

**Chapter  
10****Maintaining Mathematical Proficiency**

Find the product.

1.  $(x - 4)(x - 9)$

2.  $(k + 6)(k - 7)$

3.  $(y + 5)(y - 13)$

4.  $(2r + 3)(3r + 1)$

5.  $(4m - 5)(2 - 3m)$

6.  $(7w - 1)(6w + 5)$

Solve the equation by completing the square. Round your answer to the nearest hundredth, if necessary.

7.  $x^2 + 6x = 10$

8.  $p^2 - 14p = 5$

9.  $z^2 + 16z + 7 = 0$

10.  $z^2 + 5z - 2 = 0$

11.  $x^2 + 2x - 5 = 0$

12.  $c^2 - c - 1 = 0$

# 10.1

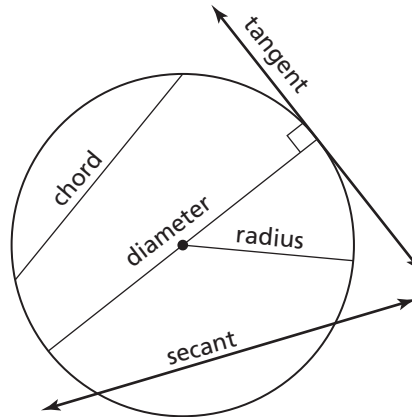
## Lines and Segments That Intersect Circles

For use with Exploration 10.1

**Essential Question** What are the definitions of the lines and segments that intersect a circle?

### 1 EXPLORATION: Lines and Line Segments That Intersect Circles

**Work with a partner.** The drawing at the right shows five lines or segments that intersect a circle. Use the relationships shown to write a definition for each type of line or segment. Then use the Internet or some other resource to verify your definitions.



Chord:

Secant:

Tangent:

Radius:

Diameter:

**10.1** Lines and Segments That Intersect Circles (continued)**2** **EXPLORATION:** Using String to Draw a Circle

**Work with a partner.** Use two pencils, a piece of string, and a piece of paper.

- a. Tie the two ends of the piece of string loosely around the two pencils.
- b. Anchor one pencil on the paper at the center of the circle. Use the other pencil to draw a circle around the anchor point while using slight pressure to keep the string taut. Do not let the string wind around either pencil.
- c. Explain how the distance between the two pencil points as you draw the circle is related to two of the lines or line segments you defined in Exploration 1.

**Communicate Your Answer**

3. What are the definitions of the lines and segments that intersect a circle?
  
  
  
  
  
  
  
  
  
  
4. Of the five types of lines and segments in Exploration 1, which one is a subset of another? Explain.
  
  
  
  
  
  
  
  
  
  
5. Explain how to draw a circle with a diameter of 8 inches.

**10.1**

**Notetaking with Vocabulary**

For use after Lesson 10.1

In your own words, write the meaning of each vocabulary term.

circle

center

radius

chord

diameter

secant

tangent

point of tangency

tangent circles

concentric circles

common tangent

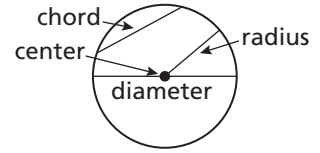
**Notes:**

**10.1** Notetaking with Vocabulary (continued)

**Core Concepts**

**Lines and Segments That Intersect Circles**

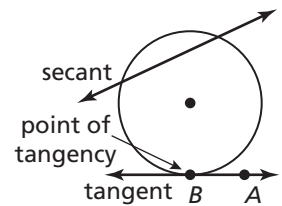
A segment whose endpoints are the center and any point on a circle is a **radius**.



A **chord** is a segment whose endpoints are on a circle. A **diameter** is a chord that contains the center of the circle.

A **secant** is a line that intersects a circle in two points.

A **tangent** is a line in the plane of a circle that intersects the circle in exactly one point, the **point of tangency**. The *tangent ray*  $\overrightarrow{AB}$  and the *tangent segment*  $\overline{AB}$  are also called tangents.



**Notes:**

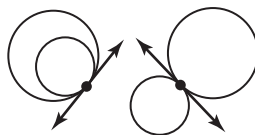
**Coplanar Circles and Common Tangents**

In a plane, two circles can intersect in two points, one point, or no points. Coplanar circles that intersect in one point are called **tangent circles**. Coplanar circles that have a common center are called **concentric circles**.

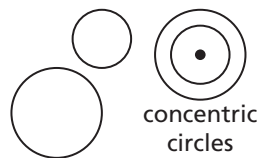
2 points of intersection



1 point of intersection (tangent circles)



no points of intersection



A line or segment that is tangent to two coplanar circles is called a **common tangent**. A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

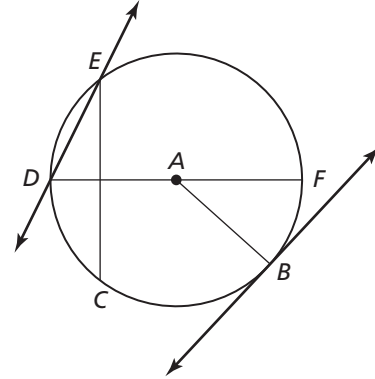
**Notes:**

**10.1** Notetaking with Vocabulary (continued)

**Extra Practice**

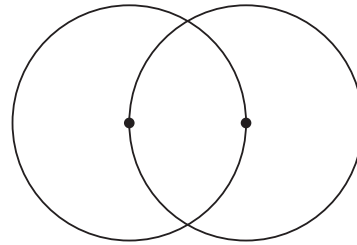
In Exercises 1–6, use the diagram.

- |                     |                              |
|---------------------|------------------------------|
| 1. Name two radii.  | 2. Name a chord.             |
| 3. Name a diameter. | 4. Name a secant.            |
| 5. Name a tangent.  | 6. Name a point of tangency. |



In Exercises 7 and 8, use the diagram.

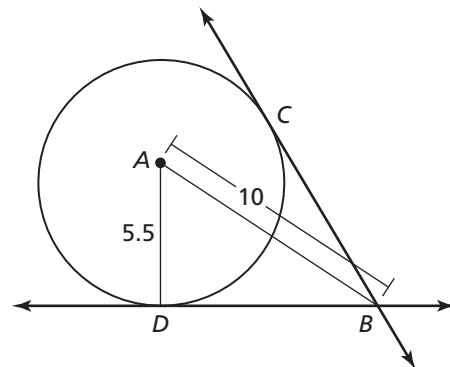
7. Tell how many common tangents the circles have and draw them.



8. Tell whether each common tangent identified in Exercise 7 is internal or external.

In Exercises 9 and 10, point *D* is a point of tangency.

9. Find *BD*.
10. Point *C* is also a point of tangency. If  $BC = 4x + 6$ , find the value of  $x$  to the nearest tenth.



# 10.2

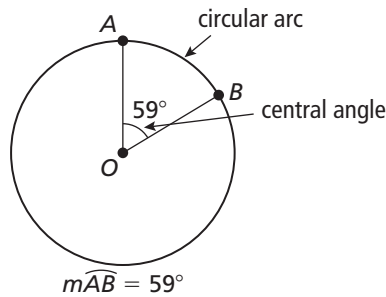
## Finding Arc Measures

For use with Exploration 10.2

### Essential Question How are circular arcs measured?

A **central angle** of a circle is an angle whose vertex is the center of the circle. A *circular arc* is a portion of a circle, as shown below. The measure of a circular arc is the measure of its central angle.

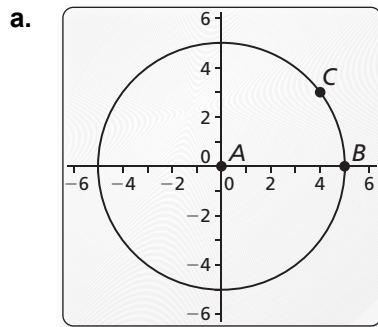
If  $m\angle AOB < 180^\circ$ , then the circular arc is called a **minor arc** and is denoted by  $\widehat{AB}$ .



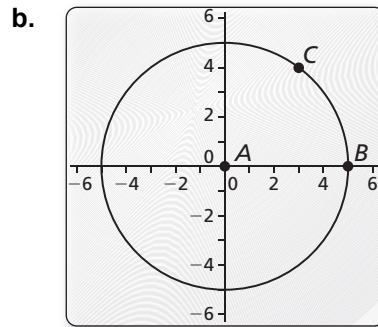
### 1 EXPLORATION: Measuring Circular Arcs

Go to [BigIdeasMath.com](http://BigIdeasMath.com) for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to find the measure of  $\widehat{BC}$ . Verify your answers using trigonometry.



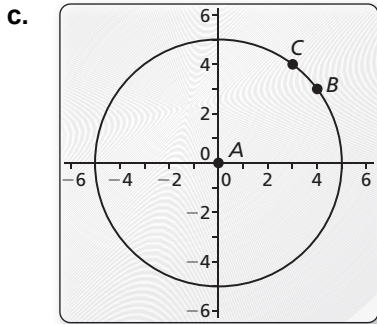
- Points  
 $A(0, 0)$   
 $B(5, 0)$   
 $C(4, 3)$



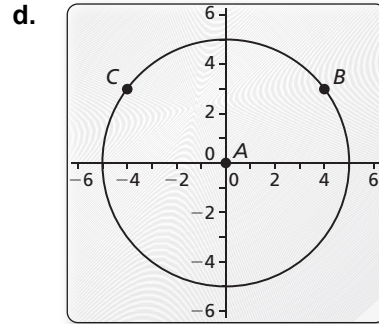
- Points  
 $A(0, 0)$   
 $B(5, 0)$   
 $C(3, 4)$

**10.2 Finding Arc Measures (continued)**

**1 EXPLORATION: Measuring Circular Arcs (continued)**



Points  
 $A(0, 0)$   
 $B(4, 3)$   
 $C(3, 4)$



Points  
 $A(0, 0)$   
 $B(4, 3)$   
 $C(-4, 3)$

**Communicate Your Answer**

2. How are circular arcs measured?
  
  
  
  
  
  
  
  
  
  
3. Use dynamic geometry software to draw a circular arc with the given measure.
 

<ol style="list-style-type: none"> <li>a. <math>30^\circ</math></li> <li>c. <math>60^\circ</math></li> </ol>	<ol style="list-style-type: none"> <li>b. <math>45^\circ</math></li> <li>d. <math>90^\circ</math></li> </ol>
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**10.2****Notetaking with Vocabulary**

For use after Lesson 10.2

In your own words, write the meaning of each vocabulary term.

central angle

minor arc

major arc

semicircle

measure of a minor arc

measure of a major arc

adjacent arcs

congruent circles

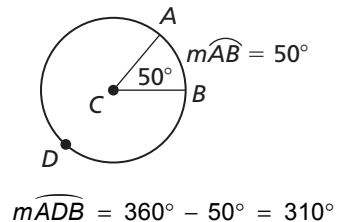
congruent arcs

similar arcs

**Core Concepts****Measuring Arcs**

The **measure of a minor arc** is the measure of its central angle. The expression  $m\widehat{AB}$  is read as “the measure of arc  $AB$ .”

The measure of the entire circle is  $360^\circ$ . The **measure of a major arc** is the difference of  $360^\circ$  and the measure of the related minor arc. The measure of a semicircle is  $180^\circ$ .

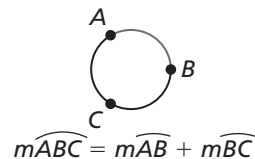
**Notes:**

**10.2** Notetaking with Vocabulary (continued)

**Postulates**

**Postulate 10.1 Arc Addition Postulate**

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

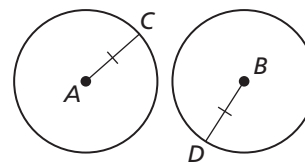


**Notes:**

**Theorems**

**Theorem 10.3 Congruent Circles Theorem**

Two circles are congruent circles if and only if they have the same radius.

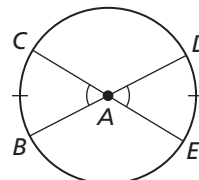


$\odot A \cong \odot B$  if and only if  $\overline{AC} \cong \overline{BD}$ .

**Notes:**

**Theorem 10.4 Congruent Central Angles Theorem**

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.



$\widehat{BC} \cong \widehat{DE}$  if and only if  $\angle BAC \cong \angle DAE$ .

**Notes:**

**Theorem 10.5 Similar Circles Theorem**

All circles are similar.

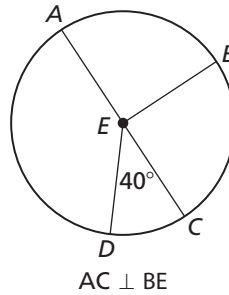
**Notes:**

**10.2** Notetaking with Vocabulary (continued)

**Extra Practice**

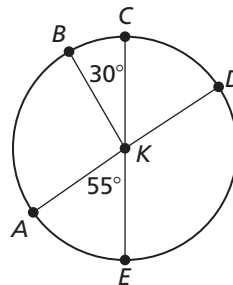
In Exercises 1–8, identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc.

- 1.  $\widehat{AB}$
- 2.  $\widehat{ABC}$
- 3.  $\widehat{ABD}$
- 4.  $\widehat{BC}$
- 5.  $\widehat{BAC}$
- 6.  $\widehat{DAB}$
- 7.  $\widehat{AD}$
- 8.  $\widehat{CD}$

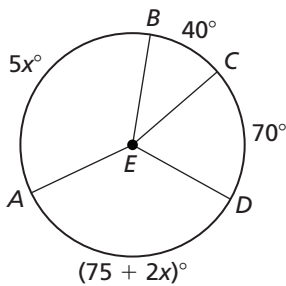


9. In  $\odot E$  above, tell whether  $\widehat{ABC} \cong \widehat{ADC}$ . Explain why or why not.

10. In  $\odot K$ , find the measure of  $\widehat{DE}$ .



11. Find the value of  $x$ . Then find the measure of  $\widehat{AB}$ .



**10.3****Using Chords**

For use with Exploration 10.3

**Essential Question** What are two ways to determine when a chord is a diameter of a circle?

**1 EXPLORATION: Drawing Diameters**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to construct a circle of radius 5 with center at the origin. Draw a diameter that has the given point as an endpoint. Explain how you know that the chord you drew is a diameter.

a.  $(4, 3)$

b.  $(0, 5)$

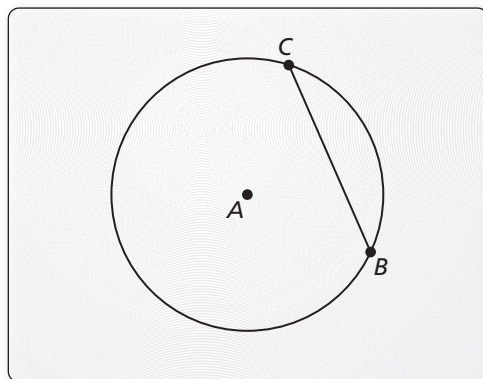
c.  $(-3, 4)$

d.  $(-5, 0)$

**2 EXPLORATION: Writing a Conjecture about Chords**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

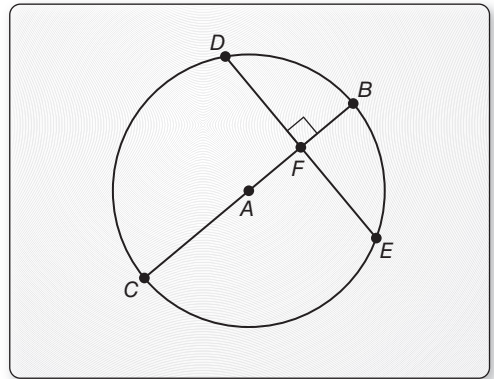
**Work with a partner.** Use dynamic geometry software to construct a chord  $\overline{BC}$  of a circle  $A$ . Construct a chord on the perpendicular bisector of  $\overline{BC}$ . What do you notice? Change the original chord and the circle several times. Are your results always the same? Use your results to write a conjecture.



**10.3** Using Chords (continued)**3** **EXPLORATION:** A Chord Perpendicular to a Diameter

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to construct a diameter  $\overline{BC}$  of a circle  $A$ . Then construct a chord  $\overline{DE}$  perpendicular to  $\overline{BC}$  at point  $F$ . Find the lengths  $DF$  and  $EF$ . What do you notice? Change the chord perpendicular to  $\overline{BC}$  and the circle several times. Do you always get the same results? Write a conjecture about a chord that is perpendicular to a diameter of a circle.

**Communicate Your Answer**

4. What are two ways to determine when a chord is a diameter of a circle?

**10.3****Notetaking with Vocabulary**

For use after Lesson 10.3

In your own words, write the meaning of each vocabulary term.

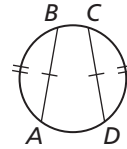
chord

arc

diameter

**Theorems****Theorem 10.6 Congruent Corresponding Chords Theorem**

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

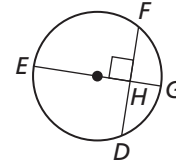
**Notes:**

$$\widehat{AB} \cong \widehat{CD} \text{ if and only if } \overline{AB} \cong \overline{CD}.$$

**10.3** Notetaking with Vocabulary (continued)

**Theorem 10.7 Perpendicular Chord Bisector Theorem**

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

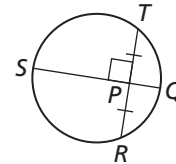


If  $\overline{EG}$  is a diameter and  $\overline{EG} \perp \overline{DF}$ ,  
then  $\overline{HD} \cong \overline{HF}$  and  $\widehat{GD} \cong \widehat{GF}$ .

**Notes:**

**Theorem 10.8 Perpendicular Chord Bisector Converse**

If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.

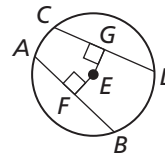


If  $\overline{QS}$  is a perpendicular bisector of  $\overline{TR}$ ,  
then  $\overline{QS}$  is a diameter of the circle.

**Notes:**

**Theorem 10.9 Equidistant Chords Theorem**

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



$\overline{AB} \cong \overline{CD}$  if and only if  $EF = EG$ .

**Notes:**

**10.3** Notetaking with Vocabulary (continued)

**Extra Practice**

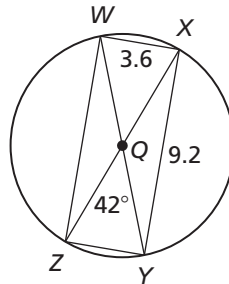
In Exercises 1–4, find the measure of the arc or chord in  $\odot Q$ .

1.  $m\widehat{WX}$

2.  $YZ$

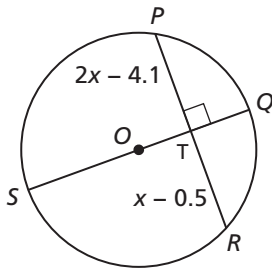
3.  $WZ$

4.  $m\widehat{XY}$

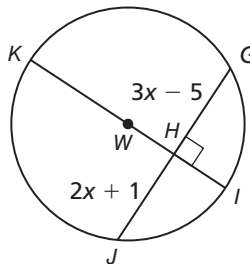


In Exercises 5 and 6, find the value of  $x$ .

5.

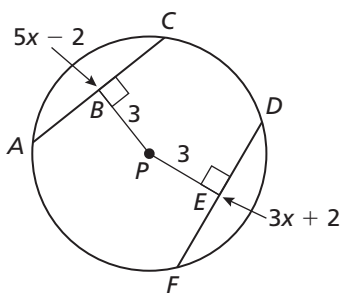


6.

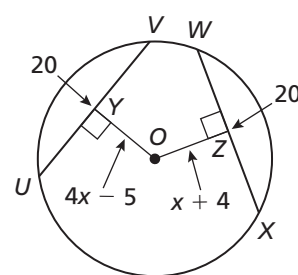


In Exercises 7 and 8, find the radius of the circle.

7.



8.





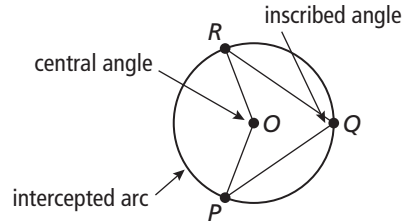
# 10.4

## Inscribed Angles and Polygons

For use with Exploration 10.4

**Essential Question** How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. A polygon is an **inscribed polygon** when all its vertices lie on a circle.



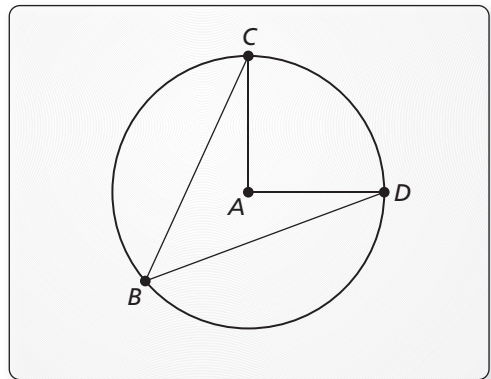
**1 EXPLORATION:** Inscribed Angles and Central Angles

Go to [BigIdeasMath.com](http://BigIdeasMath.com) for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software.

- a. Construct an inscribed angle in a circle. Then construct the corresponding central angle.
- b. Measure both angles. How is the inscribed angle related to its intercepted arc?

**Sample**



- c. Repeat parts (a) and (b) several times. Record your results in the following table. Write a conjecture about how an inscribed angle is related to its intercepted arc.

Measure of Inscribed Angle	Measure of Central Angle	Relationship

**10.4** Inscribed Angles and Polygons (continued)

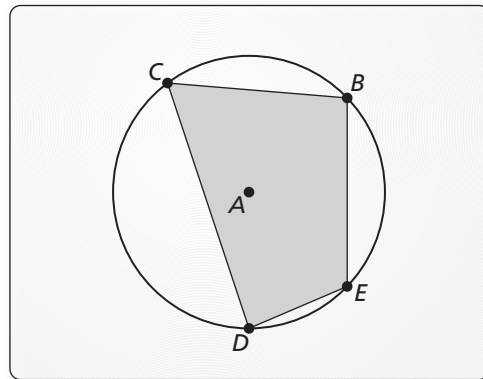
**2** **EXPLORATION:** A Quadrilateral with Inscribed Angles

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

- a. Construct a quadrilateral with each vertex on a circle.
- b. Measure all four angles.  
What relationships do you notice?

Sample



- c. Repeat parts (a) and (b) several times.  
Record your results in the following table.  
Then write a conjecture that summarizes the data.

Angle Measure 1	Angle Measure 2	Angle Measure 3	Angle Measure 4

**Communicate Your Answer**

- 3. How are inscribed angles related to their intercepted arcs? How are the angles of an inscribed quadrilateral related to each other?
- 4. Quadrilateral  $EFGH$  is inscribed in  $\odot C$ , and  $m\angle E = 80^\circ$ . What is  $m\angle G$ ?  
Explain.

**10.4****Notetaking with Vocabulary**

For use after Lesson 10.4

In your own words, write the meaning of each vocabulary term.

inscribed angle

intercepted arc

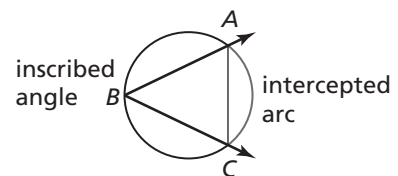
subtend

inscribed polygon

circumscribed circle

**Core Concepts****Inscribed Angle and Intercepted Arc**

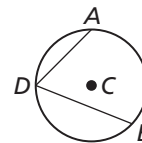
An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called an **intercepted arc**. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to **subtend** the angle.



$\angle B$  intercepts  $\widehat{AC}$ .  
 $\widehat{AC}$  subtends  $\angle B$ .  
 $\overline{AC}$  subtends  $\angle B$ .

**Notes:****Theorems****Theorem 10.10 Measure of an Inscribed Angle Theorem**

The measure of an inscribed angle is one-half the measure of its intercepted arc.



$$m\angle ADB = \frac{1}{2}m\widehat{AB}$$

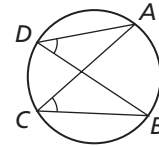
**Notes:**

**10.4** Notetaking with Vocabulary (continued)

**Theorem 10.11 Inscribed Angles of a Circle Theorem**

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

**Notes:**



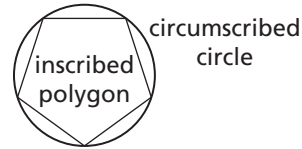
$$\angle ADB \cong \angle ACB$$

**Core Concepts**

**Inscribed Polygon**

A polygon is an **inscribed polygon** when all its vertices lie on a circle. The circle that contains the vertices is a **circumscribed circle**.

**Notes:**

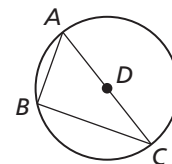


**Theorems**

**Theorem 10.12 Inscribed Right Triangle Theorem**

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

**Notes:**

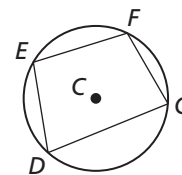


$$m\angle ABC = 90^\circ \text{ if and only if } \overline{AC} \text{ is a diameter of the circle.}$$

**Theorem 10.13 Inscribed Quadrilateral Theorem**

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

**Notes:**



$$D, E, F, \text{ and } G \text{ lie on } \odot C \text{ if and only if } m\angle D + m\angle F = m\angle E + m\angle G = 180^\circ.$$

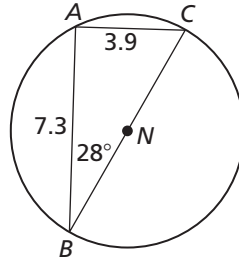
**10.4** Notetaking with Vocabulary (continued)

**Extra Practice**

In Exercises 1–5, use the diagram to find the indicated measure.

1.  $m\angle A$

2.  $m\angle C$

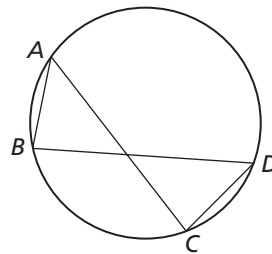


3.  $BC$

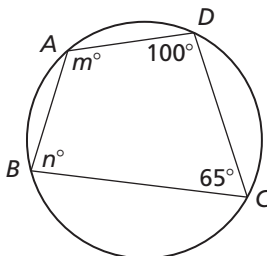
4.  $m\widehat{AC}$

5.  $m\widehat{AB}$

6. Name two pairs of congruent angles.



7. Find the value of each variable.



# 10.5

## Angle Relationships in Circles

For use with Exploration 10.5

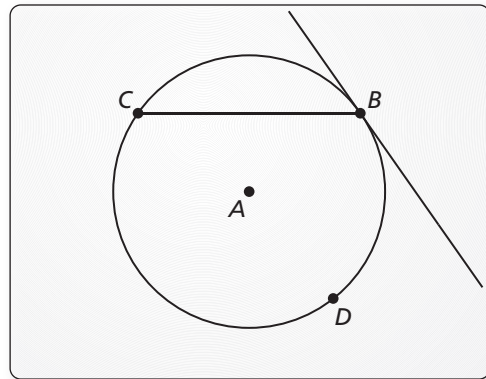
**Essential Question** When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?

### 1 EXPLORATION: Angles Formed by a Chord and Tangent Line

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software. **Sample**

- a. Construct a chord in a circle.  
At one of the endpoints of the chord, construct a tangent line to the circle.
- b. Find the measures of the two angles formed by the chord and the tangent line.
- c. Find the measures of the two circular arcs determined by the chord.



- d. Repeat parts (a)–(c) several times. Record your results in the following table. Then write a conjecture that summarizes the data.

Angle Measure 1	Angle Measure 2	Circular Arc Measure 1	Circular Arc Measure 2

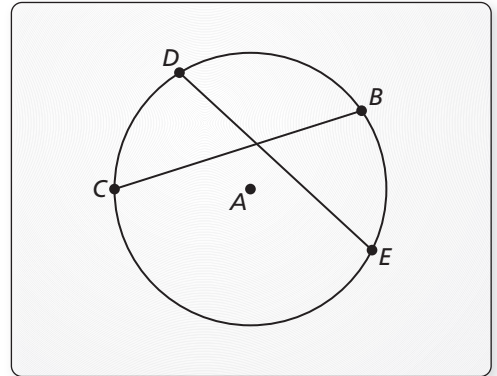
**10.5** Angle Relationships in Circles (continued)

**2** **EXPLORATION:** Angles Formed by Intersecting Chords

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software. **Sample**

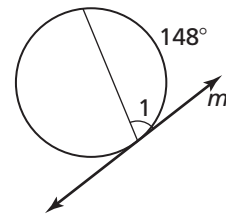
- a. Construct two chords that intersect inside a circle.
- b. Find the measure of one of the angles formed by the intersecting chords.
- c. Find the measures of the arcs intercepted by the angle in part (b) and its vertical angle. What do you observe?
- d. Repeat parts (a)–(c) several times. Record your results in the following table. Then write a conjecture that summarizes the data.



Angle Measure	Arc Measures	Observations

**Communicate Your Answer**

- 3. When a chord intersects a tangent line or another chord, what relationships exist among the angles and arcs formed?
- 4. Line  $m$  is tangent to the circle in the figure at the right. Find the measure of  $\angle 1$ .
- 5. Two chords intersect inside a circle to form a pair of vertical angles with measures of  $55^\circ$ . Find the sum of the measures of the arcs intercepted by the two angles.



# 10.5

## Notetaking with Vocabulary

For use after Lesson 10.5

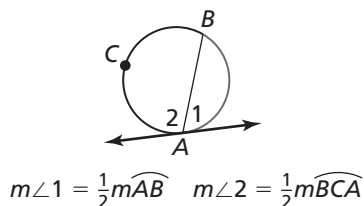
In your own words, write the meaning of each vocabulary term.

circumscribed angle

### Theorems

#### Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.

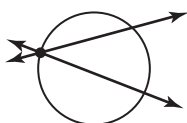


Notes:

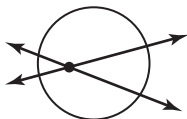
### Core Concepts

#### Intersecting Lines and Circles

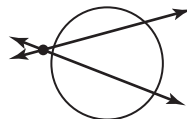
If two nonparallel lines intersect a circle, there are three places where the lines can intersect.



on the circle



inside the circle



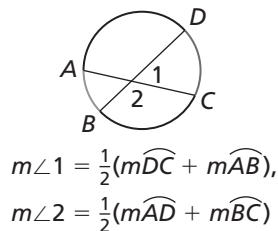
outside the circle

Notes:

### Theorems

#### Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measure of the arcs intercepted by the angle and its vertical angle.



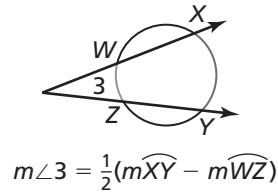
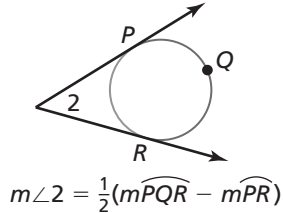
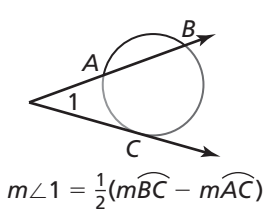
Notes:



**10.5** Notetaking with Vocabulary (continued)

**Theorem 10.16 Angles Outside the Circle Theorem**

If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



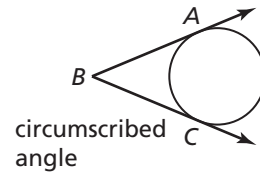
**Notes:**

**Core Concepts**

**Circumscribed Angle**

A **circumscribed angle** is an angle whose sides are tangent to a circle.

**Notes:**

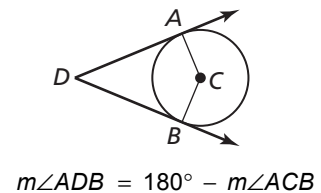


**Theorems**

**Theorem 10.17 Circumscribed Angle Theorem**

The measure of a circumscribed angle is equal to  $180^\circ$  minus the measure of the central angle that intercepts the same arc.

**Notes:**



**10.5** Notetaking with Vocabulary (continued)

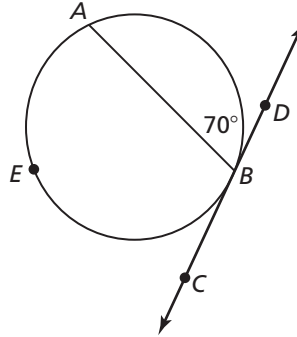
**Extra Practice**

In Exercises 1–3,  $\overline{CD}$  is tangent to the circle. Find the indicated measure.

1.  $m\angle ABC$

2.  $m\widehat{AB}$

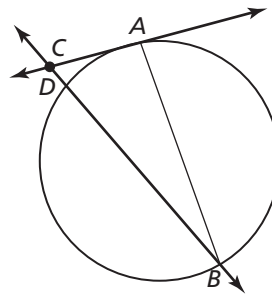
3.  $m\widehat{AEB}$



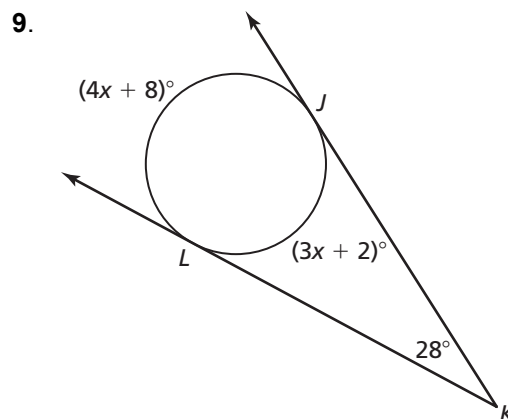
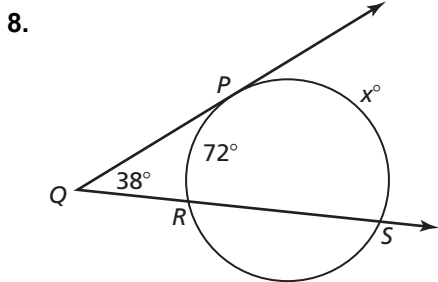
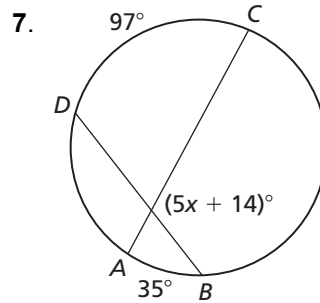
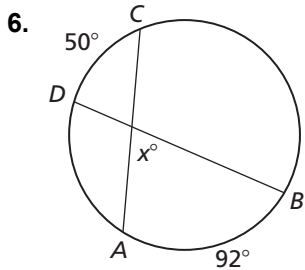
In Exercises 4 and 5,  $m\widehat{ADB} = 220^\circ$  and  $m\angle B = 21^\circ$ . Find the indicated measure.

4.  $m\widehat{AB}$

5.  $m\angle ACB$



In Exercises 6–9, find the value of  $x$ .



# 10.6

## Segment Relationships in Circles

For use with Exploration 10.6

**Essential Question** What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?

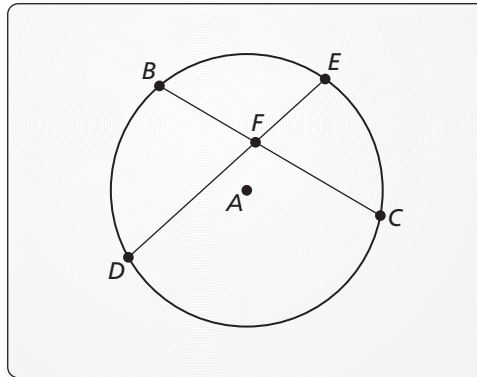
**1 EXPLORATION:** Segments Formed by Two Intersecting Chords

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software.

- a. Construct two chords  $\overline{BC}$  and  $\overline{DE}$  that intersect in the interior of a circle at point  $F$ .

**Sample**



- b. Find the segment lengths  $BF$ ,  $CF$ ,  $DF$ , and  $EF$  and complete the table. What do you observe?

$BF$	$CF$	$BF \cdot CF$
$DF$	$EF$	$DF \cdot EF$

- c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

**10.6 Segment Relationships in Circles (continued)**

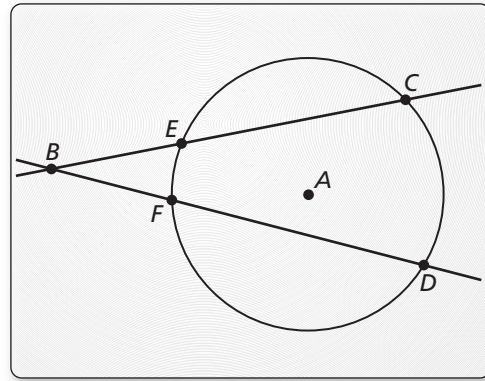
**2 EXPLORATION: Secants Intersecting Outside a Circle**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

- a. Construct two secants  $\overline{BC}$  and  $\overline{BD}$  that intersect at a point  $B$  outside a circle, as shown.
- b. Find the segment lengths  $BE$ ,  $BC$ ,  $BF$ , and  $BD$ , and complete the table. What do you observe?

**Sample**

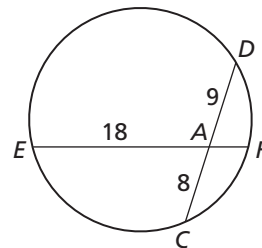


$BE$	$BC$	$BE \cdot BC$
$BF$	$BD$	$BF \cdot BD$

- c. Repeat parts (a) and (b) several times. Write a conjecture about your results.

**Communicate Your Answer**

- 3. What relationships exist among the segments formed by two intersecting chords or among segments of two secants that intersect outside a circle?
- 4. Find the segment length  $AF$  in the figure at the right.



**10.6****Notetaking with Vocabulary**

For use after Lesson 10.6

In your own words, write the meaning of each vocabulary term.

segments of a chord

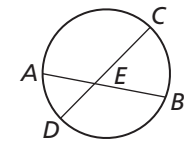
tangent segment

secant segment

external segment

**Theorems****Theorem 10.18 Segments of Chords Theorem**

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$$EA \cdot EB = EC \cdot ED$$

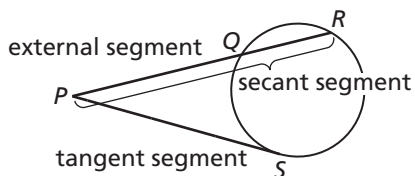
**Notes:**

**10.6** Notetaking with Vocabulary (continued)

**Core Concepts**

**Tangent Segment and Secant Segment**

A **tangent segment** is a segment that is tangent to a circle at an endpoint. A **secant segment** is a segment that contains a chord of a circle and has exactly one endpoint outside the circle. The part of a secant segment that is outside the circle is called an **external segment**.



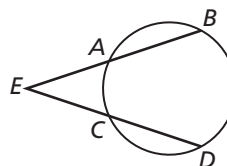
$\overline{PS}$  is a tangent segment.  
 $\overline{PR}$  is a secant segment.  
 $\overline{PQ}$  is the external segment of  $\overline{PR}$ .

**Notes:**

**Theorems**

**Theorem 10.19 Segments of Secants Theorem**

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

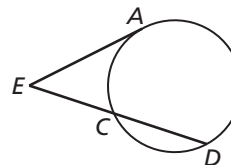


$$EA \cdot EB = EC \cdot ED$$

**Notes:**

**Theorem 10.20 Segments of Secants and Tangents Theorem**

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.



$$EA^2 = EC \cdot ED$$

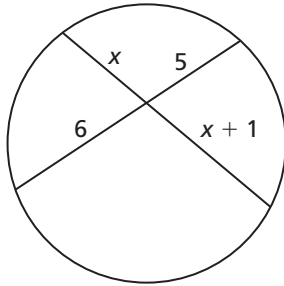
**Notes:**

**10.6** Notetaking with Vocabulary (continued)

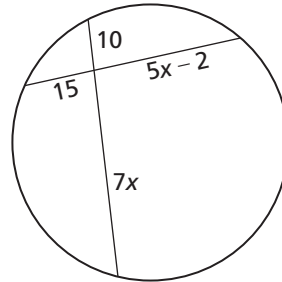
**Extra Practice**

In Exercises 1–4, find the value of  $x$ .

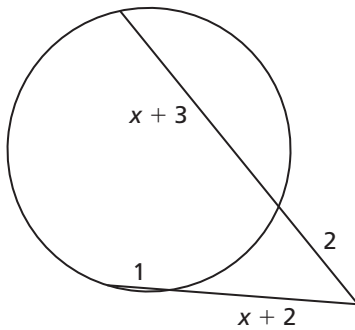
1.



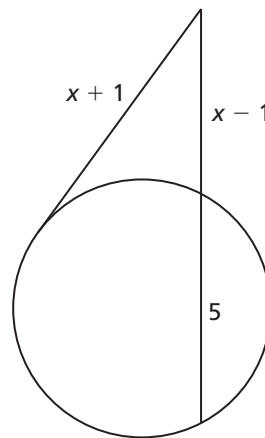
2.



3.



4.



# 10.7

## Circles in the Coordinate Plane

For use with Exploration 10.7

**Essential Question** What is the equation of a circle with center  $(h, k)$  and radius  $r$  in the coordinate plane?

### 1 EXPLORATION: The Equation of a Circle with Center at the Origin

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to construct and determine the equations of circles centered at  $(0, 0)$  in the coordinate plane, as described below.

- a. Complete the first two rows of the table for circles with the given radii. Complete the other rows for circles with radii of your choice.
- b. Write an equation of a circle with center  $(0, 0)$  and radius  $r$ .

Radius	Equation of circle
1	
2	

### 2 EXPLORATION: The Equation of a Circle with Center $(h, k)$

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to construct and determine the equations of circles of radius 2 in the coordinate plane, as described below.

- a. Complete the first two rows of the table for circles with the given centers. Complete the other rows for circles with centers of your choice.
- b. Write an equation of a circle with center  $(h, k)$  and radius 2.
- c. Write an equation of a circle with center  $(h, k)$  and radius  $r$ .

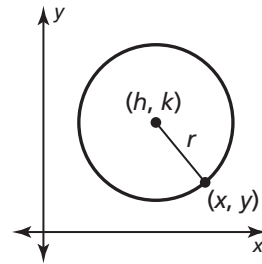
Center	Equation of circle
$(0, 0)$	
$(2, 0)$	



**10.7** Circles in the Coordinate Plane (continued)**3** **EXPLORATION:** Deriving the Standard Equation of a Circle

**Work with a partner.** Consider a circle with radius  $r$  and center  $(h, k)$ .

Write the Distance Formula to represent the distance  $d$  between a point  $(x, y)$  on the circle and the center  $(h, k)$  of the circle. Then square each side of the Distance Formula equation.



How does your result compare with the equation you wrote in part (c) of Exploration 2?

**Communicate Your Answer**

4. What is the equation of a circle with center  $(h, k)$  and radius  $r$  in the coordinate plane?
5. Write an equation of the circle with center  $(4, -1)$  and radius 3.

**10.7****Notetaking with Vocabulary**

For use after Lesson 10.7

In your own words, write the meaning of each vocabulary term.

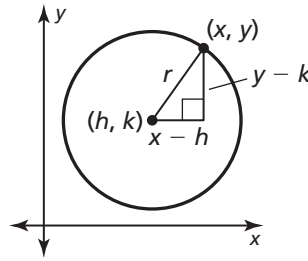
standard equation of a circle

**Core Concepts****Standard Equation of a Circle**

Let  $(x, y)$  represent any point on a circle with center  $(h, k)$  and radius  $r$ . By the Pythagorean Theorem (Theorem 9.1),

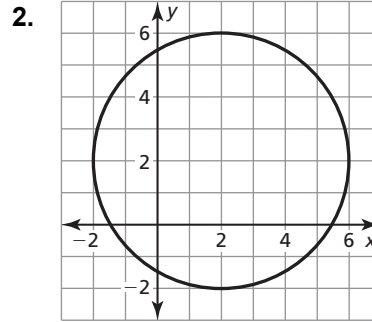
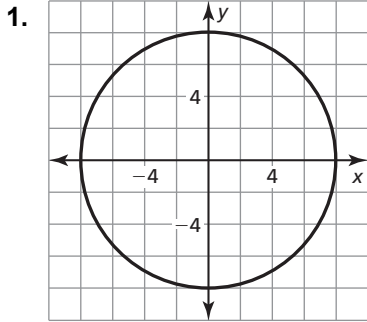
$$(x - h)^2 + (y - k)^2 = r^2.$$

This is the **standard equation of a circle** with center  $(h, k)$  and radius  $r$ .

**Notes:**

**10.7** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–4, write the standard equation of the circle.



3. a circle with center  $(0, 0)$  and radius  $\frac{1}{3}$

4. a circle with center  $(-3, -5)$  and radius 8

In Exercises 5 and 6, use the given information to write the standard equation of the circle.

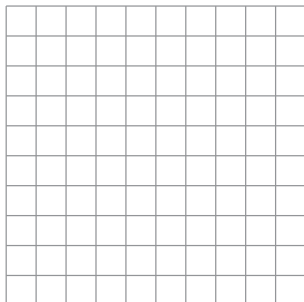
5. The center is  $(0, 0)$ , and a point on the circle is  $(4, -3)$ .

6. The center is  $(4, 5)$ , and a point on the circle is  $(0, 8)$ .

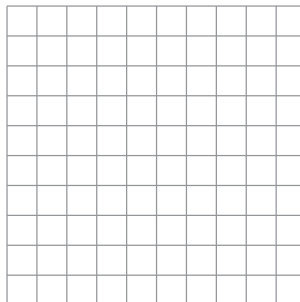
**10.7** Notetaking with Vocabulary (continued)

In Exercises 7–10, find the center and radius of the circle. Then graph the circle.

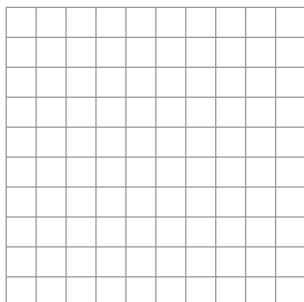
7.  $x^2 + y^2 = 225$



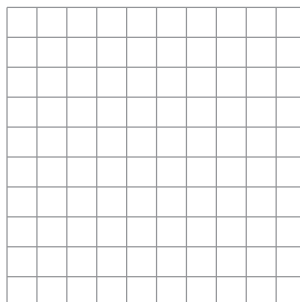
8.  $(x - 3)^2 + (y + 2)^2 = 16$



9.  $x^2 + y^2 + 2x + 2y = 2$



10.  $x^2 + y^2 - 3x + y = \frac{5}{2}$



In Exercises 11 and 12, prove or disprove the statement.

11. The point  $(-4, 4)$  lies on the circle centered at the origin with radius 6.

12. The point  $(-1, 2)$  lies on the circle centered at  $(-4, -1)$  with radius  $3\sqrt{2}$ .