Essential Question How can you prove that a quadrilateral is a parallelogram?

EXPLORATION: Proving That a Quadrilateral Is a Parallelogram

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

Sample

Points

\[ A(1, -1) \]
\[ B(0, 2) \]
\[ C(4, 4) \]
\[ D(5, 1) \]

Segments

\[ AB = 3.16 \]
\[ BC = 4.47 \]
\[ CD = 3.16 \]
\[ DA = 4.47 \]

a. Construct any quadrilateral \( ABCD \) whose opposite sides are congruent.

b. Is the quadrilateral a parallelogram? Justify your answer.

c. Repeat parts (a) and (b) for several other quadrilaterals. Then write a conjecture based on your results.

d. Write the converse of your conjecture. Is the converse true? Explain.
2 EXPLORATION: Proving That a Quadrilateral Is a Parallelogram

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct any quadrilateral $ABCD$ whose opposite angles are congruent.

b. Is the quadrilateral a parallelogram? Justify your answer.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Points</th>
<th>Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(0, 0)$</td>
<td>$\angle A = 60.26^\circ$</td>
<td></td>
</tr>
<tr>
<td>$B(1, 3)$</td>
<td>$\angle B = 119.74^\circ$</td>
<td></td>
</tr>
<tr>
<td>$C(6, 4)$</td>
<td>$\angle C = 60.26^\circ$</td>
<td></td>
</tr>
<tr>
<td>$D(5, 1)$</td>
<td>$\angle D = 119.74^\circ$</td>
<td></td>
</tr>
</tbody>
</table>

c. Repeat parts (a) and (b) for several other quadrilaterals. Then write a conjecture based on your results.

d. Write the converse of your conjecture. Is the converse true? Explain.

Communicate Your Answer

3. How can you prove that a quadrilateral is a parallelogram?

4. Is the quadrilateral at the right a parallelogram? Explain your reasoning.
7.3 Notetaking with Vocabulary
For use after Lesson 7.3

In your own words, write the meaning of each vocabulary term.

diagonal

parallelogram

Theorems

Theorem 7.7 Parallelogram Opposite Sides Converse
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$, then $ABCD$ is a parallelogram.

Notes:

Theorem 7.8 Parallelogram Opposite Angles Converse
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Notes:

Theorem 7.9 Opposite Sides Parallel and Congruent Theorem
If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Notes:
Theorem 7.10  Parallelogram Diagonals Converse

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If $BD$ and $AC$ bisect each other, then $ABCD$ is a parallelogram.

Notes:

**Core Concepts**

Ways to Prove a Quadrilateral Is a Parallelogram

1. Show that both pairs of opposite sides are parallel. *(Definition)*

2. Show that both pairs of opposite sides are congruent. *(Parallelogram Opposite Sides Converse)*

3. Show that both pairs of opposite angles are congruent. *(Parallelogram Opposite Angles Converse)*

4. Show that one pair of opposite sides are congruent and parallel. *(Opposite Sides Parallel and Congruent Theorem)*

5. Show that the diagonals bisect each other. *(Parallelogram Diagonals Converse)*
7.3 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–3, state which theorem you can use to show that the quadrilateral is a parallelogram.

1. \[ \begin{array}{c}
    a \\
    a
\end{array} \]

2. \[ \begin{array}{c}
    61^\circ \\
    119^\circ \\
    119^\circ \\
    61^\circ
\end{array} \]

3. \[ \begin{array}{c}
    \quad \\
    \quad
\end{array} \]

In Exercises 4–7, find the values of \( x \) and \( y \) that make the quadrilateral a parallelogram.

4. \[ \begin{array}{c}
    (3x - 20)^\circ \\
    (x + 40)^\circ \\
    (4y)^\circ
\end{array} \]

5. \[ \begin{array}{c}
    6x \\
    2y \\
    y + 1 \\
    4x + 10
\end{array} \]

6. \[ \begin{array}{c}
    10x + 7 \\
    12x - 9
\end{array} \]

7. \[ \begin{array}{c}
    70 \\
    5x + 16 \\
    6x - 1 \\
    4y + 2
\end{array} \]