabulary
For use after Lesson 11.7

In your own words, write the meaning of each vocabulary term.

lateral surface of a cone

Notes:

Core Concepts

Surface Area of a Right Cone

The surface area $S$ of a right cone is

$$S = \pi r^2 + \pi r \ell$$

where $r$ is the radius of the base and $\ell$ is the slant height.

Notes:
Volume of a Cone

The volume $V$ of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$

where $B$ is the area of a base, $h$ is the height, and $r$ is the radius of the base.

Notes:

Extra Practice

In Exercises 1 and 2, find the surface area of the right cone.

1.

2. A right cone has a diameter of 1.8 inches and a height of 3 inches.
11.7 Notetaking with Vocabulary (continued)

In Exercises 3 and 4, find the volume of the cone.

3. [Diagram of a cone with dimensions 4 cm, 5 cm, 3 cm]

4. A right cone has a radius of 5 feet and a slant height of 13 feet.

In Exercises 5–7, find the indicated measure.

5. A right cone has a surface area of 440 square inches and a radius of 7 inches. Find its slant height.

6. A right cone has a volume of 528 cubic meters and a diameter of 12 meters. Find its height.

7. Cone A and cone B are similar. The radius of cone A is 4 cm and the radius of cone B is 10 cm. The volume of cone A is 134 cm$^3$, find the volume of cone B.

8. Find the volume of the composite solid.
11.8 Surface Areas and Volumes of Spheres

For use with Exploration 11.8

Essential Question: How can you find the surface area and the volume of a sphere?

1 EXPLORATION: Finding the Surface Area of a Sphere

Work with a partner. Remove the covering from a baseball or softball.

You will end up with two “figure 8” pieces of material, as shown above. From the amount of material it takes to cover the ball, what would you estimate the surface area $S$ of the ball to be? Express your answer in terms of the radius $r$ of the ball.

$S = \quad$ Surface area of a sphere

Use the Internet or some other resource to confirm that the formula you wrote for the surface area of a sphere is correct.
11.8 Surface Areas and Volumes of Spheres (continued)

2 EXPLORATION: Finding the Volume of a Sphere

Work with a partner. A cylinder is circumscribed about a sphere, as shown. Write a formula for the volume $V$ of the cylinder in terms of the radius $r$.

$$V = \text{Volume of cylinder}$$

When half of the sphere (a hemisphere) is filled with sand and poured into the cylinder, it takes three hemispheres to fill the cylinder. Use this information to write a formula for the volume $V$ of a sphere in terms of the radius $r$.

$$V = \text{Volume of a sphere}$$

Communicate Your Answer

3. How can you find the surface area and the volume of a sphere?

4. Use the results of Explorations 1 and 2 to find the surface area and the volume of a sphere with a radius of (a) 3 inches and (b) 2 centimeters.
11.8 Notetaking with Vocabulary
For use after Lesson 11.8

In your own words, write the meaning of each vocabulary term.

chord of a sphere

great circle

Core Concepts

Surface Area of a Sphere
The surface area $S$ of a sphere is

$$S = 4\pi r^2$$

where $r$ is the radius of the sphere.

Notes:
11.8 Notetaking with Vocabulary (continued)

Volume of a Sphere

The volume \( V \) of a sphere is

\[
V = \frac{4}{3}\pi r^3
\]

where \( r \) is the radius of the sphere.

Notes:

Extra Practice

In Exercises 1–4, find the surface area of the solid.

1. \[ \text{radius} = 5 \text{ cm} \]

2. \[ \text{radius} = 9 \text{ in.} \]

3. \[ \text{radius} = 2 \text{ ft} \]

4. \[ \text{radius} = 11.2 \text{ m} \]
In Exercises 5–8, find the volume of the sphere.

5. \[
\text{Volume} = \frac{4}{3} \pi (15)^3
\]

6. \[
\text{Volume} = \frac{4}{3} \pi (13)^3
\]

7. \[
\text{Volume} = \frac{4}{3} \pi (3.5)^3
\]

8. \[
\text{Volume} = \frac{4}{3} \pi (10)^3
\]

In Exercises 9–11, find the indicated measure.

9. Find the diameter of a sphere with a surface area of \(144\pi\) square centimeters.

10. Find the volume of a sphere with a surface area of \(256\pi\) square inches.

11. Find the volume of a sphere with a surface area of \(400\pi\) square feet.