Name ____ Date __________

Chapter 5 Maintaining Mathematical Proficiency

Find the coordinates of the midpoint *M* of the segment with the given endpoints. Then find the distance between the two points.

1. $A(3, 1)$ and $B(5, 5)$

2. $F(0, -6)$ and $G(8, -4)$

3. $P(-2, -7)$ and $B(-4, 5)$

4. $S(10, -5)$ and $T(7, -9)$

Solve the equation.

5. $9x - 6 = 7x$

6. $2r + 6 = 5r - 9$

7. $20 - 3n = 2n + 30$

8. $8t - 5 = 6t - 4$

Name____ Date__________

5.1 Angles of Triangles

For use with Exploration 5.1

Essential Question

How are the angle measures of a triangle related?

1 EXPLORATION: Writing a Conjecture

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

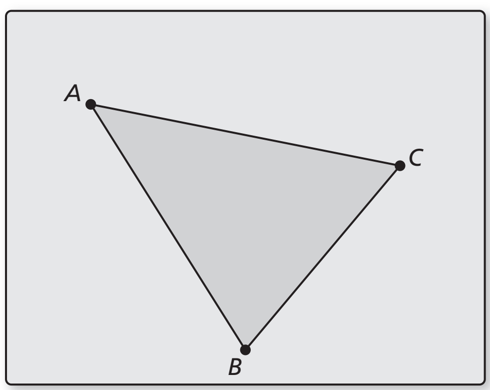
Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. Find the measures of the interior angles of the triangle.

c. Find the sum of the interior angle measures.

d. Repeat parts (a)–(c) with several other triangles. Then write a conjecture about the sum of the measures of the interior angles of a triangle.



Sample
Angles
$m\angle A = 43.67°$
$m\angle B = 81.87°$
$m\angle C = 54.46°$

Name ____ Date __________

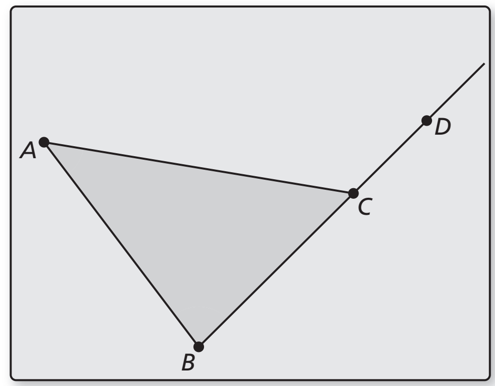
2 EXPLORATION: Writing a Conjecture

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. Draw an exterior angle at any vertex and find its measure.



c. Find the measures of the two nonadjacent interior angles of the triangle.

d. Find the sum of the measures of the two nonadjacent interior angles. Compare this sum to the measure of the exterior angle.

Sample
Angles
$m\angle A = 43.67^\circ$
$m\angle B = 81.87^\circ$
$m\angle ACD = 125.54^\circ$

e. Repeat parts (a)–(d) with several other triangles. Then write a conjecture that compares the measure of an exterior angle with the sum of the measures of the two nonadjacent interior angles.

Communicate Your Answer

3. How are the angle measures of a triangle related?

4. An exterior angle of a triangle measures 32°. What do you know about the measures of the interior angles? Explain your reasoning.

Name____ Date__________

5.1 Notetaking with Vocabulary
For use after Lesson 5.1

In your own words, write the meaning of each vocabulary term.

interior angles

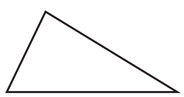
exterior angles

corollary to a theorem

Core Concepts

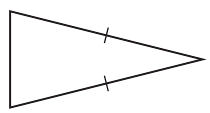
Classifying Triangles by Sides

Scalene Triangle



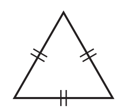
no congruent sides

Isosceles Triangle



at least 2 congruent sides

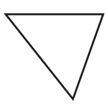
Equilateral Triangle



3 congruent sides

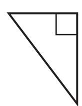
Classifying Triangles by Angles

Acute Triangle



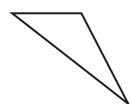
3 acute angles

Right Triangle



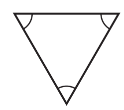
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle



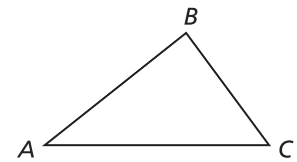
3 congruent angles

Notes:

Theorems

Theorem 5.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

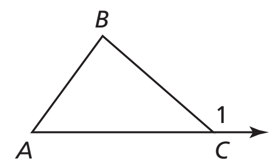


$m\angle A + m\angle B + m\angle C = 180^\circ$

Notes:

Theorem 5.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

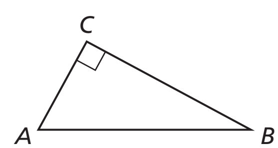


$m\angle 1 = m\angle A + m\angle B$

Notes:

Corollary 5.1 Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.



$m\angle A + m\angle B = 90^\circ$

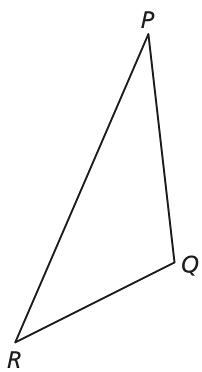
Notes:

Name____ Date__________

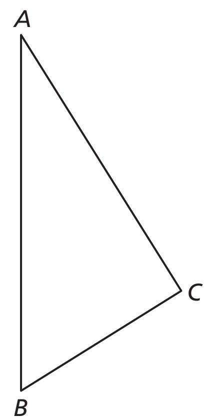
Extra Practice

In Exercises 1–3, classify the triangle by its sides and by measuring its angles.

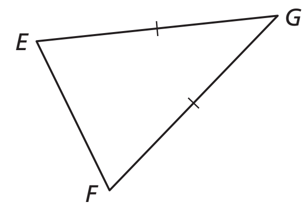
1.



2.



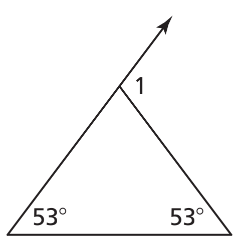
3.



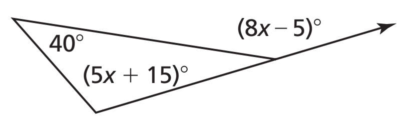
4. Classify $\triangle ABC$ by its sides. Then determine whether it is a right triangle.
$A(6, 6)$, $B(9, 3)$, $C(2, 2)$

In Exercises 5 and 6, find the measure of the exterior angle.

5.



6.



7. In a right triangle, the measure of one acute angle is twice the sum of the measure of the other acute angle and 30. Find the measure of each acute angle in the right triangle.

Name ____ Date __________

5.2 Congruent Polygons

For use with Exploration 5.2

Essential Question Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

1 EXPLORATION: Describing Rigid Motions

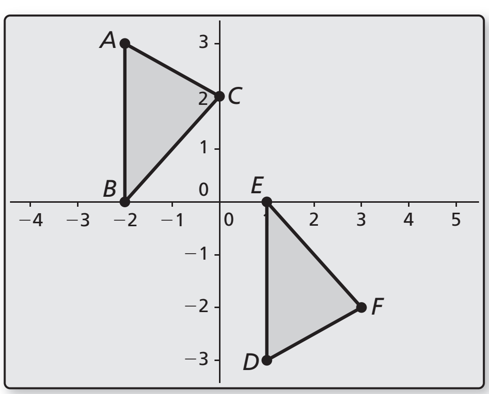
Work with a partner. Of the four transformations you studied in Chapter 4, which are rigid motions? Under a rigid motion, why is the image of a triangle always congruent to the original triangle? Explain you reasoning.

2 EXPLORATION: Finding a Composition of Rigid Motions

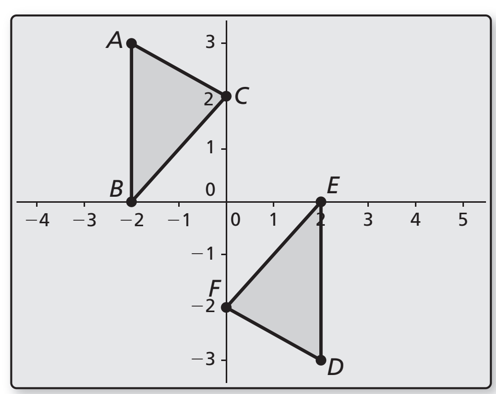
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Describe a composition of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Use dynamic geometry software to verify your answer.

a. $\triangle ABC \cong \triangle DEF$



b. $\triangle ABC \cong \triangle DEF$

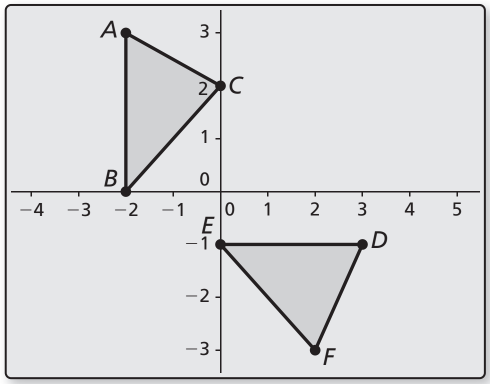


Name____ Date__________

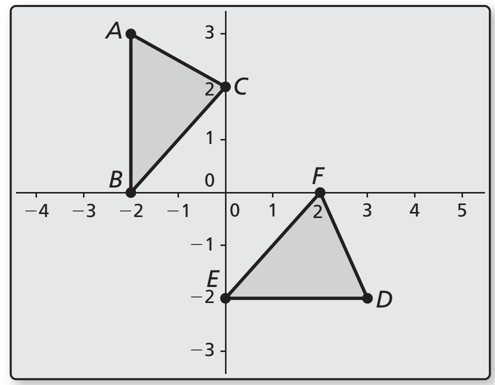
5.2 Congruent Polygons (continued)

2 EXPLORATION: Finding a Composition of Rigid Motions (continued)

c. $\triangle ABC \cong \triangle DEF$



d. $\triangle ABC \cong \triangle DEF$



Communicate Your Answer

3. Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

4. The vertices of $\triangle ABC$ are $A(1, 1)$, $B(3, 2)$, and $C(4, 4)$. The vertices of $\triangle DEF$ are $D(2, -1)$, $E(0, 0)$, and $F(-1, 2)$. Describe a composition of rigid motions that maps $\triangle ABC$ to $\triangle DEF$.

Name ____ Date __________

5.2 Notetaking with Vocabulary
For use after Lesson 5.2

In your own words, write the meaning of each vocabulary term.

corresponding parts

Theorems

Theorem 5.3 Properties of Triangle Congruence

Triangle congruence is reflexive, symmetric, and transitive.

Reflexive For any triangle $\triangle ABC, \triangle ABC \cong \triangle ABC$.

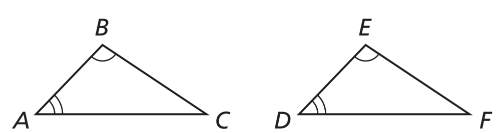
Symmetric If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

Transitive If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Notes:

Theorem 5.4 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.



If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

Notes:

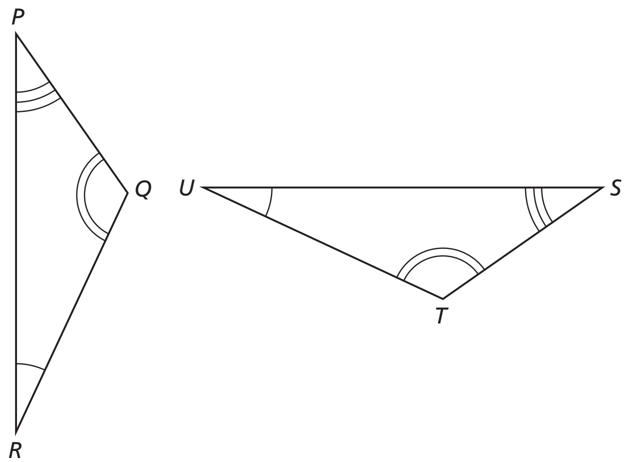
Name____________________________ Date__________

5.2 Notetaking with Vocabulary (continued)

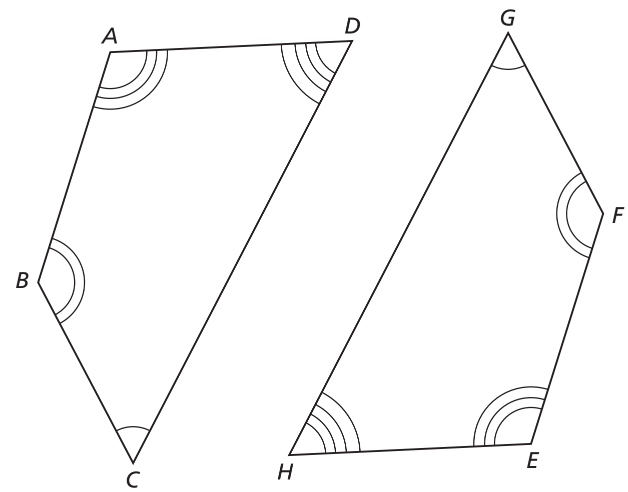
Extra Practice

In Exercises 1 and 2, identify all pairs of congruent corresponding parts. Then write another congruence statement for the polygons.

1. $\triangle PQR \cong \triangle STU$

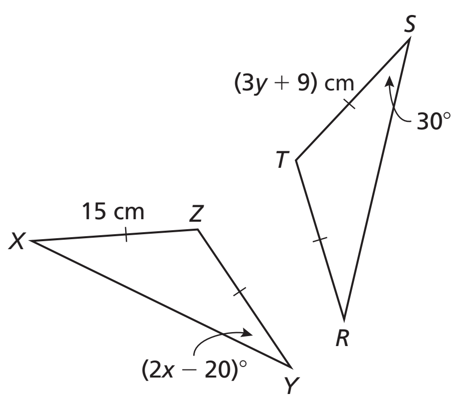


2. $ABCD \cong EFGH$

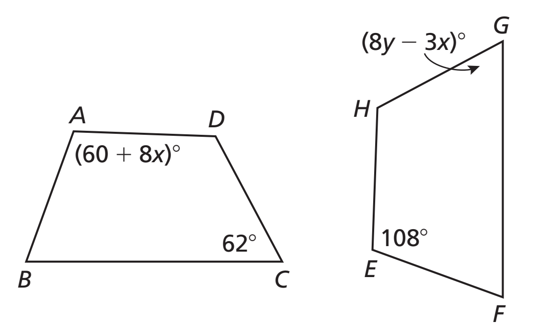


In Exercises 3 and 4, find the values of *x* and *y*.

3. $\triangle XYZ \cong \triangle RST$



4. $ABCD \cong EFGH$

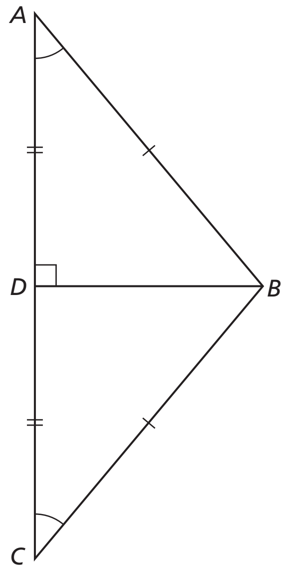


Name ____ Date __________

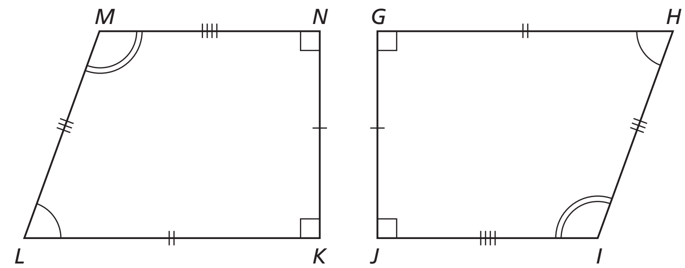
5.2 Notetaking with Vocabulary (continued)

In Exercises 5 and 6, show that the polygons are congruent. Explain your reasoning.

5.

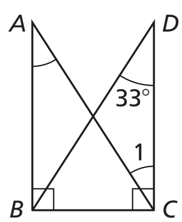


6.

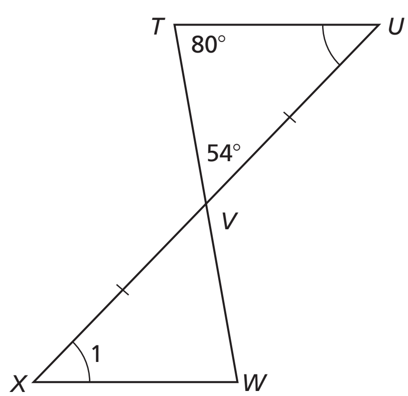


In Exercises 7 and 8, find $m\angle 1$.

7.



8.



Name____ Date__________

5.3 Proving Triangle Congruence by SAS

For use with Exploration 5.3

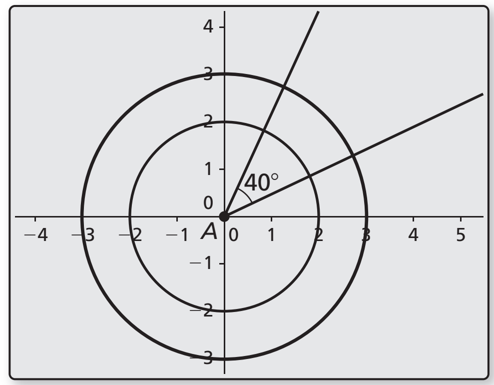
Essential Question What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

1 EXPLORATION: Drawing Triangles

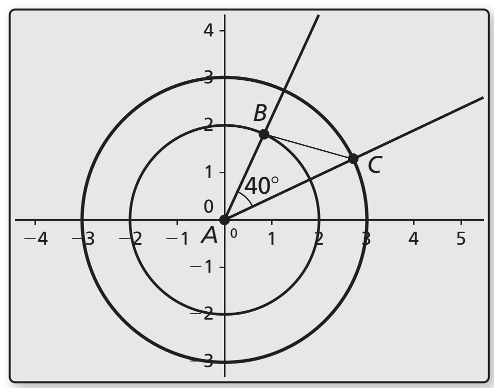
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Construct a 40° angle with its vertex at the origin. Label the vertex A.



b. Locate the point where one ray of the angle intersects the smaller circle and label this point B. Locate the point where the other ray of the angle intersects the larger circle and label this point C. Then draw $\triangle ABC$.



c. Find BC, $m\angle B$, and $m\angle C$.

d. Repeat parts (a)–(c) several times, redrawing the angle in different positions. Keep track of your results by completing the table on the next page. What can you conclude?

Name ____ Date __________

5.3 **Proving Triangle Congruence by SAS (continued)**

1 **EXPLORATION:** Drawing Triangles (continued)

	A	*B*	*C*	*AB*	*AC*	*BC*	$m\angle A$	$m\angle B$	$m\angle C$
1.	(0, 0)			2	3		40°		
2.	(0, 0)			2	3		40°		
3.	(0, 0)			2	3		40°		
4.	(0, 0)			2	3		40°		
5.	(0, 0)			2	3		40°		

Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

3. How would you prove your conclusion in Exploration 1(d)?

Name____ Date __________

5.3 Notetaking with Vocabulary

For use after Lesson 5.3

In your own words, write the meaning of each vocabulary term.

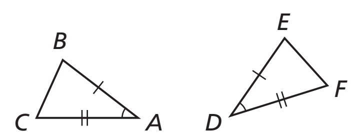
congruent figures

rigid motion

Theorems

Theorem 5.5 Side-Angle-Side (SAS) Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.



If $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

Notes:

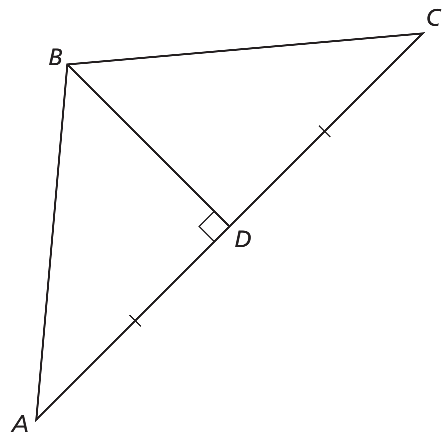
Name ____ Date __________

Extra Practice

In Exercises 1 and 2, write a proof.

1. Given $\overline{BD} \perp \overline{AC}$, $\overline{AD} \cong \overline{CD}$

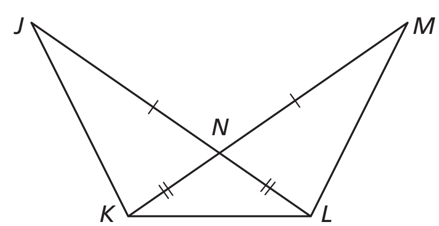
Prove $\triangle ABD \cong \triangle CBD$



STATEMENTS	REASONS

2. Given $\overline{JN} \cong \overline{MN}$, $\overline{NK} \cong \overline{NL}$

Prove $\triangle JNK \cong \triangle MNL$



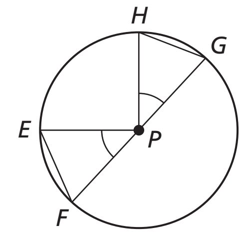
STATEMENTS	REASONS

Name______________________________ Date__________

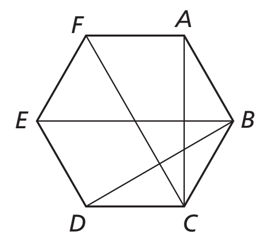
5.3 Notetaking with Vocabulary (continued)

In Exercises 3 and 4, use the given information to name two triangles that are congruent. Explain your reasoning.

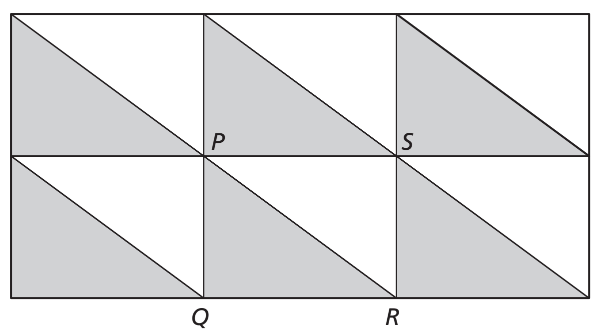
3. $\angle EPF \cong \angle GPH$, and P is the center of the circle.



4. $ABCDEF$ is a regular hexagon.



5. A quilt is made of triangles. You know $\overline{PS} \parallel \overline{QR}$ and $\overline{PS} \cong \overline{QR}$. Use the SAS Congruence Theorem (Theorem 5.5) to show that $\triangle PQR \cong \triangle RSP$.



Name ____ Date __________

5.4 Equilateral and Isosceles Triangles

For use with Exploration 5.4

Essential Question What conjectures can you make about the side lengths and angle measures of an isosceles triangle?

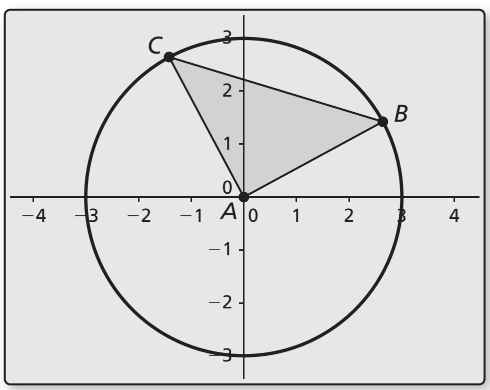
1 EXPLORATION: Writing a Conjecture about Isosceles Triangles

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct a circle with a radius of 3 units centered at the origin.

b. Construct $\triangle ABC$ so that B and C are on the circle and A is at the origin.



Sample

Points
$A(0, 0)$
$B(2.64, 1.42)$
$C(-1.42, 2.64)$

Segments
$AB = 3$
$AC = 3$
$BC = 4.24$

Angles
$m\angle A = 90°$
$m\angle B = 45°$
$m\angle C = 45°$

c. Recall that a triangle is *isosceles* if it has at least two congruent sides. Explain why $\triangle ABC$ is an isosceles triangle.

d. What do you observe about the angles of $\triangle ABC$?

e. Repeat parts (a)–(d) with several other isosceles triangles using circles of different radii. Keep track of your observations by completing the table on the next page. Then write a conjecture about the angle measures of an isosceles triangle.

1 EXPLORATION: Writing a Conjecture about Isosceles Triangles (continued)

		A	*B*	*C*	*AB*	*AC*	*BC*	$m\angle A$	$m\angle B$	$m\angle C$
Sample	**1.**	(0, 0)	(2.64, 1.42)	(–1.42, 2.64)	3	3	4.24	90°	45°	45°
	2.	(0, 0)								
	3.	(0, 0)								
	4.	(0, 0)								
	5.	(0, 0)								

f. Write the converse of the conjecture you wrote in part (e). Is the converse true?

Communicate Your Answer

2. What conjectures can you make about the side lengths and angle measures of an isosceles triangle?

3. How would you prove your conclusion in Exploration 1(e)? in Exploration 1(f)?

Name ____ Date _________

5.4 Notetaking with Vocabulary
For use after Lesson 5.4

In your own words, write the meaning of each vocabulary term.

legs

vertex angle

base

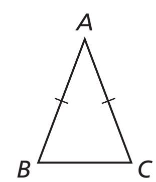
base angles

Theorems

Theorem 5.6 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

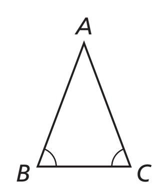
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



Theorem 5.7 Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

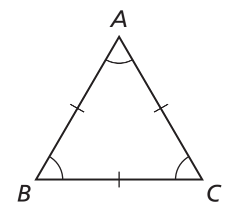


Notes:

Corollaries

Corollary 5.2 Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.



Corollary 5.3 Corollary to the Converse of the Base Angles Theorem

If a triangle is equiangular, then it is equilateral.

Notes:

Extra Practice

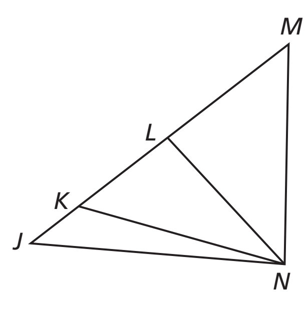
In Exercises 1–4, complete the statement. State which theorem you used.

1. If $\overline{NJ} \cong \overline{NM}$, then $\angle$_______ $\cong$ $\angle$_______.

2. If $\overline{LM} \cong \overline{LN}$, then $\angle$_______ $\cong$ $\angle$_______.

3. If $\angle NKM \cong \angle NMK$, then _______ $\cong$ _______.

4. If $\angle LJN \cong \angle LNJ$, then _______ $\cong$ _______.

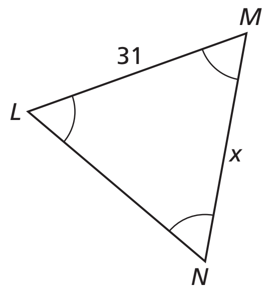


Name ____ Date __________

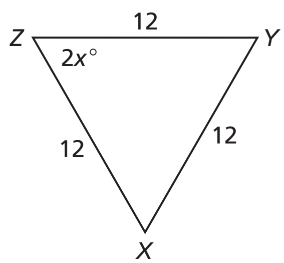
5.4 Notetaking with Vocabulary (continued)

In Exercises 5 and 6, find the value of *x*.

5.

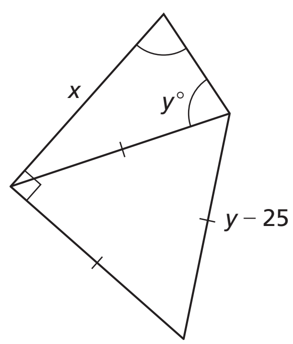


6.

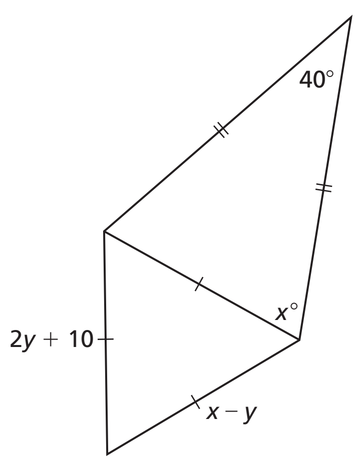


In Exercises 7 and 8, find the values of *x* and *y*.

7.



8.



Name____ Date__________

5.5 Proving Triangle Congruence by SSS

For use with Exploration 5.5

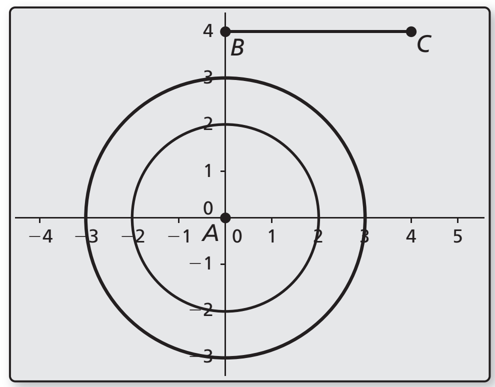
Essential Question What can you conclude about two triangles when you know the corresponding sides are congruent?

1 EXPLORATION: Drawing Triangles

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

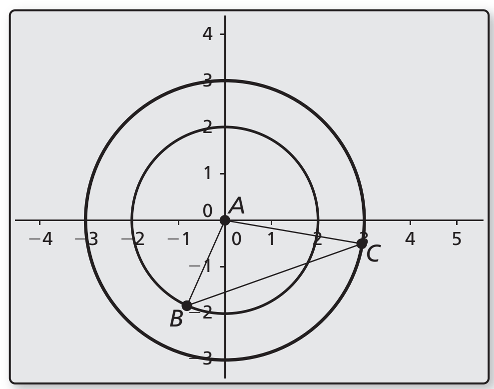
Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Label the origin A. Then draw $\overline{BC}$ of length 4 units.



b. Move $\overline{BC}$ so that B is on the smaller circle and C is on the larger circle. Then draw $\triangle ABC$.

c. Explain why the side lengths of $\triangle ABC$ are 2, 3, and 4 units.



d. Find $m\angle A, m\angle B,$ and $m\angle C.$

e. Repeat parts (b) and (d) several times, moving $\overline{BC}$ to different locations. Keep track of your results by completing the table on the next page. What can you conclude?

5.5 Proving Triangle Congruence by SSS (continued)

1 EXPLORATION: Drawing Triangles (continued)

	A	B	C	AB	AC	BC	$m\angle A$	$m\angle B$	$m\angle C$
1.	(0, 0)			2	3	4			
2.	(0, 0)			2	3	4			
3.	(0, 0)			2	3	4			
4.	(0, 0)			2	3	4			
5.	(0, 0)			2	3	4			

Communicate Your Answer

2. What can you conclude about two triangles when you know the corresponding sides are congruent?

3. How would you prove your conclusion in Exploration 1(e)?

Name____________________________________ Date__________

5.5 Notetaking with Vocabulary

For use after Lesson 5.5

In your own words, write the meaning of each vocabulary term.

legs

hypotenuse

Theorems

Theorem 5.8 Side-Side-Side (SSS) Congruence Theorem

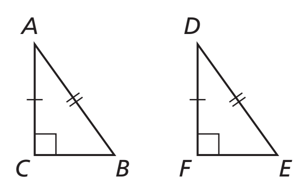
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

Notes:

Theorem 5.9 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.



If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $m\angle C = m\angle F = 90°$, then $\triangle ABC \cong \triangle DEF$.

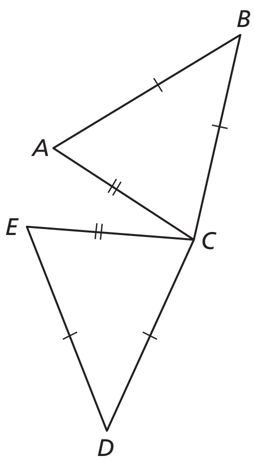
Notes:

Name ____ Date __________

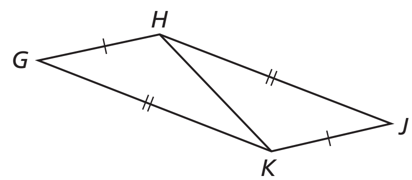
Extra Practice

In Exercises 1–4, decide whether the congruence statement is true. Explain your reasoning.

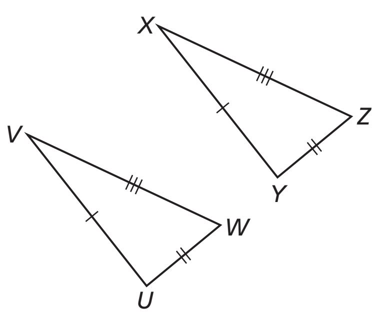
1. $\triangle ABC \cong \triangle EDC$



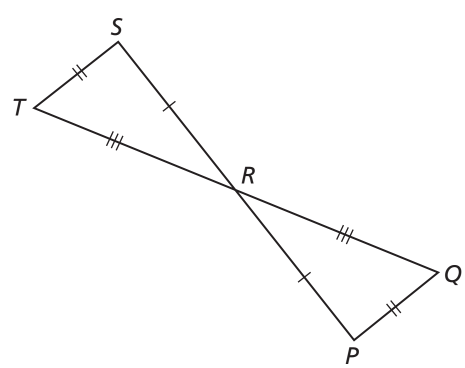
2. $\triangle KGH \cong \triangle HJK$



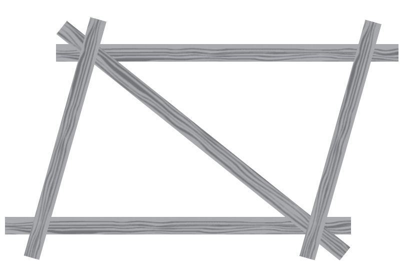
3. $\triangle UVW \cong \triangle XYZ$



4. $\triangle RST \cong \triangle RPQ$



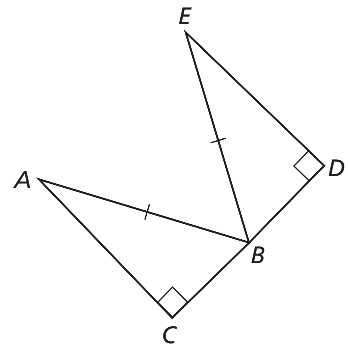
5. Determine whether the figure is stable. Explain your reasoning.



Name____ Date__________

5.5 Notetaking with Vocabulary (continued)

6. Redraw the triangles so they are side by side with corresponding parts in the same position. Then write a proof.

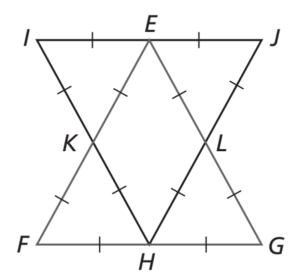


Given B is the midpoint of $\overline{CD}$,
$\overline{AB} \cong \overline{EB}$, $\angle C$ and $\angle D$ are right angles.

Prove $\triangle ABC \cong \triangle EBD$

STATEMENTS	REASONS

7. Write a proof.



Given $\overline{IE} \cong \overline{EJ} \cong \overline{JL} \cong \overline{LH} \cong \overline{HK} \cong \overline{KI} \cong \overline{EK} \cong \overline{KF} \cong \overline{FH} \cong \overline{HG} \cong \overline{GL} \cong \overline{LE}$

Prove $\triangle EFG \cong \triangle HIJ$

STATEMENTS	REASONS

Name ____ Date __________

5.6 Proving Triangle Congruence by ASA and AAS

For use with Exploration 5.6

Essential Question What information is sufficient to determine whether two triangles are congruent?

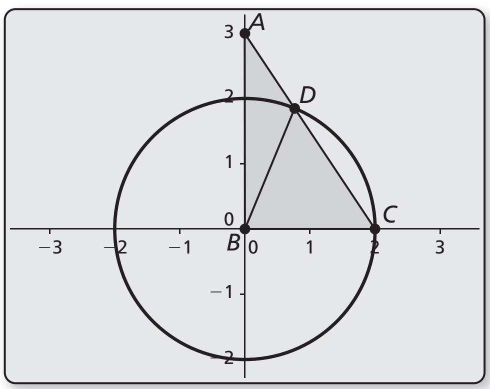
1 EXPLORATION: Determining Whether SSA Is Sufficient

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to construct $\triangle ABC$. Construct the triangle so that vertex B is at the origin, $\overline{AB}$ has a length of 3 units, and $\overline{BC}$ has a length of 2 units.

b. Construct a circle with a radius of 2 units centered at the origin. Locate point D where the circle intersects $\overline{AC}$. Draw $\overline{BD}$.



Sample

Points
$A(0, 3)$
$B(0, 0)$
$C(2, 0)$
$D(0.77, 1.85)$
Segments
$AB = 3$
$AC = 3.61$
$BC = 2$
$AD = 1.38$
Angle
$m\angle A = 33.69°$

c. $\triangle ABC$ and $\triangle ABD$ have two congruent sides and a nonincluded congruent angle. Name them.

d. Is $\triangle ABC \cong \triangle ABD$? Explain your reasoning.

e. Is SSA sufficient to determine whether two triangles are congruent? Explain your reasoning.

2 EXPLORATION: Determining Valid Congruence Theorems

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to determine which of the following are valid triangle congruence theorems. For those that are not valid, write a counterexample. Explain your reasoning.

Possible Congruence Theorem	Valid or not valid?
SSS	
SSA	
SAS	
AAS	
ASA	
AAA	

Communicate Your Answer

3. What information is sufficient to determine whether two triangles are congruent?

4. Is it possible to show that two triangles are congruent using more than one congruence theorem? If so, give an example.

Name ____ Date _________

5.6 Notetaking with Vocabulary
For use after Lesson 5.6

In your own words, write the meaning of each vocabulary term.

congruent figures

rigid motion

Theorems

Theorem 5.10 Angle-Side-Angle (ASA) Congruence Theorem

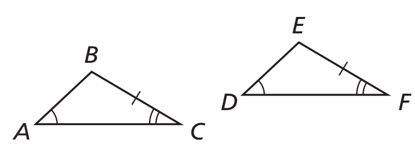
If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, and $\angle C \cong \angle F$, then $\triangle ABC \cong \triangle DEF$.

Notes:

Theorem 5.11 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.



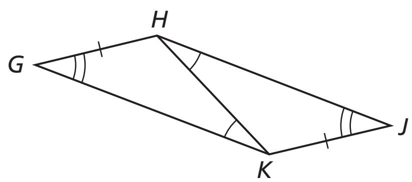
If $\angle A \cong \angle D$, $\angle C \cong \angle F$, and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$.

Notes:

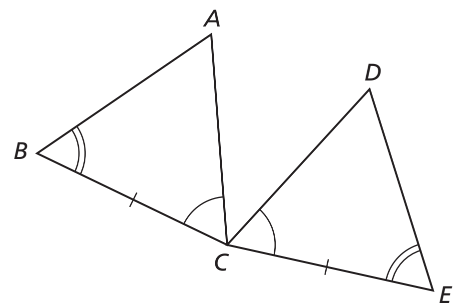
Extra Practice

In Exercises 1–4, decide whether enough information is given to prove that the triangles are congruent. If so, state the theorem you would use.

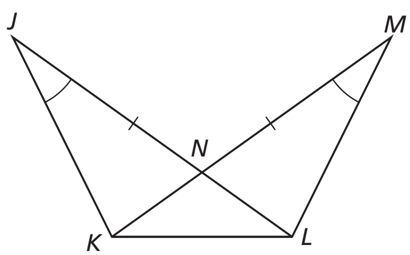
1. $\triangle GHK, \triangle JKH$



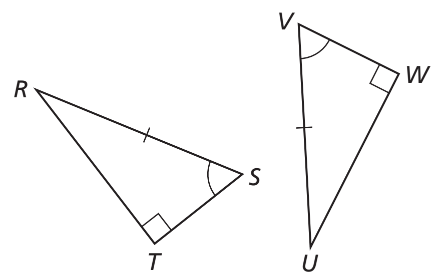
2. $\triangle ABC, \triangle DEC$



3. $\triangle JKL, \triangle MLK$



4. $\triangle RST, \triangle UVW$

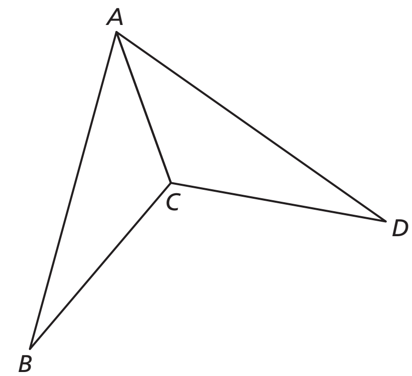


In Exercises 5 and 6, decide whether you can use the given information to prove that $\triangle LMN \cong \triangle PQR$. Explain your reasoning.

5. $\angle M \cong \angle Q, \angle N \cong \angle R, \overline{NL} \cong \overline{RP}$

6. $\angle L \cong \angle R, \angle M \cong \angle Q, \overline{LM} \cong \overline{PQ}$

7. Prove that the triangles are congruent using the ASA Congruence Theorem (Theorem 5.10).

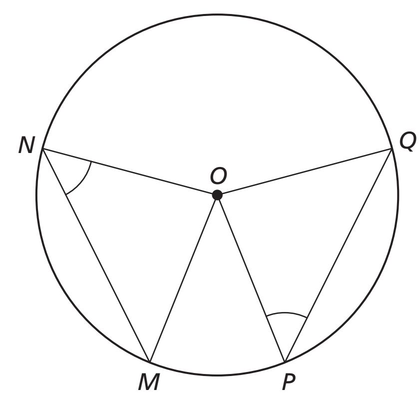


Given $\overline{AC}$ bisects $\angle DAB$ and $\angle DCB$.

Prove $\triangle ABC \cong \triangle ADC$

STATEMENTS	REASONS

8. Prove that the triangles are congruent using the AAS Congruence Theorem (Theorem 5.11).



Given O is the center of the circle and $\angle N \cong \angle P$.

Prove $\triangle MNO \cong \triangle PQO$

STATEMENTS	REASONS

Name____ Date__________

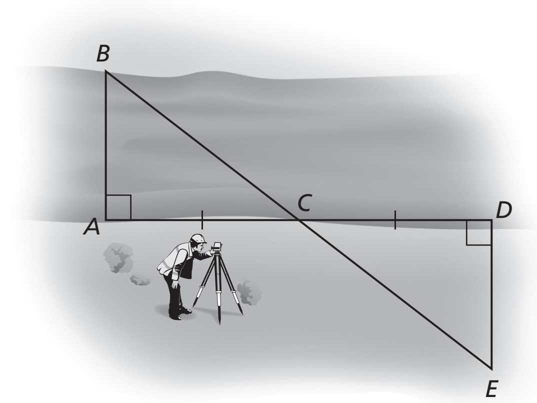
5.7 Using Congruent Triangles

For use with Exploration 5.7

Essential Question How can you use congruent triangles to make an indirect measurement?

1 EXPLORATION: Measuring the Width of a River

Work with a partner. The figure shows how a surveyor can measure the width of a river by making measurements on only one side of the river.



a. Study the figure. Then explain how the surveyor can find the width of the river.

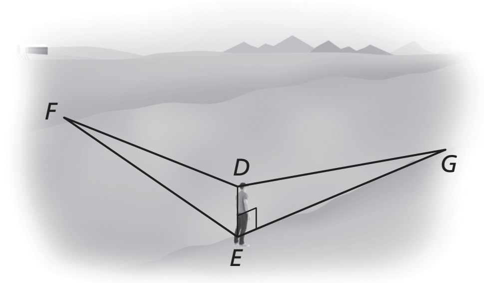
b. Write a proof to verify that the method you described in part (a) is valid.

Given $\angle A$ is a right angle, $\angle D$ is a right angle, $\overline{AC} \cong \overline{CD}$

c. Exchange proofs with your partner and discuss the reasoning used.

2 EXPLORATION: Measuring the Width of a River

Work with a partner. It was reported that one of Napoleon's officers estimated the width of a river as follows. The officer stood on the bank of the river and lowered the visor on his cap until the farthest thing visible was the edge of the bank on the other side. He then turned and noted the point on his side that was in line with the tip of his visor and his eye. The officer then paced the distance to this point and concluded that distance was the width of the river.



a. Study the figure. Then explain how the officer concluded that the width of the river is EG.

b. Write a proof to verify that the conclusion the officer made is correct.

Given $\angle DEG$ is a right angle, $\angle DEF$ is a right angle, $\angle EDG \cong \angle EDF$

c. Exchange proofs with your partner and discuss the reasoning used.

Communicate Your Answer

3. How can you use congruent triangles to make an indirect measurement?

4. Why do you think the types of measurements described in Explorations 1 and 2 are called *indirect* measurements?

Name____ Date __________

5.7 Notetaking with Vocabulary

For use after Lesson 5.7

In your own words, write the meaning of each vocabulary term.

congruent figures

corresponding parts

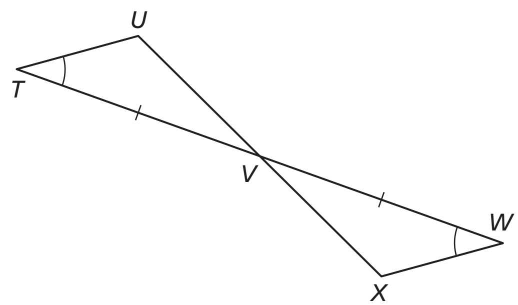
construction

Notes:

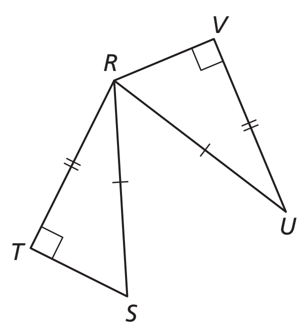
Extra Practice

In Exercises 1–3, explain how to prove that the statement is true.

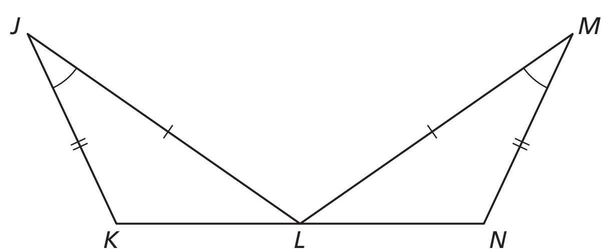
1. $\overline{UV} \cong \overline{XV}$



2. $\overline{TS} \cong \overline{VR}$

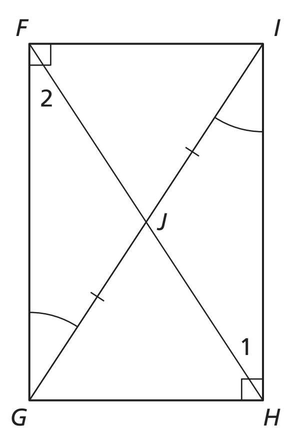


3. $\angle JLK \cong \angle MLN$

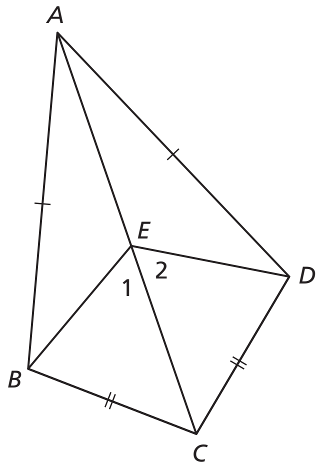


In Exercises 4 and 5, write a plan to prove that $\angle 1 \cong \angle 2$.

4.



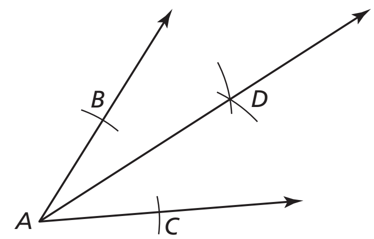
5.



6. Write a proof to verify that the construction is valid.

Ray bisects an angle

Plan for Proof Show that $\triangle ABD \cong \triangle ACD$ by the SSS Congruence Theorem (Thm. 5.8). Use corresponding parts of congruent triangles to show that $\angle BAD \cong \angle CAD$.



STATEMENTS	REASONS

Name ______________________________ Date __________

5.8 Coordinate Proofs

For use with Exploration 5.8

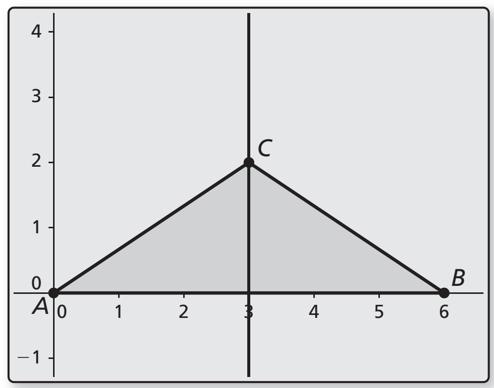
Essential Question How can you use a coordinate plane to write a proof?

1 EXPLORATION: Writing a Coordinate Proof

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw $\overline{AB}$ with endpoints $A(0, 0)$ and $B(6, 0)$.



Sample
Points
$A(0, 0)$
$B(6, 0)$
$C(3, y)$
Segments
$AB = 6$
Line
$x = 3$

b. Draw the vertical line $x = 3$.

c. Draw $\triangle ABC$ so that C lies on the line $x = 3$.

d. Use your drawing to prove that $\triangle ABC$ is an isosceles triangle.

2 EXPLORATION: Writing a Coordinate Proof

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

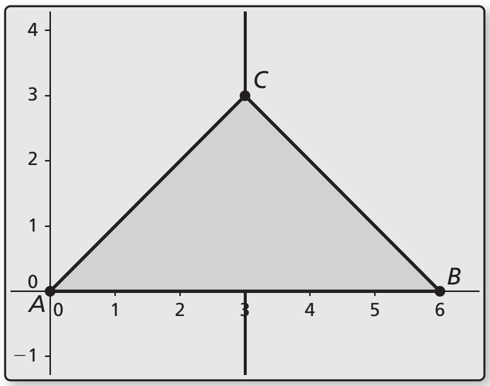
Work with a partner.

a. Use dynamic geometry software to draw $\overline{AB}$ with endpoints $A(0, 0)$ and $B(6, 0)$.

b. Draw the vertical line $x = 3$.

c. Plot the point $C(3, 3)$ and draw $\triangle ABC$. Then use your drawing to prove that $\triangle ABC$ is an isosceles right triangle.

2 EXPLORATION: Writing a Coordinate Proof (continued)



Sample
Points
$A(0, 0)$
$B(6, 0)$
$C(3, 3)$
Segments
$AB = 6$
$BC = 4.24$
$AC = 4.24$
Line
$x = 3$

d. Change the coordinates of C so that C lies below the x-axis and $\triangle ABC$ is an isosceles right triangle.

e. Write a coordinate proof to show that if C lies on the line $x = 3$ and $\triangle ABC$ is an isosceles right triangle, then C must be the point (3, 3) or the point found in part (d).

Communicate Your Answer

3. How can you use a coordinate plane to write a proof?

4. Write a coordinate proof to prove that $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$, and $C\left(3, 3\sqrt{3}\right)$ is an equilateral triangle.

Name ____ Date __________

5.8 Notetaking with Vocabulary
For use after Lesson 5.8

In your own words, write the meaning of each vocabulary term.

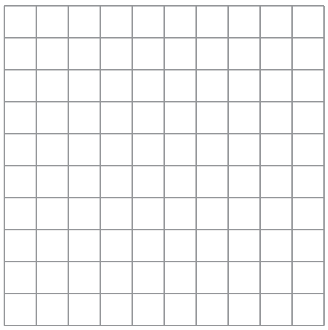
coordinate proof

Notes:

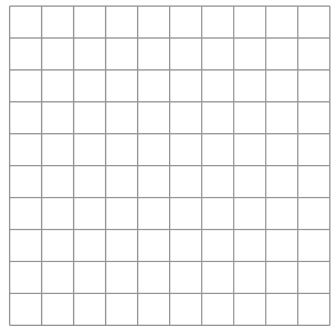
Extra Practice

In Exercises 1 and 2, place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex. Explain the advantages of your placement.

1. an obtuse triangle with height of 3 units and base of 2 units



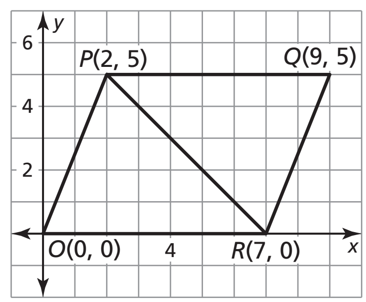
2. a rectangle with length of $2w$



In Exercises 3 and 4, write a plan for the proof.

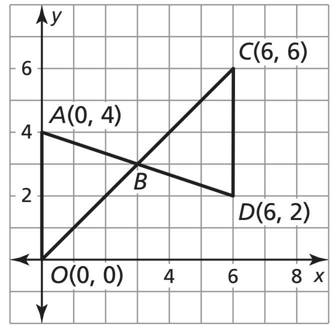
3. **Given** Coordinates of vertices of $\triangle OPR$ and $\triangle QRP$

Proof $\triangle OPR \cong \triangle QRP$

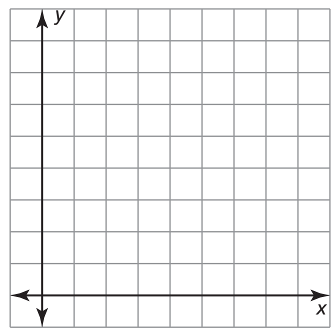


4. Given Coordinates of vertices of $\triangle OAB$ and $\triangle CDB$

Prove B is the midpoint of $\overline{AD}$ and $\overline{OC}$.



5. Graph the triangle with vertices $A(0, 0)$, $B(3m, m)$, and $C(0, 3m)$. Find the length and the slope of each side of the triangle. Then find the coordinates of the midpoint of each side. Is the triangle a right triangle? isosceles? Explain. (Assume all variables are positive.)



6. Write a coordinate proof.

Given Coordinates of vertices of $\triangle OEF$ and $\triangle OGF$

Prove $\triangle OEF \cong \triangle OGF$