Name ______________________________ Date __________

6.5 Indirect Proof and Inequalities in One Triangle

For use with Exploration 6.5

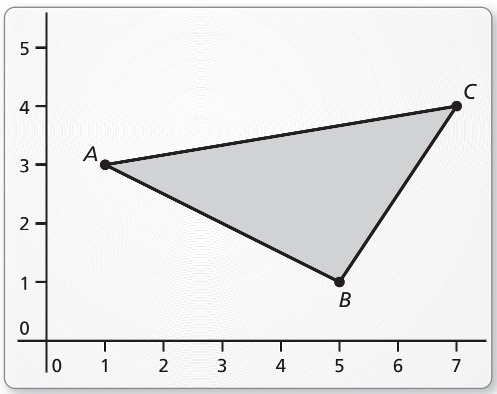
Essential Question How are the sides related to the angles of a triangle? How are any two sides of a triangle related to the third side?

1 EXPLORATION: Comparing Angle Measures and Side Lengths

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software. Draw any scalene $\triangle ABC$.

a. Find the side lengths and angle measures of the triangle.



Sample

Points
$A(1, 3)$
$B(5, 1)$
$C(7, 4)$

Segments
$BC = ?$
$AC = ?$
$AB = ?$

Angles
$m\angle A = ?$
$m\angle B = ?$
$m\angle C = ?$

b. Order the side lengths. Order the angle measures. What do you observe?

c. Drag the vertices of $\triangle ABC$ to form new triangles. Record the side lengths and angle measures in the following table. Write a conjecture about your findings.

BC	*AC*	*AB*	$m\angle A$	$m\angle B$	$m\angle C$

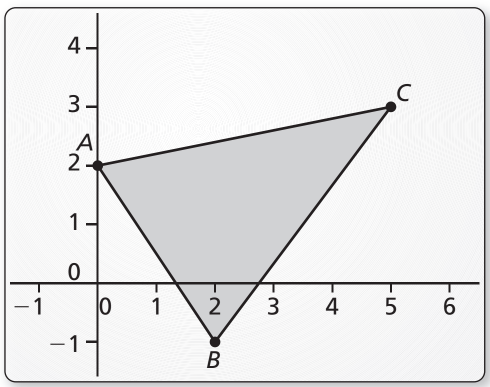
2 EXPLORATION: A Relationship of the Side Lengths of a Triangle

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software. Draw any $\triangle ABC$.

a. Find the side lengths of the triangle.

b. Compare each side length with the sum of the other two side lengths.



Sample
Points
$A(0, 2)$
$B(2, -1)$
$C(5, 3)$
Segments
$BC = ?$
$AC = ?$
$AB = ?$

c. Drag the vertices of $\triangle ABC$ to form new triangles and repeat parts (a) and (b). Organize your results in a table. Write a conjecture about your findings.

BC	*AC*	*AB*	Comparisons

Communicate Your Answer

3. How are the sides related to the angles of a triangle? How are any two sides of a triangle related to the third side?

4. Is it possible for a triangle to have side lengths of 3, 4, and 10? Explain.

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6.5 Notetaking with Vocabulary

For use after Lesson 6.5

In your own words, write the meaning each vocabulary term.

indirect proof

Core Concepts

How to Write an Indirect Proof (Proof by Contradiction)

Step 1 Identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true.

Step 2 Reason logically until you reach a contradiction.

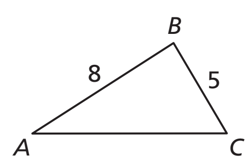
Step 3 Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.

Notes:

Theorems

Theorem 6.9 Triangle Longer Side Theorem

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



$AB > BC$, so $m\angle C > m\angle A$.

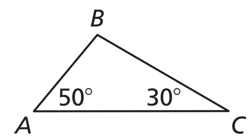
Notes:

Name____ Date__________

6.5 Notetaking with Vocabulary (continued)

Theorem 6.10 Triangle Larger Angle Theorem

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



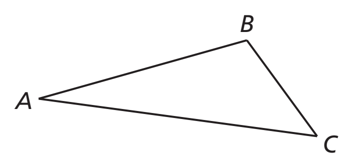
$m\angle A > m\angle C$, so $BC > AB$.

Notes:

Theorem 6.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AB + BC > AC$ $\quad$ $AC + BC > AB$ $\quad$ $AB + AC > BC$



Notes:

Extra Practice

In Exercises 1–3, write the first step in an indirect proof of the statement.

1. Not all the students in a given class can be above average.

2. No number equals another number divided by zero.

3. The square root of 2 is not equal to the quotient of any two integers.

In Exercises 4 and 5, determine which two statements contradict each other. Explain your reasoning.

4. **A** $\triangle LMN$ is equilateral.

B $LM \neq MN$

C $\angle L = \angle M$

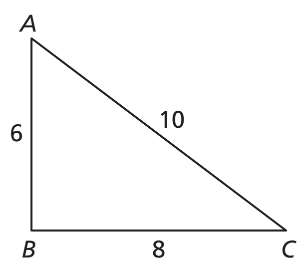
5. **A** $\triangle ABC$ is a right triangle.

B $\angle A$ is acute.

C $\angle C$ is obtuse.

In Exercises 6–8, list the angles of the given triangle from smallest to largest.

6.



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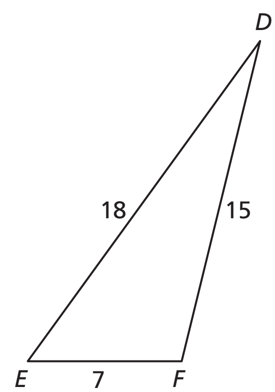
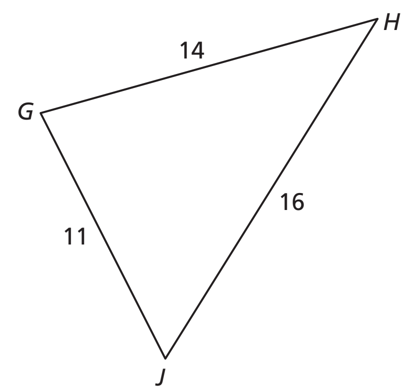


8.



In Exercises 9–12, is it possible to construct a triangle with the given side lengths? If not, explain why not.

9. 3, 12, 17 **10.** 5, 21, 16 **11.** 8, 5, 7 **12.** 10, 3, 11

13. A triangle has two sides with lengths 5 inches and 13 inches. Describe the possible lengths of the third side of the triangle.