Name ____ Date __________

Chapter 8 Maintaining Mathematical Proficiency

Tell whether the ratios form a proportion.

1. $\frac{3}{4}, \frac{16}{12}$

2. $\frac{35}{63}, \frac{45}{81}$

3. $\frac{12}{96}, \frac{16}{100}$

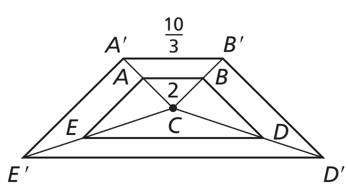
4. $\frac{15}{24}, \frac{75}{100}$

5. $\frac{17}{68}, \frac{32}{128}$

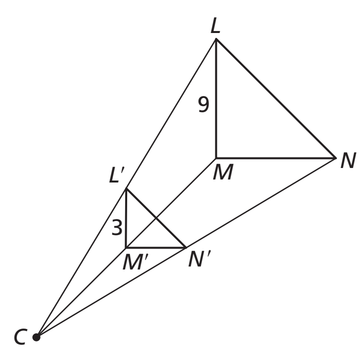
6. $\frac{65}{105}, \frac{156}{252}$

Find the scale factor of the dilation.

7.



8.



Name____ Date___________

8.1 Similar Polygons

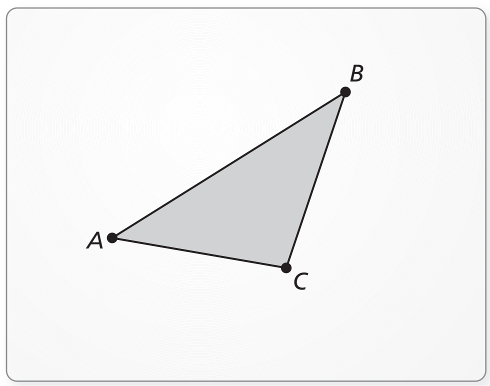
For use with Exploration 8.1

Essential Question How are similar polygons related?

1 EXPLORATION: Comparing Triangles after a Dilation

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



a. Compare the corresponding angles of $\triangle A'B'C'$ and $\triangle ABC$.

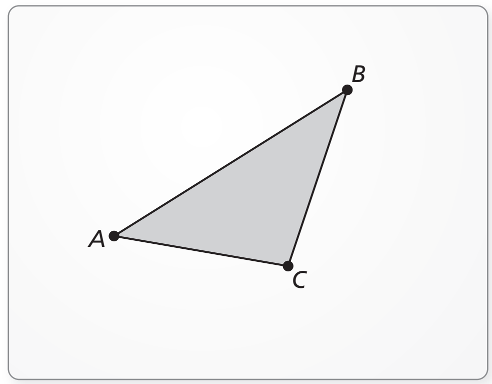
b. Find the ratios of the lengths of the sides of $\triangle A'B'C'$ to the lengths of the corresponding sides of $\triangle ABC$. What do you observe?

c. Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

2 EXPLORATION: Comparing Triangles after a Dilation

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



a. Compare the perimeters of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?

b. Compare the areas of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?

c. Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

Communicate Your Answer

3. How are similar polygons related?

4. A $\triangle RST$ is dilated by a scale factor of 3 to form $\triangle R'S'T'$. The area of $\triangle RST$ is 1 square inch. What is the area of $\triangle R'S'T'$?

Name____ Date__________

8.1 Notetaking with Vocabulary

For use after Lesson 8.1

In your own words, write the meaning of each vocabulary term.

similar figures

similarity transformation

corresponding parts

Core Concepts

Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write "$\triangle ABC$ is similar to $\triangle DEF$" as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor k. So, corresponding side lengths are proportional.

Corresponding angles

$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$

Ratios of corresponding side lengths

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$$

Notes:

Name ____ Date __________

8.1 Notetaking with Vocabulary (continued)

Corresponding Lengths in Similar Polygons

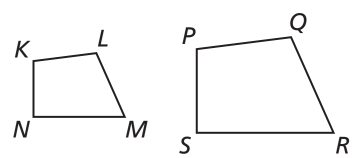
If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

Notes:

Theorems

Theorem 8.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



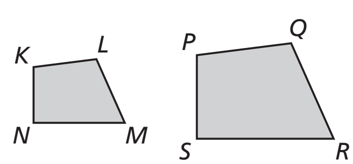
If $KLMN \sim PQRS$, then

$$\frac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \frac{PQ}{KL} = \frac{QR}{LM} = \frac{RS}{MN} = \frac{SP}{NK}.$$

Notes:

Theorem 8.2 Areas of Similar Polygons

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then

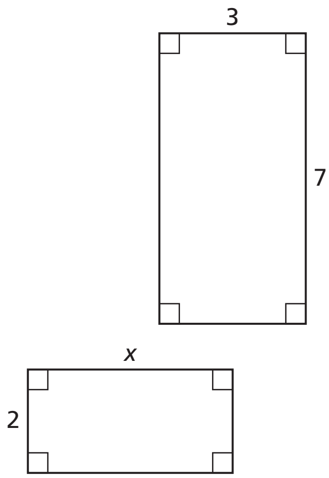
$$\frac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\frac{PQ}{KL}\right)^2 = \left(\frac{QR}{LM}\right)^2 = \left(\frac{RS}{MN}\right)^2 = \left(\frac{SP}{NK}\right)^2.$$

Notes:

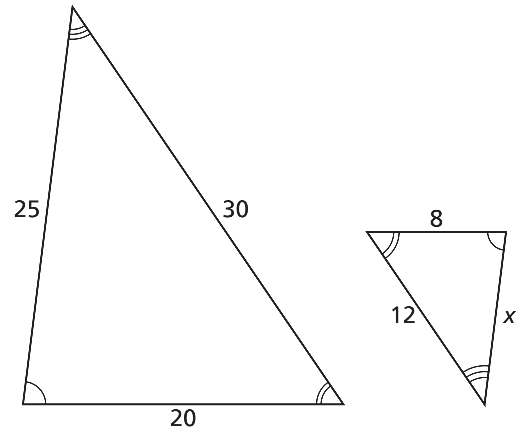
Extra Practice

In Exercises 1 and 2, the polygons are similar. Find the value of *x*.

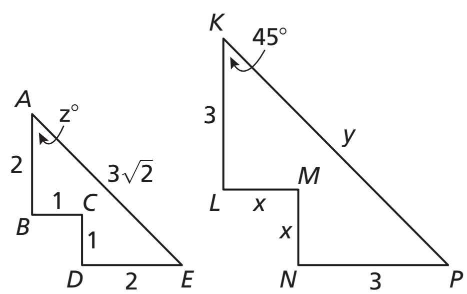
1.



2.



In Exercises 3–8, *ABCDE* ~ *KLMNP*.



3. Find the scale factor from *ABCDE* to *KLMNP*.

4. Find the scale factor from *KLMNP* to *ABCDE*.

5. Find the values of *x*, *y*, and *z*.

6. Find the perimeter of each polygon.

7. Find the ratio of the perimeters of *ABCDE* to *KLMNP*.

8. Find the ratio of the areas of *ABCDE* to *KLMNP*.

Name ____ Date __________

8.2 Proving Triangle Similarity by AA

For use with Exploration 8.2

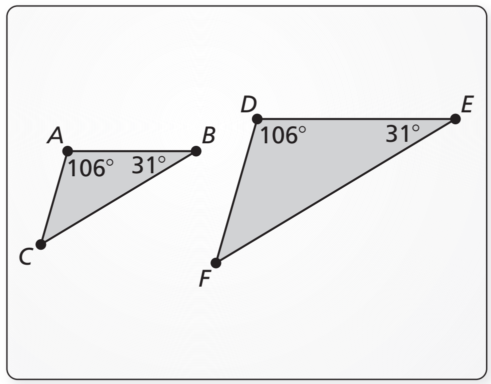
Essential Question What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?

1 EXPLORATION: Comparing Triangles

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$ and $\triangle DEF$ so that $m\angle A = m\angle D = 106°$, $m\angle B = m\angle E = 31°$, and $\triangle DEF$ is not congruent to $\triangle ABC$.



b. Find the third angle measure and the side lengths of each triangle. Record your results in column 1 of the table below.

	1.	2.	3.	4.	5.	6.
$m\angle A$, $m\angle D$	106°	88°	40°			
$m\angle B$, $m\angle E$	31°	42°	65°			
$m\angle C$						
$m\angle F$						
AB						
DE						
BC						
EF						
AC						
DF						

1 EXPLORATION: Comparing Triangles (continued)

c. Are the two triangles similar? Explain.

d. Repeat parts (a)–(c) to complete columns 2 and 3 of the table for the given angle measures.

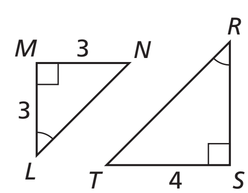
e. Complete each remaining column of the table using your own choice of two pairs of equal corresponding angle measures. Can you construct two triangles in this way that are *not* similar?

f. Make a conjecture about any two triangles with two pairs of congruent corresponding angles.

Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?

3. Find RS in the figure at the right.



Name ____ Date __________

8.2 Notetaking with Vocabulary
For use after Lesson 8.2

In your own words, write the meaning of each vocabulary term.

similar figures

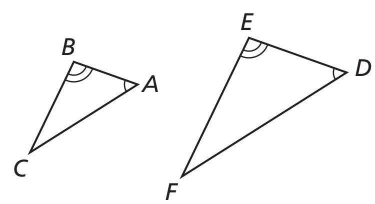
similarity transformation

Theorems

Theorem 8.3 Angle-Angle (AA) Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.



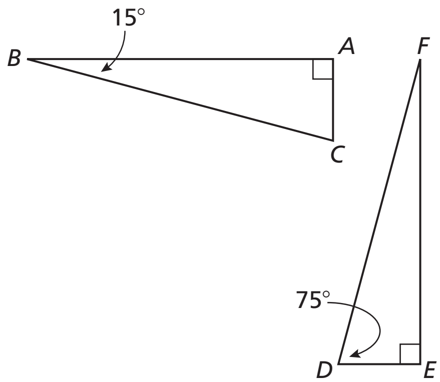
Notes:

Name____ Date__________

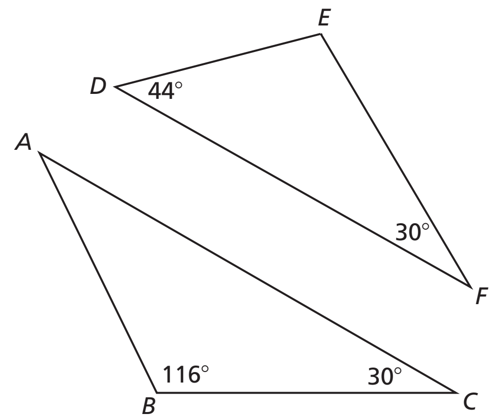
Extra Practice

In Exercises 1 and 2, determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

1.

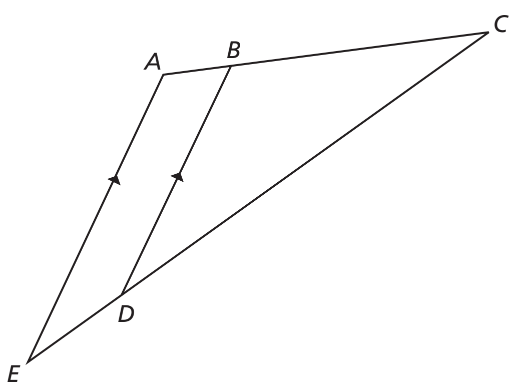


2.

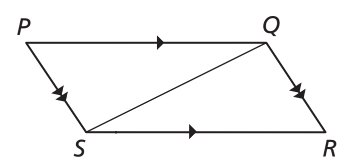


In Exercises 3 and 4, show that the two triangles are similar.

3.



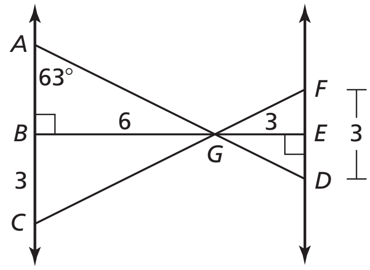
4.



Name ______________________________ Date __________

8.2 Notetaking with Vocabulary (continued)

In Exercises 5–13, use the diagram to complete the statement.



5. $m\angle AGB =$ ______ **6.** $m\angle EGD =$ ______ **7.** $m\angle BCG =$ ______

8. $AG =$ ______ **9.** $AB =$ ______ **10.** $FE =$ ______

11. $ED =$ ______ **12.** $GF =$ ______ **13.** $\triangle AGC \sim$ ______

14. Using the diagram for Exercises 5–13, write similarity statements for each triangle similar to $\triangle EFG$.

15. Determine if it is possible for $\triangle HJK$ and $\triangle PQR$ to be similar. Explain your reasoning.

$$m\angle H = 100°, m\angle K = 46°, m\angle P = 44°, \text{and } m\angle Q = 46°$$

Name____ Date__________

8.3 Proving Triangle Similarity by SSS and SAS

For use with Exploration 8.3

Essential Question What are two ways to use corresponding sides of two triangles to determine that the triangles are similar?

1 EXPLORATION: Deciding Whether Triangles Are Similar

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$ and $\triangle DEF$ with the side lengths given in column 1 of the table below.

	1.	2.	3.	4.	5.	6.	7.
AB	5	5	6	15	9	24	
BC	8	8	8	20	12	18	
AC	10	10	10	10	8	16	
DE	10	15	9	12	12	8	
EF	16	24	12	16	15	6	
DF	20	30	15	8	10	8	
$m\angle A$							
$m\angle B$							
$m\angle C$							
$m\angle D$							
$m\angle E$							
$m\angle F$							

b. Complete column 1 in the table above.

c. Are the triangles similar? Explain your reasoning.

d. Repeat parts (a)–(c) for columns 2–6 in the table.

e. How are the corresponding side lengths related in each pair of triangles that are similar? Is this true for each pair of triangles that are not similar?

1 EXPLORATION: Deciding Whether Triangles Are Similar (continued)

f. Make a conjecture about the similarity of two triangles based on their corresponding side lengths.

g. Use your conjecture to write another set of side lengths of two similar triangles. Use the side lengths to complete column 7 of the table.

2 EXPLORATION: Deciding Whether Triangles Are Similar

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software. Construct any $\triangle ABC$.

a. Find AB, AC, and $m\angle A$. Choose any positive rational number k and construct $\triangle DEF$ so that $DE = k \bullet AB$, $DF = k \bullet AC$, and $m\angle D = m\angle A$.

b. Is $\triangle DEF$ similar to $\triangle ABC$? Explain your reasoning.

c. Repeat parts (a) and (b) several times by changing $\triangle ABC$ and k. Describe your results.

Communicate Your Answer

3. What are two ways to use corresponding sides of two triangles to determine that the triangles are similar?

Name____ Date__________

8.3 Notetaking with Vocabulary
For use after Lesson 8.3

In your own words, write the meaning of each vocabulary term.

similar figures

corresponding parts

slope

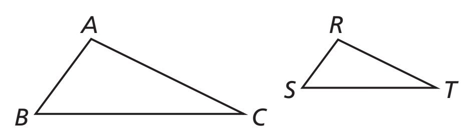
parallel lines

perpendicular lines

Theorems

Theorem 8.4 Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.



If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

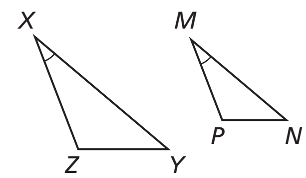
Notes:

Name ______________________________ Date __________

8.3 **Notetaking with Vocabulary (continued)**

Theorem 8.5 Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

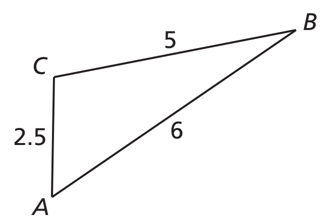


If $\angle X \cong \angle M$ and $\dfrac{ZX}{PM} = \dfrac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

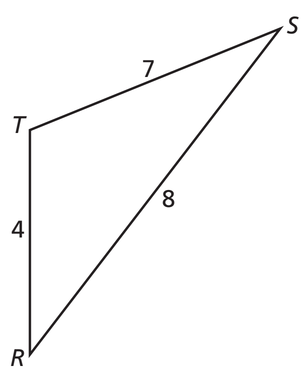
Notes:

Extra Practice

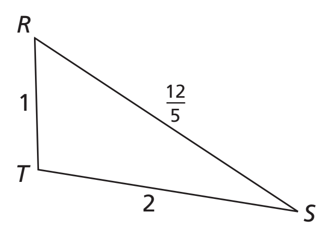
In Exercises 1 and 2, determine whether $\triangle RST$ is similar to $\triangle ABC$.



1.

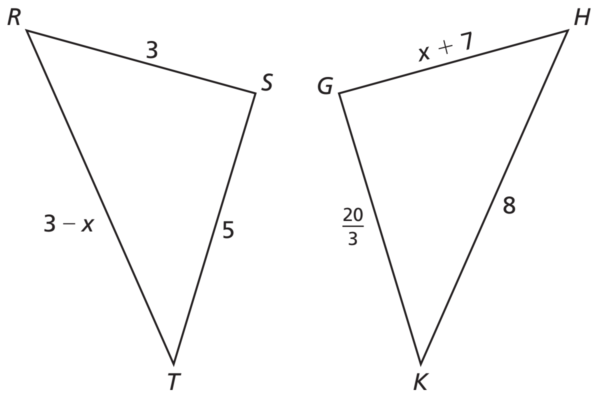


2.



8.3 Notetaking with Vocabulary (continued)

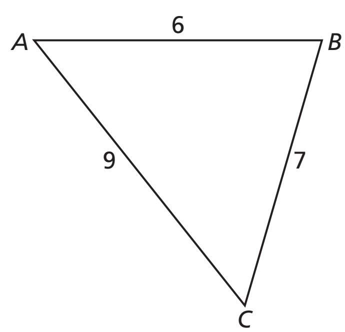
3. Find the value of x that makes $\triangle RST \sim \triangle HGK$.



4. Verify that $\triangle RST \sim \triangle XYZ$. Find the scale factor of $\triangle RST$ to $\triangle XYZ$.

$$\triangle RST : RS = 12, ST = 15, TR = 24$$
$$\triangle XYZ : XY = 28, YZ = 35, ZX = 56$$

In Exercises 5 and 6, use $\triangle ABC$.



5. The shortest side of a triangle similar to $\triangle ABC$ is 15 units long. Find the other side lengths of the triangle.

6. The longest side of a triangle similar to $\triangle ABC$ is 6 units long. Find the other side lengths of the triangle.

Name ____ Date __________

8.4 Proportionality Theorems

For use with Exploration 8.4

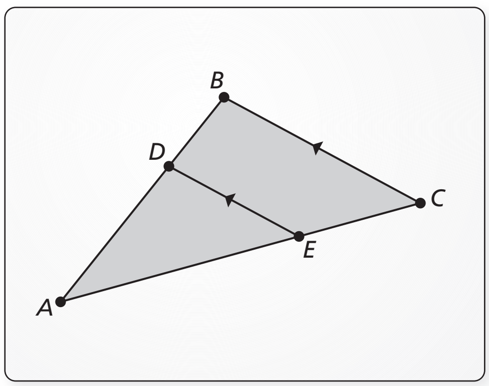
Essential Question What proportionality relationships exist in a triangle intersected by an angle bisector or by a line parallel to one of the sides?

1 EXPLORATION: Discovering a Proportionality Relationship

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$.

a. Construct $\overline{DE}$ parallel to $\overline{BC}$ with endpoints on $\overline{AB}$ and $\overline{AC}$, respectively.



b. Compare the ratios of AD to BD and AE to CE.

c. Move $\overline{DE}$ to other locations parallel to $\overline{BC}$ with endpoints on $\overline{AB}$ and $\overline{AC}$, and repeat part (b).

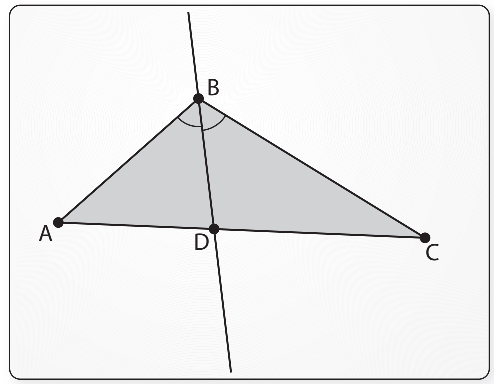
d. Change $\triangle ABC$ and repeat parts (a)–(c) several times. Write a conjecture that summarizes your results.

2 EXPLORATION: Discovering a Proportionality Relationship

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$.

a. Bisect $\angle B$ and plot point D at the intersection of the angle bisector and $\overline{AC}$.



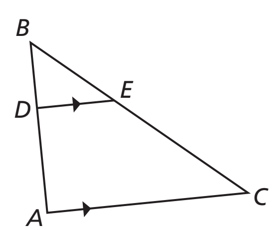
b. Compare the ratios of AD to DC and BA to BC.

c. Change $\triangle ABC$ and repeat parts (a) and (b) several times. Write a conjecture that summarizes your results.

Communicate Your Answer

3. What proportionality relationships exist in a triangle intersected by an angle bisector or by a line parallel to one of the sides?

4. Use the figure at the right to write a proportion.



Name ____ Date __________

8.4 Notetaking with Vocabulary

For use after Lesson 8.4

In your own words, write the meaning of each vocabulary term.

corresponding angles

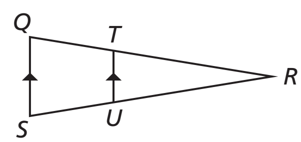
ratio

proportion

Theorems

Theorem 8.6 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

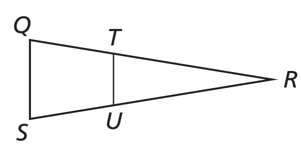


If $\overline{TU} \parallel \overline{QS}$, then $\dfrac{RT}{TQ} = \dfrac{RU}{US}$.

Notes:

Theorem 8.7 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



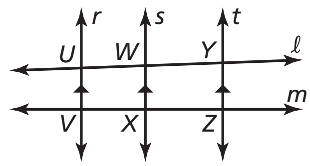
If $\dfrac{RT}{TQ} = \dfrac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Notes:

Name____ Date__________

Theorem 8.8 Three Parallel Lines Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

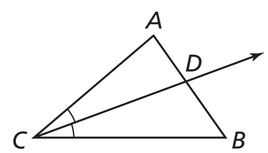


$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Notes:

Theorem 8.9 Triangle Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.



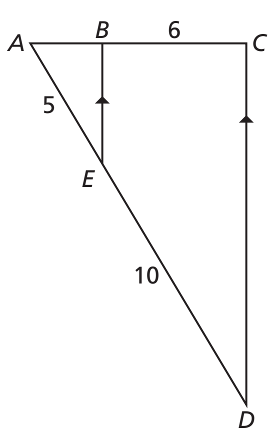
$$\frac{AD}{DB} = \frac{CA}{CB}$$

Notes:

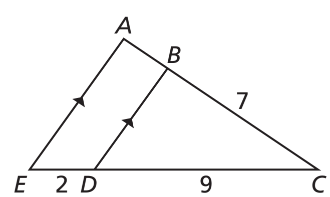
Extra Practice

In Exercises 1 and 2, find the length of $\overline{AB}$.

1.



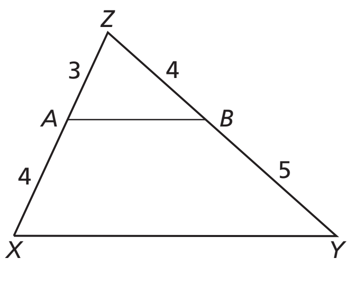
2.



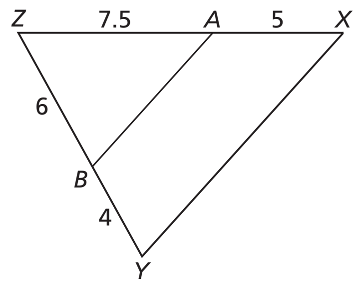
8.4 Notetaking with Vocabulary (continued)

In Exercises 3 and 4, determine whether $\overline{AB} \parallel \overline{XY}$.

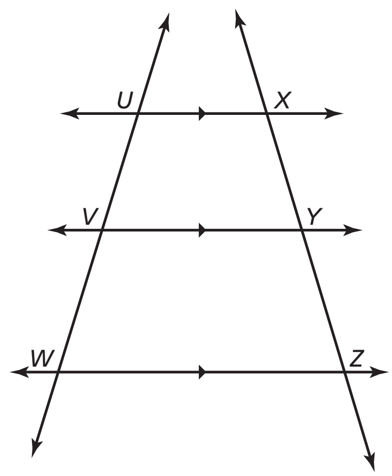
3.



4.



In Exercises 5–7, use the diagram to complete the proportion.



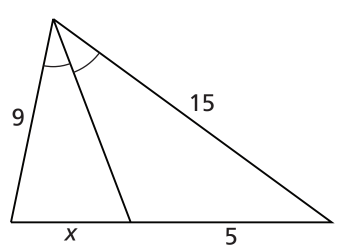
5. $\dfrac{UV}{UW} = \dfrac{XY}{\square}$

6. $\dfrac{XY}{YZ} = \dfrac{\square}{VW}$

7. $\dfrac{\square}{ZY} = \dfrac{WU}{WV}$

In Exercises 8 and 9, find the value of the variable.

8.



9.