

# 9.7

## Law of Sines and Law of Cosines

For use with Exploration 9.7

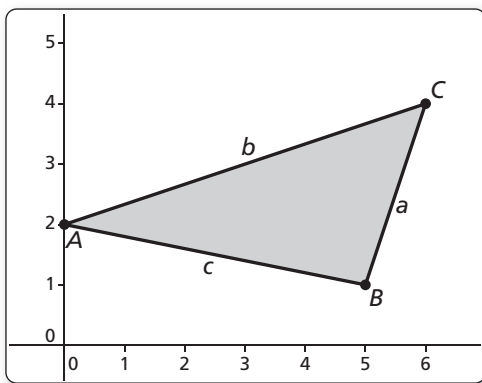
**Essential Question** What are the Law of Sines and the Law of Cosines?

**1 EXPLORATION:** Discovering the Law of Sines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- a. Complete the table for the triangle shown. What can you conclude?



**Sample**  
 Segments  
 $a = 3.16$   
 $b = 6.32$   
 $c = 5.10$   
 Angles  
 $m\angle A = 29.74^\circ$   
 $m\angle B = 97.13^\circ$   
 $m\angle C = 53.13^\circ$

$m\angle A$	$a$	$\frac{\sin A}{a}$	$m\angle B$	$b$	$\frac{\sin B}{b}$	$m\angle C$	$c$	$\frac{\sin C}{c}$

- b. Use dynamic geometry software to draw two other triangles. Complete a table for each triangle. Use your results to write a conjecture about the relationship between the sines of the angles and the lengths of the sides of a triangle.

$m\angle A$	$a$	$\frac{\sin A}{a}$	$m\angle B$	$b$	$\frac{\sin B}{b}$	$m\angle C$	$c$	$\frac{\sin C}{c}$

$m\angle A$	$a$	$\frac{\sin A}{a}$	$m\angle B$	$b$	$\frac{\sin B}{b}$	$m\angle C$	$c$	$\frac{\sin C}{c}$

**9.7 Law of Sines and Law of Cosines (continued)****2 EXPLORATION:** Discovering the Law of Cosines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- a. Complete the table for the triangle in Exploration 1(a). What can you conclude?

$c$	$c^2$	$a$	$a^2$	$b$	$b^2$	$m\angle C$	$a^2 + b^2 - 2ab \cos C$

- b. Use dynamic geometry software to draw two other triangles. Complete a table for each triangle. Use your results to write a conjecture about what you observe in the completed tables.

$c$	$c^2$	$a$	$a^2$	$b$	$b^2$	$m\angle C$	$a^2 + b^2 - 2ab \cos C$

$c$	$c^2$	$a$	$a^2$	$b$	$b^2$	$m\angle C$	$a^2 + b^2 - 2ab \cos C$

**Communicate Your Answer**

3. What are the Law of Sines and the Law of Cosines?
4. When would you use the Law of Sines to solve a triangle? When would you use the Law of Cosines to solve a triangle?

**9.7****Notetaking with Vocabulary**

For use after Lesson 9.7

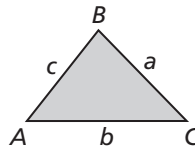
In your own words, write the meaning of each vocabulary term.

Law of Sines

Law of Cosines

**Core Concepts****Area of a Triangle**

The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their included angle. For  $\triangle ABC$  shown, there are three ways to calculate the area.

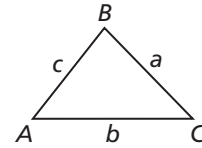


$$\text{Area} = \frac{1}{2}bc \sin A \quad \text{Area} = \frac{1}{2}ac \sin B \quad \text{Area} = \frac{1}{2}ab \sin C$$

**Notes:**

**9.7** Notetaking with Vocabulary (continued)**Theorems****Theorem 9.9 Law of Sines**

The Law of Sines can be written in either of the following forms for  $\triangle ABC$  with sides of length  $a$ ,  $b$ , and  $c$ .

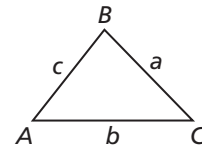


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

**Notes:**

**Theorem 9.10 Law of Cosines**

If  $\triangle ABC$  has sides of length  $a$ ,  $b$ , and  $c$ , as shown, then the following are true.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

**Notes:**

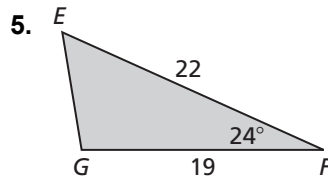
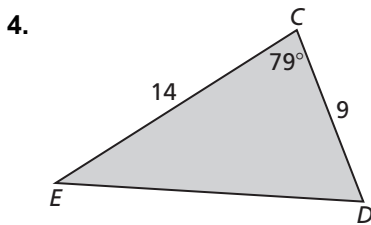
**9.7** Notetaking with Vocabulary (continued)

**Extra Practice**

In Exercises 1–3, use a calculator to find the trigonometric ratio. Round your answer to four decimal places.

1.  $\sin 225^\circ$                       2.  $\cos 111^\circ$                       3.  $\tan 96^\circ$

In Exercises 4 and 5, find the area of the triangle. Round your answer to the nearest tenth.



In Exercises 6-8, solve the triangle. Round decimal answers to the nearest tenth.

