9.4 The Tangent Ratio

For use with Exploration 9.4

Essential Question  How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?

Let \( \triangle ABC \) be a right triangle with acute \( \angle A \).
The tangent of \( \angle A \) (written as \( \tan A \)) is defined as follows.

\[
\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}
\]

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct \( \triangle ABC \), as shown. Construct segments perpendicular to \( AC \) to form right triangles that share vertex \( A \) and are similar to \( \triangle ABC \) with vertices, as shown.

b. Calculate each given ratio to complete the table for the decimal value of \( \tan A \) for each right triangle. What can you conclude?

<table>
<thead>
<tr>
<th>Ratio</th>
<th>( \frac{BC}{AC} )</th>
<th>( \frac{KD}{AD} )</th>
<th>( \frac{LE}{AE} )</th>
<th>( \frac{MF}{AF} )</th>
<th>( \frac{NG}{AG} )</th>
<th>( \frac{OH}{AH} )</th>
<th>( \frac{PI}{AI} )</th>
<th>( \frac{QJ}{AJ} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tan A )</td>
<td></td>
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</tbody>
</table>
Work with a partner. Use a calculator that has a tangent key to calculate the tangent of $36.87^\circ$. Do you get the same result as in Exploration 1? Explain.

Communicate Your Answer

3. Repeat Exploration 1 for $\triangle ABC$ with vertices $A(0, 0)$, $B(8, 5)$, and $C(8, 0)$.
   Construct the seven perpendicular segments so that not all of them intersect $\overline{AC}$ at integer values of $x$. Discuss your results.

4. How is a right triangle used to find the tangent of an acute angle? Is there a unique right triangle that must be used?
In your own words, write the meaning of each vocabulary term.

trigonometric ratio

tangent

angle of elevation

**Core Concepts**

**Tangent Ratio**

Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The tangent of $\angle A$ (written as $\tan A$) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

**Notes:**
Extra Practice

In Exercises 1–3, find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places.

1. \[ \tan \alpha = \frac{24}{51} \] 
2. \[ \tan \beta = \frac{\sqrt{4}}{7} \] 
3. \[ \tan \gamma = \frac{\sqrt{6}}{2} \]

In Exercises 4–6, find the value of \( x \). Round your answer to the nearest tenth.

4. \[ \tan 10^\circ = \frac{5}{x} \] 
5. \[ \tan 64^\circ = \frac{13}{x} \] 
6. \[ \tan 31^\circ = \frac{x}{24} \]

7. In \( \triangle CDE \), \( \angle E = 90^\circ \) and \( \tan C = \frac{4}{3} \). Find \( \tan D \). Write your answer as a fraction.
9.4 Notetaking with Vocabulary (continued)

8. An environmentalist wants to measure the width of a river to monitor its erosion. From point $A$, she walks downstream 100 feet and measures the angle from this point to point $C$ to be $40^\circ$.

a. How wide is the river? Round to the nearest tenth.

![Diagram](https://via.placeholder.com/150)

b. One year later, the environmentalist returns to measure the same river. From point $A$, she again walks downstream 100 feet and measures the angle from this point to point $C$ to be now $51^\circ$. By how many feet has the width of the river increased?

9. A boy flies a kite at an angle of elevation of $18^\circ$. The kite reaches its maximum height 300 feet away from the boy. What is the maximum height of the kite? Round to the nearest tenth.

10. Find the perimeter of the figure.

![Diagram](https://via.placeholder.com/150)