

# 9.5

## The Sine and Cosine Ratios

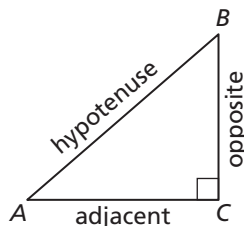
For use with Exploration 9.5

**Essential Question** How is a right triangle used to find the sine and cosine of an acute angle? Is there a unique right triangle that must be used?

Let  $\triangle ABC$  be a right triangle with acute  $\angle A$ . The *sine* of  $\angle A$  and *cosine* of  $\angle A$  (written as  $\sin A$  and  $\cos A$ , respectively) are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

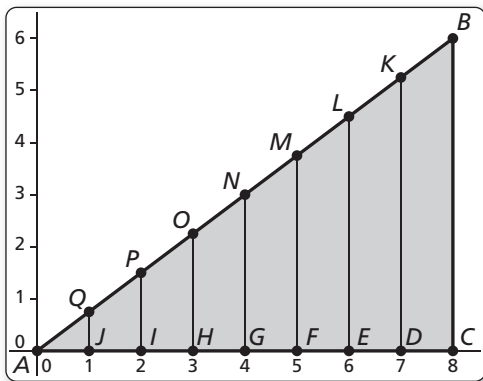


**1 EXPLORATION:** Calculating Sine and Cosine Ratios

Go to [BigIdeasMath.com](http://BigIdeasMath.com) for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

- a. Construct  $\triangle ABC$ , as shown. Construct segments perpendicular to  $\overline{AC}$  to form right triangles that share vertex  $A$  and are similar to  $\triangle ABC$  with vertices, as shown.



**Sample**

Points

$A(0, 0)$

$B(8, 6)$

$C(8, 0)$

Angle

$m\angle BAC = 36.87^\circ$

**9.5** The Sine and Cosine Ratios (continued)

**1** **EXPLORATION:** Calculating Sine and Cosine Ratios (continued)

- b. Calculate each given ratio to complete the table for the decimal values of  $\sin A$  and  $\cos A$  for each right triangle. What can you conclude?

<b>Sine ratio</b>	$\frac{BC}{AB}$	$\frac{KD}{AK}$	$\frac{LE}{AL}$	$\frac{MF}{AM}$	$\frac{NG}{AN}$	$\frac{OH}{AO}$	$\frac{PI}{AP}$	$\frac{QJ}{AQ}$
<b>sin A</b>								
<b>Cosine ratio</b>	$\frac{AC}{AB}$	$\frac{AD}{AK}$	$\frac{AE}{AL}$	$\frac{AF}{AM}$	$\frac{AG}{AN}$	$\frac{AH}{AO}$	$\frac{AI}{AP}$	$\frac{AJ}{AQ}$
<b>cos A</b>								

**Communicate Your Answer**

- How is a right triangle used to find the sine and cosine of an acute angle? Is there a unique right triangle that must be used?
  
- In Exploration 1, what is the relationship between  $\angle A$  and  $\angle B$  in terms of their measures? Find  $\sin B$  and  $\cos B$ . How are these two values related to  $\sin A$  and  $\cos A$ ? Explain why these relationships exist.

**9.5****Notetaking with Vocabulary**

For use after Lesson 9.5

In your own words, write the meaning of each vocabulary term.

sine

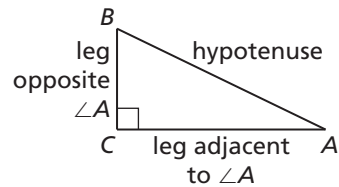
cosine

angle of depression

**Core Concepts****Sine and Cosine Ratios**Let  $\triangle ABC$  be a right triangle with acute  $\angle A$ .The sine of  $\angle A$  and cosine of  $\angle A$  (written as  $\sin A$  and  $\cos A$ ) are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$

**Notes:**

**9.5** Notetaking with Vocabulary (continued)

**Sine and Cosine of Complementary Angles**

The sine of an acute angle is equal to the cosine of its complement. The cosine of an acute angle is equal to the sine of its complement.

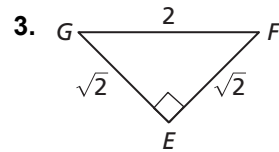
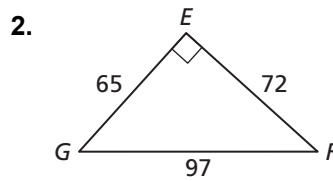
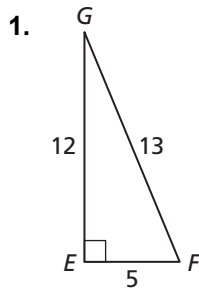
Let  $A$  and  $B$  be complementary angles. Then the following statements are true.

$$\begin{aligned} \sin A &= \cos(90^\circ - A) = \cos B & \sin B &= \cos(90^\circ - B) = \cos A \\ \cos A &= \sin(90^\circ - A) = \sin B & \cos B &= \sin(90^\circ - B) = \sin A \end{aligned}$$

**Notes:**

**Extra Practice**

In Exercises 1–3, find  $\sin F$ ,  $\sin G$ ,  $\cos F$ , and  $\cos G$ . Write each answer as a fraction and as a decimal rounded to four places.



In Exercises 4–6, write the expression in terms of cosine.

4.  $\sin 9^\circ$

5.  $\sin 30^\circ$

6.  $\sin 77^\circ$

**9.5** Notetaking with Vocabulary (continued)

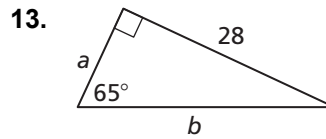
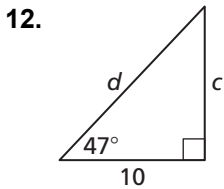
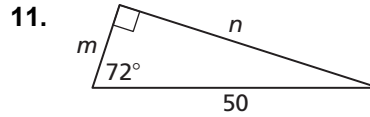
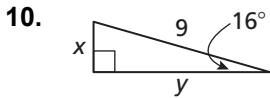
In Exercises 7–9, write the expression in terms of sine.

7.  $\cos 15^\circ$

8.  $\cos 83^\circ$

9.  $\cos 45^\circ$

In Exercises 10–13, find the value of each variable using sine and cosine. Round your answers to the nearest tenth.



14. A camera attached to a kite is filming the damage caused by a brush fire in a closed-off area. The camera is directly above the center of the closed-off area.

- a. A person is standing 100 feet away from the center of the closed-off area. The angle of depression from the camera to the person flying the kite is  $25^\circ$ . How long is the string on the kite?
  
- b. If the string on the kite is 200 feet long, how far away must the person flying the kite stand from the center of the closed-off area, assuming the same angle of depression of  $25^\circ$ , to film the damage?