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Chapter 7 Maintaining Mathematical Proficiency

Solve the equation by interpreting the expression in parentheses as a single quantity.

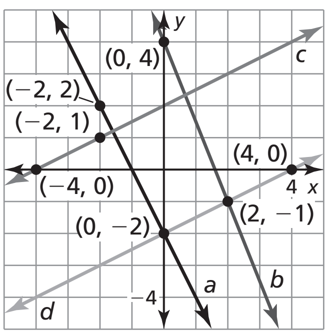
1. $5(10 - x) = 100$

2. $6(x + 8) - 12 = -48$

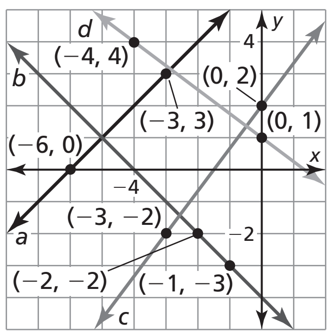
3. $3(2 - x) + 4(2 - x) = 56$

Determine which lines are parallel and which are perpendicular.

4.



5.



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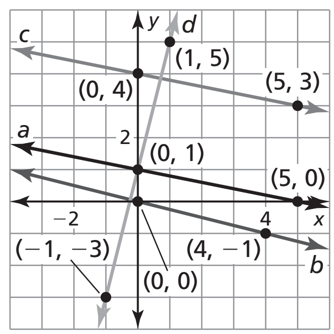

7. Explain why you can rewrite $4(x - 9) + 5(9 - x) = 11$ as $-(x - 9) = 11$? Then solve the equation.

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7.1 Angles of Polygons
For use with Exploration 7.1

Essential Question What is the sum of the measures of the interior angles of a polygon?

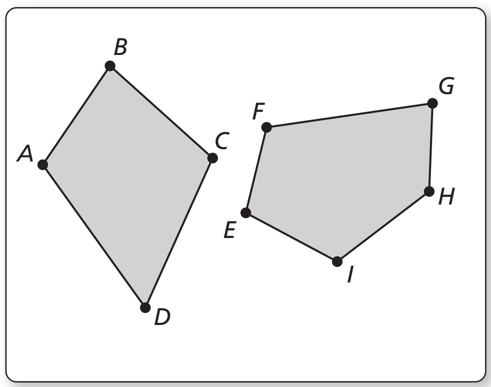
1 EXPLORATION: The Sum of the Angle Measures of a Polygon

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Draw a quadrilateral and a pentagon. Find the sum of the measures of the interior angles of each polygon.

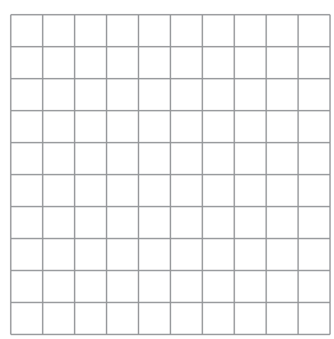
Sample



b. Draw other polygons and find the sums of the measures of their interior angles. Record your results in the table below.

Number of sides, *n*	3	4	5	6	7	8	9
Sum of angle measures, *S*							

c. Plot the data from your table in a coordinate plane.



d. Write a function that fits the data. Explain what the function represents.

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2 EXPLORATION: Measure of One Angle in a Regular Polygon

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use the function you found in Exploration 1 to write a new function that gives the measure of one interior angle in a regular polygon with *n* sides.

b. Use the function in part (a) to find the measure of one interior angle of a regular pentagon. Use dynamic geometry software to check your result by constructing a regular pentagon and finding the measure of one of its interior angles.

c. Copy your table from Exploration 1 and add a row for the measure of one interior angle in a regular polygon with *n* sides. Complete the table. Use dynamic geometry software to check your results.

Number of sides, *n*	3	4	5	6	7	8	9
Sum of angle measures, *S*							
Measure of one interior angle							

Communicate Your Answer

3. What is the sum of the measures of the interior angles of a polygon?

4. Find the measure of one interior angle in a regular dodecagon (a polygon with 12 sides).

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7.1 Notetaking with Vocabulary

For use after Lesson 7.1

In your own words, write the meaning of each vocabulary term.

diagonal

equilateral polygon

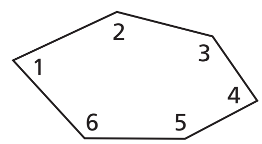
equiangular polygon

regular polygon

Theorems

Theorem 7.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \bullet 180°$.



$n = 6$

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \bullet 180°$$

Notes:

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7.1 Notetaking with Vocabulary (continued)

Corollary 7.1 Corollary to the Polygon Interior Angles Theorem

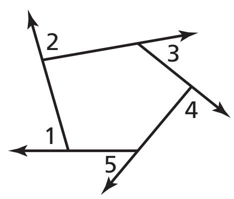
The sum of the measures of the interior angles of a quadrilateral is $360°$.

Notes:

Theorem 7.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is $360°$.

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$$



$n = 5$

Notes:

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Extra Practice

In Exercises 1–3, find the sum of the measures of the interior angles of the indicated convex polygon.

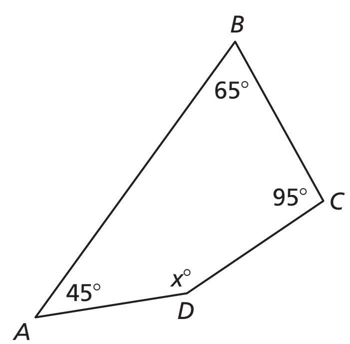
1. octagon **2.** 15-gon **3.** 24-gon

In Exercises 4–6, the sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.

4. 900° **5.** 1620° **6.** 2880°

In Exercises 7–10, find the value of *x*.

7.



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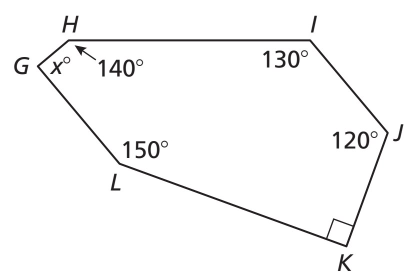

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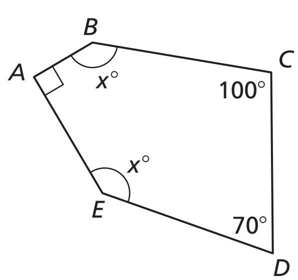
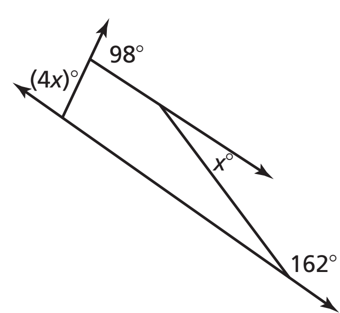


10.



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7.2 Properties of Parallelograms

For use with Exploration 7.2

Essential Question What are the properties of parallelograms?

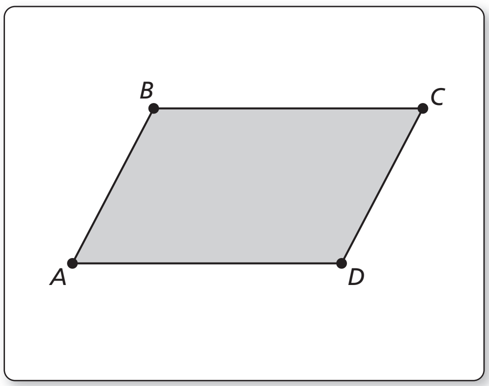
1 EXPLORATION: Discovering Properties of Parallelograms

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct any parallelogram and label it *ABCD*. Explain your process.

Sample



b. Find the angle measures of the parallelogram. What do you observe?

c. Find the side lengths of the parallelogram. What do you observe?

d. Repeat parts (a)–(c) for several other parallelograms. Use your results to write conjectures about the angle measures and side lengths of a parallelogram.

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2 EXPLORATION: Discovering a Property of Parallelograms

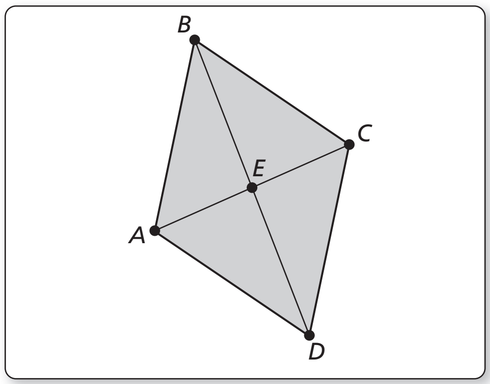
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct any parallelogram and label it *ABCD*.

b. Draw the two diagonals of the parallelogram. Label the point of intersection *E*.

Sample



c. Find the segment lengths *AE, BE, CE,* and *DE*. What do you observe?

d. Repeat parts (a)–(c) for several other parallelograms. Use your results to write a conjecture about the diagonals of a parallelogram.

Communicate Your Answer

3. What are the properties of parallelograms?

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7.2 Notetaking with Vocabulary
For use after Lesson 7.2

In your own words, write the meaning of each vocabulary term.

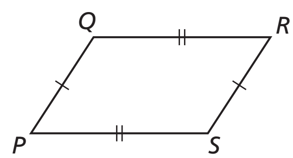
parallelogram

Theorems

Theorem 7.3 Parallelogram Opposite Sides Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If $PQRS$ is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$.

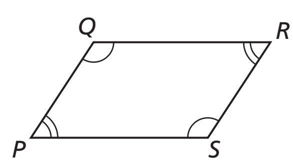


Notes:

Theorem 7.4 Parallelogram Opposite Angles Theorem

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If $PQRS$ is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.



Notes:

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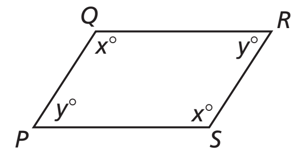
7.2 Notetaking with Vocabulary (continued)

Theorem 7.5 Parallelogram Consecutive Angles Theorem

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

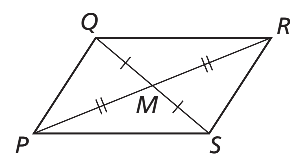


If *PQRS* is a parallelogram, then $x° + y° = 180°$.

Notes:

Theorem 7.6 Parallelogram Diagonals Theorem

If a quadrilateral is a parallelogram, then its diagonals bisect each other.



If *PQRS* is a parallelogram, then $\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$.

Notes:

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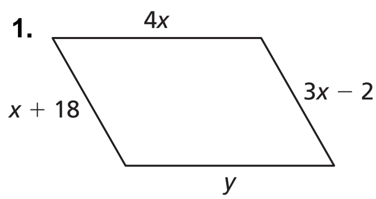
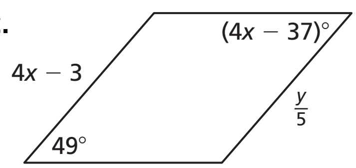
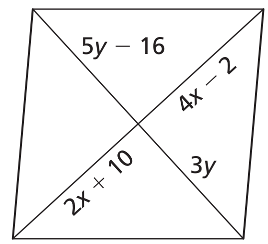
Extra Practice

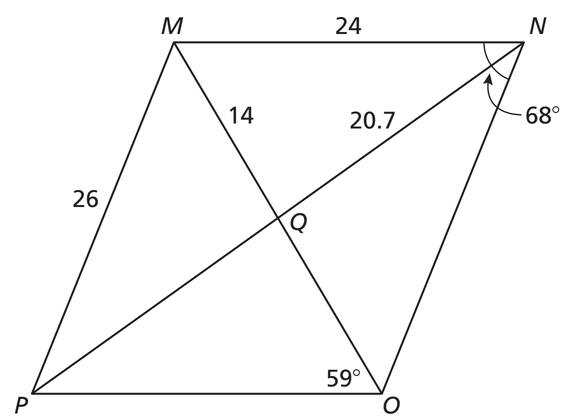
In Exercises 1–3, find the value of each variable in the parallelogram.

1. 

2. 

3. 

In Exercises 4–11, find the indicated measure in ▱*MNOP*. Explain your reasoning.



4. PO

5. OQ

6. NO

7. PQ

8. $m\angle PMN$

9. $m\angle NOP$

10. $m\angle OPM$

11. $m\angle NMO$

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7.3 Proving That a Quadrilateral Is a Parallelogram

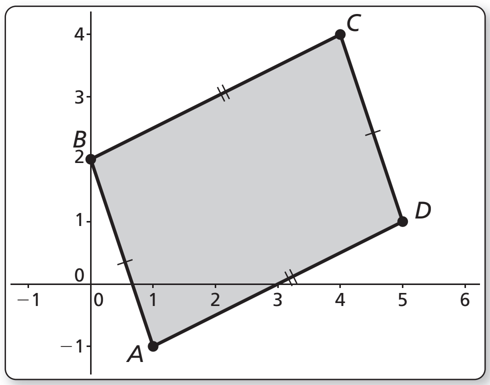
For use with Exploration 7.3

Essential Question How can you prove that a quadrilateral is a parallelogram?

1 EXPLORATION: Proving That a Quadrilateral Is a Parallelogram

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.



Sample

Points

$A(1, -1)$

$B(0, 2)$

$C(4, 4)$

$D(5, 1)$

Segments

$AB = 3.16$

$BC = 4.47$

$CD = 3.16$

$DA = 4.47$

a. Construct any quadrilateral *ABCD* whose opposite sides are congruent.

b. Is the quadrilateral a parallelogram? Justify your answer.

c. Repeat parts (a) and (b) for several other quadrilaterals. Then write a conjecture based on your results.

d. Write the converse of your conjecture. Is the converse true? Explain.

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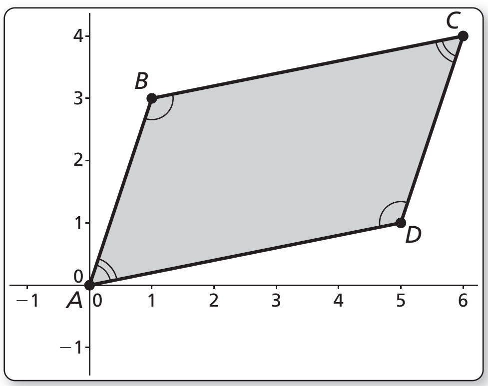
2 EXPLORATION: Proving That a Quadrilateral Is a Parallelogram

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct any quadrilateral *ABCD* whose opposite angles are congruent.

b. Is the quadrilateral a parallelogram? Justify your answer.



Sample

Points	Angles
$A(0, 0)$	$\angle A = 60.26°$
$B(1, 3)$	$\angle B = 119.74°$
$C(6, 4)$	$\angle C = 60.26°$
$D(5, 1)$	$\angle D = 119.74°$

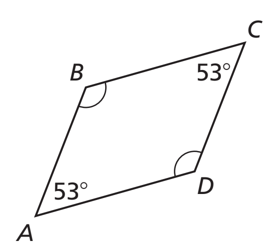
c. Repeat parts (a) and (b) for several other quadrilaterals. Then write a conjecture based on your results.

d. Write the converse of your conjecture. Is the converse true? Explain.

Communicate Your Answer

3. How can you prove that a quadrilateral is a parallelogram?

4. Is the quadrilateral at the right a parallelogram? Explain your reasoning.



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7.3 Notetaking with Vocabulary
For use after Lesson 7.3

In your own words, write the meaning of each vocabulary term.

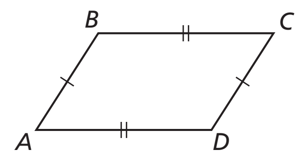
diagonal

parallelogram

Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

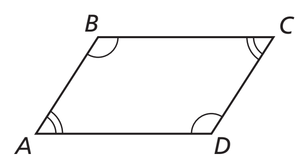


If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

Notes:

Theorem 7.8 Parallelogram Opposite Angles Converse

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

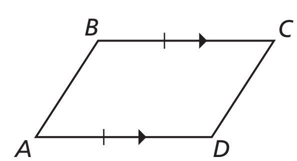


If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Notes:

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Notes:

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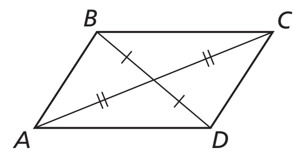
7.3 Notetaking with Vocabulary (continued)

Theorem 7.10 Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

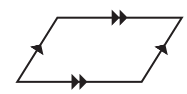
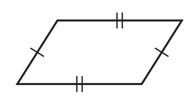
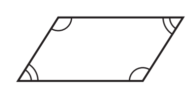
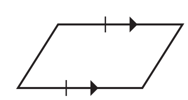
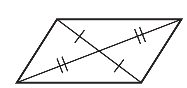


If $\overline{BD}$ and $\overline{AC}$ bisect each other, then *ABCD* is a parallelogram.

Notes:

Core Concepts

Ways to Prove a Quadrilateral Is a Parallelogram

1. Show that both pairs of opposite sides are parallel. *(Definition)*	
2. Show that both pairs of opposite sides are congruent. *(Parallelogram Opposite Sides Converse)*	
3. Show that both pairs of opposite angles are congruent. *(Parallelogram Opposite Angles Converse)*	
4. Show that one pair of opposite sides are congruent and parallel. *(Opposite Sides Parallel and Congruent Theorem)*	
5. Show that the diagonals bisect each other. *(Parallelogram Diagonals Converse)*	

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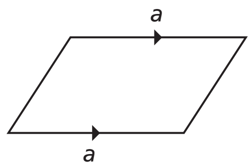
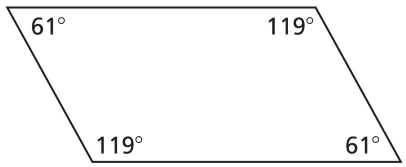
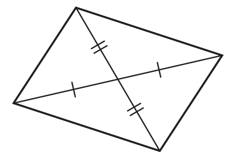
Extra Practice

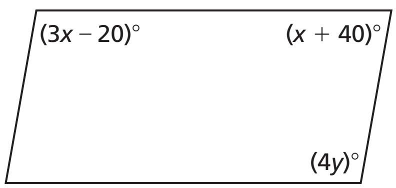
In Exercises 1–3, state which theorem you can use to show that the quadrilateral is a parallelogram.

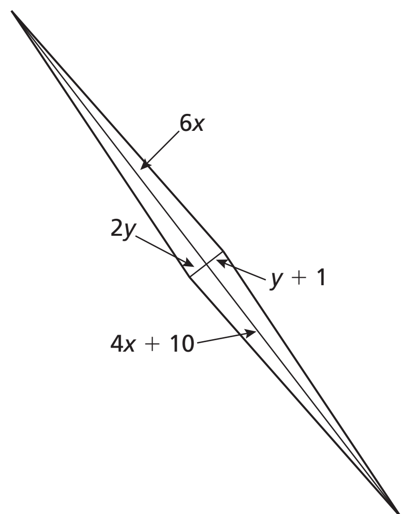
1. 

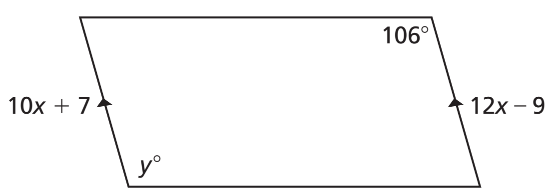
2. 

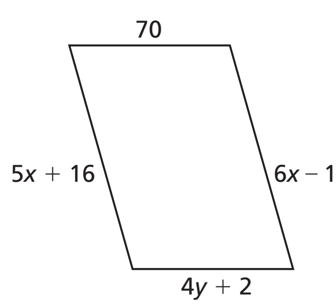
3. 

In Exercises 4–7, find the values of *x* and *y* that make the quadrilateral a parallelogram.

4. 

5. 

6. 

7. 

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7.4 Properties of Special Parallelograms

For use with Exploration 7.4

Essential Question What are the properties of the diagonals of rectangles, rhombuses, and squares?

1 EXPLORATION: Identifying Special Quadrilaterals

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

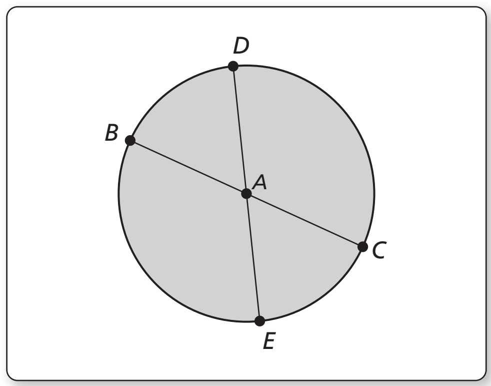
Work with a partner. Use dynamic geometry software.

a. Draw a circle with center *A*.

b. Draw two diameters of the circle. Label the endpoints *B, C, D,* and *E*.

c. Draw quadrilateral *BDCE*.

Sample



d. Is *BDCE* a parallelogram? rectangle? rhombus? square? Explain your reasoning.

e. Repeat parts (a) – (d) for several other circles. Write a conjecture based on your results.

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2 EXPLORATION: Identifying Special Quadrilaterals

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

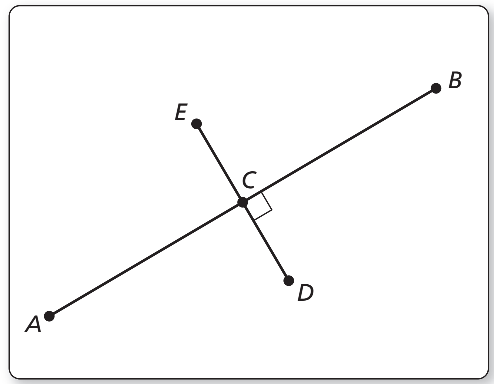
Work with a partner. Use dynamic geometry software.

a. Construct two segments that are perpendicular bisectors of each other. Label the endpoints *A, B, D,* and *E.* Label the intersection *C.*

b. Draw quadrilateral *AEBD.*

c. Is *AEBD* a parallelogram? rectangle? rhombus? square? Explain your reasoning.

Sample

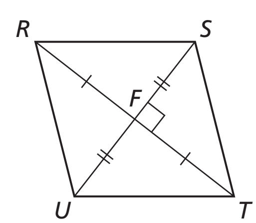


d. Repeat parts (a) – (c) for several other segments. Write a conjecture based on your results.

Communicate Your Answer

3. What are the properties of the diagonals of rectangles, rhombuses, and squares?

4. Is *RSTU* a parallelogram? rectangle? rhombus? square? Explain your reasoning.



5. What type of quadrilateral has congruent diagonals that bisect each other?

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7.4 Notetaking with Vocabulary

For use after Lesson 7.4

In your own words, write the meaning of each vocabulary term.

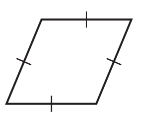
rhombus

rectangle

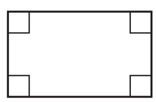
square

Core Concepts

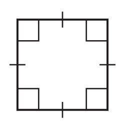
Rhombuses, Rectangles, and Squares



A **rhombus** is a parallelogram with four congruent sides.



A **rectangle** is a parallelogram with four right angles.



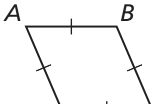
A **square** is a parallelogram with four congruent sides and four right angles.

Notes:

Corollary 7.2 Rhombus Corollary

A quadrilateral is a rhombus if and only if it has four congruent sides.

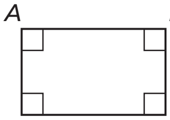
$ABCD$ is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.



Corollary 7.3 Rectangle Corollary

A quadrilateral is a rectangle if and only if it has four right angles.

$ABCD$ is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.



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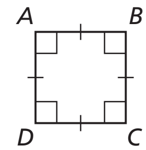
Corollary 7.4 Square Corollary

A quadrilateral is a square if and only if it is a rhombus and a rectangle.

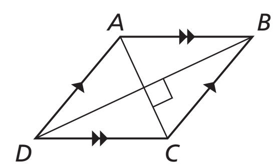


ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Notes:

Theorem 7.11 Rhombus Diagonals Theorem

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

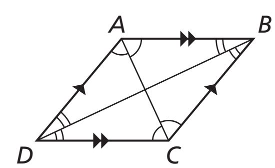


$\square ABCD$ is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Notes:

Theorem 7.12 Rhombus Opposite Angles Theorem

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

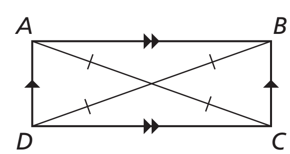


$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$, and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Notes:

Theorem 7.13 Rectangle Diagonals Theorem

A parallelogram is a rectangle if and only if its diagonals are congruent.



$\square ABCD$ is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

Notes:

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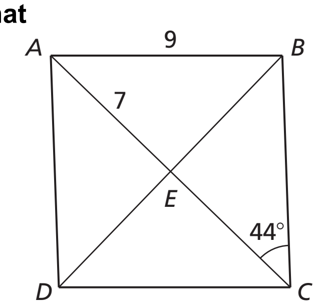
Extra Practice

1. For any rhombus *MNOP,* decide whether the statement $\overline{MO} \cong \overline{NP}$ is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

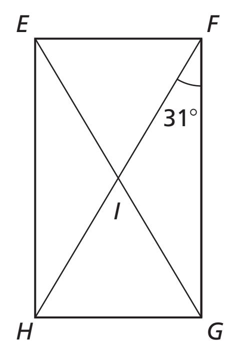
2. For any rectangle *PQRS,* decide whether the statement $\angle PQS \cong \angle RSQ$ is *always* or *sometimes* true. Draw a diagram and explain your reasoning.

In Exercises 3–5, the diagonals of rhombus *ABCD* intersect at *E*. Given that $m\angle BCA = 44°$, $AB = 9$, and $AE = 7$, find the indicated measure.



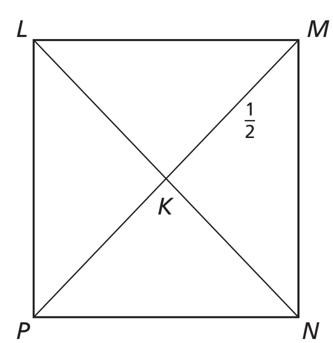
3. BC 4. AC 5. $m\angle ADC$

In Exercises 6–8, the diagonals of rectangle *EFGH* intersect at *I*. Given that $m\angle HFG = 31°$ and $EG = 17$, find the indicated measure.



6. $m\angle FHG$ 7. HF 8. $m\angle EFH$

In Exercises 9–11, the diagonals of square *LMNP* intersect at *K*. Given that $MK = \frac{1}{2}$, find the indicated measure.



9. PK 10. $m\angle PKN$ 11. $m\angle MNK$

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7.5 Properties of Trapezoids and Kites

For use with Exploration 7.5

Essential Question What are some properties of trapezoids and kites?

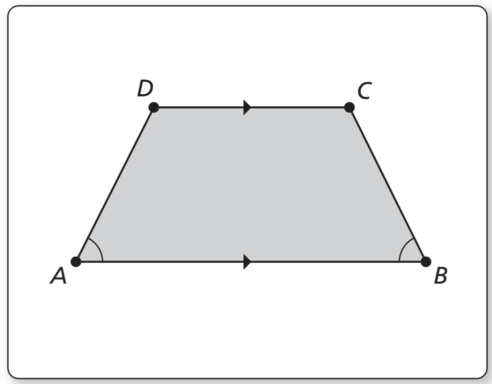
1 EXPLORATION: Making a Conjecture about Trapezoids

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct a trapezoid whose base angles are congruent. Explain your process.

Sample



b. Is the trapezoid isosceles? Justify your answer.

c. Repeat parts (a) and (b) for several other trapezoids. Write a conjecture based on your results.

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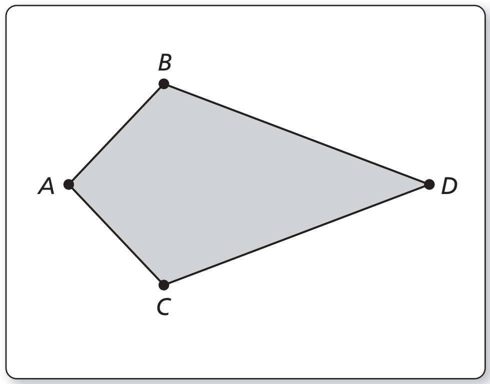
2 EXPLORATION: Discovering a Property of Kites

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct a kite. Explain your process.

Sample



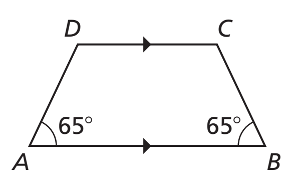
b. Measure the angles of the kite. What do you observe?

c. Repeat parts (a) and (b) for several other kites. Write a conjecture based on your results.

Communicate Your Answer

3. What are some properties of trapezoids and kites?

4. Is the trapezoid at the right isosceles? Explain.



5. A quadrilateral has angle measures of $70°$, $70°$, $110°$, and $110°$. Is the quadrilateral a kite? Explain.

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7.5 Notetaking with Vocabulary
For use after Lesson 7.5

In your own words, write the meaning of each vocabulary term.

trapezoid

bases

base angles

legs

isosceles trapezoid

midsegment of a trapezoid

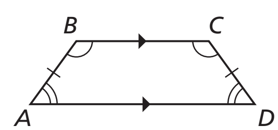
kite

Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

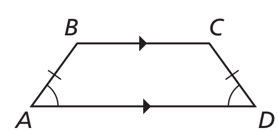
If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.



Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.



If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

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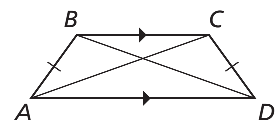
Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

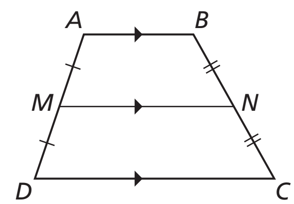
Trapezoid *ABCD* is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



Theorem 7.17 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

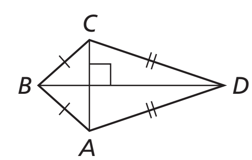
If $\overline{MN}$ is the midsegment of trapezoid *ABCD,* then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.



Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

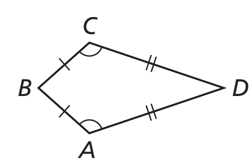
If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.



Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral *ABCD* is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\cong \angle D$.



Notes:

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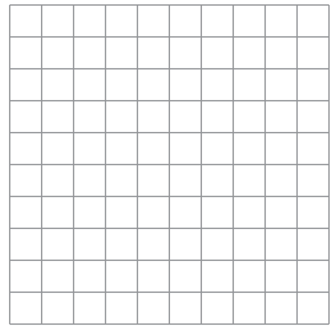
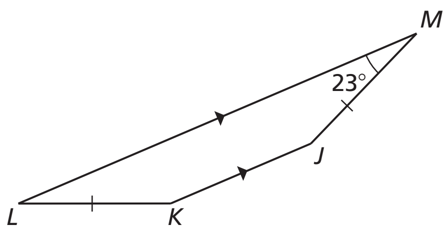
7.5 **Notetaking with Vocabulary (continued)**

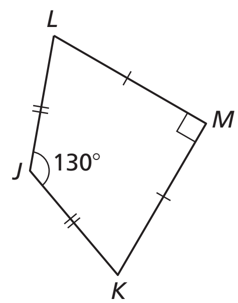
Extra Practice

1. Show that the quadrilateral with vertices at $Q(0, 3)$, $R(0, 6)$, $S(-6, 0)$, and $T(-3, 0)$ is a trapezoid. Decide whether the trapezoid is isosceles. Then find the length of the midsegment of the trapezoid.

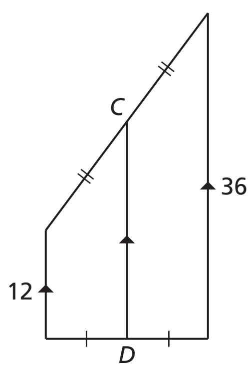


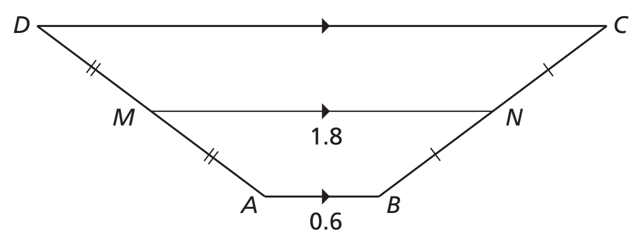
In Exercises 2 and 3, find $m\angle K$ and $m\angle L$.

2. 

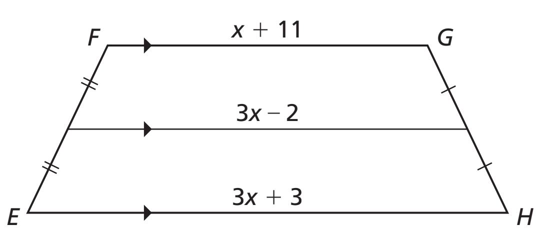
3. 

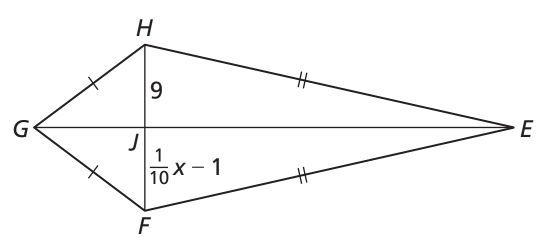
In Exercises 4 and 5, find *CD*.

4. 

5. 

In Exercises 6 and 7, find the value of *x*.

6. 

7. 

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