Find the coordinates of the midpoint $M$ of the segment with the given endpoints. Then find the distance between the two points.

1. $A(3, 1)$ and $B(5, 5)$
2. $F(0, -6)$ and $G(8, -4)$

3. $P(-2, -7)$ and $B(-4, 5)$
4. $S(10, -5)$ and $T(7, -9)$

Solve the equation.

5. $9x - 6 = 7x$
6. $2r + 6 = 5r - 9$

7. $20 - 3n = 2n + 30$
8. $8t - 5 = 6t - 4$
5.1 Angles of Triangles
For use with Exploration 5.1

Essential Question  How are the angle measures of a triangle related?

1 EXPLORATION: Writing a Conjecture

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it \( \triangle ABC \).

b. Find the measures of the interior angles of the triangle.

c. Find the sum of the interior angle measures.

d. Repeat parts (a)–(c) with several other triangles. Then write a conjecture about the sum of the measures of the interior angles of a triangle.

Sample
Angles
\( m\angle A = 43.67^\circ \)
\( m\angle B = 81.87^\circ \)
\( m\angle C = 54.46^\circ \)
2 EXPLORATION: Writing a Conjecture

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it \( \triangle ABC \).

b. Draw an exterior angle at any vertex and find its measure.

c. Find the measures of the two nonadjacent interior angles of the triangle.

d. Find the sum of the measures of the two nonadjacent interior angles. Compare this sum to the measure of the exterior angle.

e. Repeat parts (a)–(d) with several other triangles. Then write a conjecture that compares the measure of an exterior angle with the sum of the measures of the two nonadjacent interior angles.

Communicate Your Answer

3. How are the angle measures of a triangle related?

4. An exterior angle of a triangle measures 32°. What do you know about the measures of the interior angles? Explain your reasoning.
5.1 Notetaking with Vocabulary
For use after Lesson 5.1

In your own words, write the meaning of each vocabulary term.

interior angles

exterior angles

corollary to a theorem

Core Concepts

Classifying Triangles by Sides

Scalene Triangle  Isosceles Triangle  Equilateral Triangle

no congruent sides  at least 2 congruent sides  3 congruent sides

Classifying Triangles by Angles

Acute Triangle  Right Triangle  Obtuse Triangle  Equiangular Triangle

3 acute angles  1 right angle  1 obtuse angle  3 congruent angles

Notes:
5.1 Notetaking with Vocabulary (continued)

Theorems

Theorem 5.1  Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

Notes:

\[ m\angle A + m\angle B + m\angle C = 180^\circ \]

Theorem 5.2  Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Notes:

\[ m\angle 1 = m\angle A + m\angle B \]

Corollary 5.1  Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

Notes:

\[ m\angle A + m\angle B = 90^\circ \]
Extra Practice

In Exercises 1–3, classify the triangle by its sides and by measuring its angles.

1. 

2. 

3. 

4. Classify $\triangle ABC$ by its sides. Then determine whether it is a right triangle.
   $A(6, 6), B(9, 3), C(2, 2)$

In Exercises 5 and 6, find the measure of the exterior angle.

5. 

6. 

7. In a right triangle, the measure of one acute angle is twice the sum of the measure of the other acute angle and 30. Find the measure of each acute angle in the right triangle.
Essential Question  Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

1  EXPLORATION: Describing Rigid Motions

Work with a partner. Of the four transformations you studied in Chapter 4, which are rigid motions? Under a rigid motion, why is the image of a triangle always congruent to the original triangle? Explain you reasoning.

2  EXPLORATION: Finding a Composition of Rigid Motions

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Describe a composition of rigid motions that maps \( \triangle ABC \) to \( \triangle DEF \). Use dynamic geometry software to verify your answer.

a. \( \triangle ABC \cong \triangle DEF \)  

\[
\begin{align*}
A & \rightarrow (3, 3) \\
B & \rightarrow (-1, 0) \\
C & \rightarrow (1, 1) \\
D & \rightarrow (2, -1) \\
E & \rightarrow (2, 2) \\
F & \rightarrow (4, 3)
\end{align*}
\]

b. \( \triangle ABC \cong \triangle DEF \)  

\[
\begin{align*}
A & \rightarrow (3, 3) \\
B & \rightarrow (-1, 0) \\
C & \rightarrow (1, 1) \\
D & \rightarrow (2, -1) \\
E & \rightarrow (2, 2) \\
F & \rightarrow (4, 3)
\end{align*}
\]
3. Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

4. The vertices of $\triangle ABC$ are $A(1, 1), B(3, 2)$, and $C(4, 4)$. The vertices of $\triangle DEF$ are $D(2, -1), E(0, 0)$, and $F(-1, 2)$. Describe a composition of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. 
In your own words, write the meaning of each vocabulary term.

corresponding parts

Theorems

**Theorem 5.3  Properties of Triangle Congruence**

Triangle congruence is reflexive, symmetric, and transitive.

**Reflexive**  For any triangle \( \triangle ABC \), \( \triangle ABC \cong \triangle ABC \).

**Symmetric**  If \( \triangle ABC \cong \triangle DEF \), then \( \triangle DEF \cong \triangle ABC \).

**Transitive**  If \( \triangle ABC \cong \triangle DEF \) and \( \triangle DEF \cong \triangle JKL \), then \( \triangle ABC \cong \triangle JKL \).

**Notes:**

**Theorem 5.4  Third Angles Theorem**

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

**Notes:**

If \( \angle A \cong \angle D \) and \( \angle B \cong \angle E \), then \( \angle C \cong \angle F \).
Extra Practice

In Exercises 1 and 2, identify all pairs of congruent corresponding parts. Then write another congruence statement for the polygons.

1. $\triangle PQR \cong \triangle STU$

2. $ABCD \cong EFGH$

In Exercises 3 and 4, find the values of $x$ and $y$.

3. $\triangle XYZ \cong \triangle RST$

4. $ABCD \cong EFGH$
In Exercises 5 and 6, show that the polygons are congruent. Explain your reasoning.

5. \[\triangle ABC\]

6. \[\quad\]

In Exercises 7 and 8, find \(m\angle 1\).

7. \[\triangle ABC\]

8. \[\triangle UTV\]
5.3 Proving Triangle Congruence by SAS
For use with Exploration 5.3

Essential Question What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

1 EXPLORATION: Drawing Triangles

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Construct a 40° angle with its vertex at the origin. Label the vertex \( A \).

b. Locate the point where one ray of the angle intersects the smaller circle and label this point \( B \). Locate the point where the other ray of the angle intersects the larger circle and label this point \( C \). Then draw \( \triangle ABC \).

c. Find \( BC \), \( m\angle B \), and \( m\angle C \).

d. Repeat parts (a)–(c) several times, redrawing the angle in different positions. Keep track of your results by completing the table on the next page. What can you conclude?
Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

3. How would you prove your conclusion in Exploration 1(d)?
In your own words, write the meaning of each vocabulary term.

congruent figures

rigid motion

Theorems

Theorem 5.5 Side-Angle-Side (SAS) Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$. 

Notes:
Extra Practice

In Exercises 1 and 2, write a proof.

1. Given $BD \perp AC, AD \cong CD$
   Prove $\triangle ABD \cong \triangle CBD$

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<th>STATEMENTS</th>
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2. Given $JN \cong MN, NK \cong NL$
   Prove $\triangle JNK \cong \triangle MNL$

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</table>
In Exercises 3 and 4, use the given information to name two triangles that are congruent. Explain your reasoning.

3. $\angle EPF \cong \angle GPH$, and $P$ is the center of the circle.

4. $ABCDEF$ is a regular hexagon.

5. A quilt is made of triangles. You know $\overline{PS} \parallel \overline{QR}$ and $\overline{PS} \cong \overline{QR}$. Use the SAS Congruence Theorem (Theorem 5.5) to show that $\triangle PQR \cong \triangle RSP$. 
5.4 Equilateral and Isosceles Triangles
For use with Exploration 5.4

Essential Question  What conjectures can you make about the side lengths and angle measures of an isosceles triangle?

EXPLORATION: Writing a Conjecture about Isosceles Triangles

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct a circle with a radius of 3 units centered at the origin.

b. Construct $\triangle ABC$ so that $B$ and $C$ are on the circle and $A$ is at the origin.

![Diagram of a circle with a triangle constructed from it]

Sample

Points
$A(0, 0)$
$B(2.64, 1.42)$
$C(-1.42, 2.64)$

Segments
$AB = 3$
$AC = 3$
$BC = 4.24$

Angles
$m\angle A = 90^\circ$
$m\angle B = 45^\circ$
$m\angle C = 45^\circ$

c. Recall that a triangle is isosceles if it has at least two congruent sides. Explain why $\triangle ABC$ is an isosceles triangle.

d. What do you observe about the angles of $\triangle ABC$?

e. Repeat parts (a)–(d) with several other isosceles triangles using circles of different radii. Keep track of your observations by completing the table on the next page. Then write a conjecture about the angle measures of an isosceles triangle.
5.4 Equilateral and Isosceles Triangles (continued)

1 EXPLORATION: Writing a Conjecture about Isosceles Triangles (continued)

<table>
<thead>
<tr>
<th>Sample</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>AB</th>
<th>AC</th>
<th>BC</th>
<th>$m \angle A$</th>
<th>$m \angle B$</th>
<th>$m \angle C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(0, 0)</td>
<td>(2.64, 1.42)</td>
<td>(–1.42, 2.64)</td>
<td>3</td>
<td>3</td>
<td>4.24</td>
<td>90°</td>
<td>45°</td>
<td>45°</td>
</tr>
<tr>
<td>2.</td>
<td>(0, 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>(0, 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>(0, 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>5.</td>
<td>(0, 0)</td>
<td></td>
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</table>

f. Write the converse of the conjecture you wrote in part (e). Is the converse true?

Communicate Your Answer

2. What conjectures can you make about the side lengths and angle measures of an isosceles triangle?

3. How would you prove your conclusion in Exploration 1(e)? in Exploration 1(f)?
5.4 Notetaking with Vocabulary
For use after Lesson 5.4

In your own words, write the meaning of each vocabulary term.

legs

vertex angle

base

base angles

Theorems

Theorem 5.6 Base Angles Theorem
If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $AB \cong AC$, then $\angle B \cong \angle C$.

Theorem 5.7 Converse of the Base Angles Theorem
If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $AB \cong AC$.

Notes:
5.4 Notetaking with Vocabulary (continued)

Corollaries

Corollary 5.2 Corollary to the Base Angles Theorem
If a triangle is equilateral, then it is equiangular.

Corollary 5.3 Corollary to the Converse of the Base Angles Theorem
If a triangle is equiangular, then it is equilateral.

Notes:

Extra Practice
In Exercises 1–4, complete the statement. State which theorem you used.

1. If $\overline{NJ} \cong \overline{NM}$, then $\angle \equiv \angle$.

2. If $\overline{LM} \cong \overline{LN}$, then $\angle \equiv \angle$.

3. If $\angle NKM \cong \angle NMK$, then $\angle \equiv \angle$.

4. If $\angle LNJ \cong \angle LNJ$, then $\angle \equiv \angle$. 
In Exercises 5 and 6, find the value of $x$.

5.

In Exercises 7 and 8, find the values of $x$ and $y$.

7.

8.
Essential Question  What can you conclude about two triangles when you know the corresponding sides are congruent?

1  EXPLORATION:  Drawing Triangles

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Label the origin \( A \). Then draw \( BC \) of length 4 units.

b. Move \( BC \) so that \( B \) is on the smaller circle and \( C \) is on the larger circle. Then draw \( \triangle ABC \).

c. Explain why the side lengths of \( \triangle ABC \) are 2, 3, and 4 units.

d. Find \( m\angle A, m\angle B, \) and \( m\angle C \).

e. Repeat parts (b) and (d) several times, moving \( BC \) to different locations. Keep track of your results by completing the table on the next page.

What can you conclude?
2. What can you conclude about two triangles when you know the corresponding sides are congruent?

3. How would you prove your conclusion in Exploration 1(e)?
5.5 Notetaking with Vocabulary
For use after Lesson 5.5

In your own words, write the meaning of each vocabulary term.

legs

hypotenuse

**Theorems**

**Theorem 5.8 Side-Side-Side (SSS) Congruence Theorem**

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

**Notes:**

**Theorem 5.9 Hypotenuse-Leg (HL) Congruence Theorem**

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $m\angle C = m\angle F = 90^\circ$, then $\triangle ABC \cong \triangle DEF$.

**Notes:**
Extra Practice

In Exercises 1–4, decide whether the congruence statement is true. Explain your reasoning.

1. \( \triangle ABC \cong \triangle EDC \)

2. \( \triangle KGH \cong \triangle HJK \)

3. \( \triangle UVW \cong \triangle XYZ \)

4. \( \triangle RST \cong \triangle RPQ \)

5. Determine whether the figure is stable. Explain your reasoning.
5.5 Notetaking with Vocabulary (continued)

6. Redraw the triangles so they are side by side with corresponding parts in the same position. Then write a proof.

**Given**
- \( B \) is the midpoint of \( \overline{CD} \),
- \( \overline{AB} \cong \overline{EB} \), \( \angle C \) and \( \angle D \) are right angles.

**Prove** \( \triangle ABC \cong \triangle EBD \)

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</table>

7. Write a proof.

**Given**
- \( \overline{IE} \cong \overline{EJ} \cong \overline{JL} \cong \overline{IH} \cong \overline{HK} \cong \overline{KI} \cong \overline{EK} \cong \overline{KF} \cong \overline{FH} \cong \overline{HG} \cong \overline{GL} \cong \overline{LE} \)

**Prove** \( \triangle EFG \cong \triangle HIJ \)

<table>
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<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
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</table>
5.6 Proving Triangle Congruence by ASA and AAS
For use with Exploration 5.6

Essential Question  What information is sufficient to determine whether two triangles are congruent?

1 EXPLORATION: Determining Whether SSA Is Sufficient

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to construct \( \triangle ABC \). Construct the triangle so that vertex \( B \) is at the origin, \( AB \) has a length of 3 units, and \( BC \) has a length of 2 units.

b. Construct a circle with a radius of 2 units centered at the origin. Locate point \( D \) where the circle intersects \( AC \). Draw \( BD \).

Sample

Points
\( A(0, 3) \)
\( B(0, 0) \)
\( C(2, 0) \)
\( D(0.77, 1.85) \)

Segments
\( AB = 3 \)
\( AC = 3.61 \)
\( BC = 2 \)
\( AD = 1.38 \)

Angle
\( m \angle A = 33.69^\circ \)

c. \( \triangle ABC \) and \( \triangle ABD \) have two congruent sides and a nonincluded congruent angle. Name them.

d. Is \( \triangle ABC \cong \triangle ABD \)? Explain your reasoning.

e. Is SSA sufficient to determine whether two triangles are congruent? Explain your reasoning.
**5.6 Proving Triangle Congruence by ASA and AAS (continued)**

**EXPLORATION: Determining Valid Congruence Theorems**

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to determine which of the following are valid triangle congruence theorems. For those that are not valid, write a counterexample. Explain your reasoning.

<table>
<thead>
<tr>
<th>Possible Congruence Theorem</th>
<th>Valid or not valid?</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSS</td>
<td></td>
</tr>
<tr>
<td>SSA</td>
<td></td>
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<tr>
<td>SAS</td>
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<tr>
<td>AAS</td>
<td></td>
</tr>
<tr>
<td>ASA</td>
<td></td>
</tr>
<tr>
<td>AAA</td>
<td></td>
</tr>
</tbody>
</table>

**Communicate Your Answer**

3. What information is sufficient to determine whether two triangles are congruent?

4. Is it possible to show that two triangles are congruent using more than one congruence theorem? If so, give an example.
5.6 Notetaking with Vocabulary
For use after Lesson 5.6

In your own words, write the meaning of each vocabulary term.

congruent figures

rigid motion

Theorems

Theorem 5.10 Angle-Side-Angle (ASA) Congruence Theorem

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If \( \angle A \cong \angle D \), \( AC \cong DF \), and \( \angle C \cong \angle F \), then \( \triangle ABC \cong \triangle DEF \).

Notes:

Theorem 5.11 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If \( \angle A \cong \angle D \), \( \angle C \cong \angle F \), and \( BC \cong EF \), then \( \triangle ABC \cong \triangle DEF \).

Notes:
Extra Practice

In Exercises 1–4, decide whether enough information is given to prove that the triangles are congruent. If so, state the theorem you would use.

1. \(\triangle GHK, \triangle JKH\)

2. \(\triangle ABC, \triangle DEC\)

3. \(\triangle JKL, \triangle MLK\)

4. \(\triangle RST, \triangle UVW\)

In Exercises 5 and 6, decide whether you can use the given information to prove that \(\triangle L MN \cong \triangle PQR\). Explain your reasoning.

5. \(\angle M \cong \angle Q, \angle N \cong \angle R, \overline{NL} \cong \overline{RP}\)

6. \(\angle L \cong \angle R, \angle M \cong \angle Q, \overline{LM} \cong \overline{PQ}\)
7. Prove that the triangles are congruent using the ASA Congruence Theorem (Theorem 5.10).

**Given** \( \overline{AC} \) bisects \( \angle DAB \) and \( \angle DCB \).

**Prove** \( \triangle ABC \cong \triangle ADC \)

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8. Prove that the triangles are congruent using the AAS Congruence Theorem (Theorem 5.11).

**Given** \( O \) is the center of the circle and \( \angle N \equiv \angle P \).

**Prove** \( \triangle MNO \cong \triangle PQO \)

<table>
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<th>STATEMENTS</th>
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</table>
5.7 Using Congruent Triangles
For use with Exploration 5.7

Essential Question  How can you use congruent triangles to make an indirect measurement?

1 EXPLORATION: Measuring the Width of a River

Work with a partner. The figure shows how a surveyor can measure the width of a river by making measurements on only one side of the river.

a. Study the figure. Then explain how the surveyor can find the width of the river.

b. Write a proof to verify that the method you described in part (a) is valid.

Given  \( \angle A \) is a right angle, \( \angle D \) is a right angle, \( AC \cong CD \)

c. Exchange proofs with your partner and discuss the reasoning used.
EXPLORATION: Measuring the Width of a River

Work with a partner. It was reported that one of Napoleon’s officers estimated the width of a river as follows. The officer stood on the bank of the river and lowered the visor on his cap until the farthest thing visible was the edge of the bank on the other side. He then turned and noted the point on his side that was in line with the tip of his visor and his eye. The officer then paced the distance to this point and concluded that distance was the width of the river.

a. Study the figure. Then explain how the officer concluded that the width of the river is $EG$.

b. Write a proof to verify that the conclusion the officer made is correct.

**Given** $\angle DEG$ is a right angle, $\angle DEF$ is a right angle, $\angle EDG \cong \angle EDF$

3. How can you use congruent triangles to make an indirect measurement?

4. Why do you think the types of measurements described in Explorations 1 and 2 are called indirect measurements?
In your own words, write the meaning of each vocabulary term.

congruent figures

corresponding parts

construction

Notes:
Extra Practice

In Exercises 1–3, explain how to prove that the statement is true.

1. $\overline{UV} \cong \overline{XV}$

2. $\overline{TS} \cong \overline{VR}$

3. $\angle JLK \cong \angle MLN$

In Exercises 4 and 5, write a plan to prove that $\angle 1 \cong \angle 2$. 

4. $F \cong I$
5. Write a proof to verify that the construction is valid.

Ray bisects an angle

**Plan for Proof** Show that \( \triangle ABD \cong \triangle ACD \) by the SSS Congruence Theorem (Thm. 5.8). Use corresponding parts of congruent triangles to show that \( \angle BAD \cong \angle CAD \).

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<th>STATEMENTS</th>
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Essential Question  How can you use a coordinate plane to write a proof?

1 EXPLORATION: Writing a Coordinate Proof

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw \( AB \) with endpoints \( A(0, 0) \) and \( B(6, 0) \).

b. Draw the vertical line \( x = 3 \).

c. Draw \( \triangle ABC \) so that \( C \) lies on the line \( x = 3 \).

d. Use your drawing to prove that \( \triangle ABC \) is an isosceles triangle.

2 EXPLORATION: Writing a Coordinate Proof

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to draw \( AB \) with endpoints \( A(0, 0) \) and \( B(6, 0) \).

b. Draw the vertical line \( x = 3 \).

c. Plot the point \( C(3, 3) \) and draw \( \triangle ABC \). Then use your drawing to prove that \( \triangle ABC \) is an isosceles right triangle.
5.8 Coordinate Proofs (continued)

**EXPLORATION: Writing a Coordinate Proof (continued)**

**Sample**

Points
A(0, 0)
B(6, 0)
C(3, 3)

Segments
AB = 6
BC = 4.24
AC = 4.24

Line
x = 3

---

d. Change the coordinates of C so that C lies below the x-axis and \( \triangle ABC \) is an isosceles right triangle.

e. Write a coordinate proof to show that if C lies on the line \( x = 3 \) and \( \triangle ABC \) is an isosceles right triangle, then C must be the point (3, 3) or the point found in part (d).

**Communicate Your Answer**

3. How can you use a coordinate plane to write a proof?

4. Write a coordinate proof to prove that \( \triangle ABC \) with vertices A(0, 0), B(6, 0), and \( C(3, 3\sqrt{3}) \) is an equilateral triangle.
In your own words, write the meaning of each vocabulary term.

coordinate proof
**Extra Practice**

In Exercises 1 and 2, place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex. Explain the advantages of your placement.

1. an obtuse triangle with height of 3 units and base of 2 units
2. a rectangle with length of $2w$

In Exercises 3 and 4, write a plan for the proof.

3. **Given** Coordinates of vertices of $\triangle OPR$ and $\triangle QRP$
   
   **Proof** $\triangle OPR \cong \triangle QRP$
4. **Given** Coordinates of vertices of \( \triangle OAB \) and \( \triangle CDB \)  
   **Prove** \( B \) is the midpoint of \( AD \) and \( OC \).

5. Graph the triangle with vertices \( A(0, 0) \), \( B(3m, m) \), and \( C(0, 3m) \). Find the length and the slope of each side of the triangle. Then find the coordinates of the midpoint of each side. Is the triangle a right triangle? isosceles? Explain. (Assume all variables are positive.)

6. Write a coordinate proof.
   **Given** Coordinates of vertices of \( \triangle OEF \) and \( \triangle OGF \)  
   **Prove** \( \triangle OEF \cong \triangle OGF \)