5.6 Proving Triangle Congruence by ASA and AAS

For use with Exploration 5.6

Essential Question What information is sufficient to determine whether two triangles are congruent?

1 EXPLORATION: Determining Whether SSA Is Sufficient

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Use dynamic geometry software to construct \( \triangle ABC \). Construct the triangle so that vertex \( B \) is at the origin, \( AB \) has a length of 3 units, and \( BC \) has a length of 2 units.

b. Construct a circle with a radius of 2 units centered at the origin. Locate point \( D \) where the circle intersects \( AC \). Draw \( BD \).

Sample

Points
\( A(0, 3) \)
\( B(0, 0) \)
\( C(2, 0) \)
\( D(0.77, 1.85) \)

Segments
\( AB = 3 \)
\( AC = 3.61 \)
\( BC = 2 \)
\( AD = 1.38 \)

Angle
\( m \angle A = 33.69^\circ \)

c. \( \triangle ABC \) and \( \triangle ABD \) have two congruent sides and a nonincluded congruent angle. Name them.

d. Is \( \triangle ABC \cong \triangle ABD \)? Explain your reasoning.

e. Is SSA sufficient to determine whether two triangles are congruent? Explain your reasoning.
Work with a partner. Use dynamic geometry software to determine which of the following are valid triangle congruence theorems. For those that are not valid, write a counterexample. Explain your reasoning.

<table>
<thead>
<tr>
<th>Possible Congruence Theorem</th>
<th>Valid or not valid?</th>
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<tbody>
<tr>
<td>SSS</td>
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<td>SSA</td>
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<td>SAS</td>
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<td>ASA</td>
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<td>AAA</td>
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Communicate Your Answer

3. What information is sufficient to determine whether two triangles are congruent?

4. Is it possible to show that two triangles are congruent using more than one congruence theorem? If so, give an example.
5.6 Notetaking with Vocabulary
For use after Lesson 5.6

In your own words, write the meaning of each vocabulary term.

congruent figures

text

rigid motion

text

Theorems

Theorem 5.10 Angle-Side-Angle (ASA) Congruence Theorem

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If \( \angle A \cong \angle D, AC \cong DF, \) and \( \angle C \cong \angle F, \) then \( \triangle ABC \cong \triangle DEF. \)

Notes:

Theorem 5.11 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If \( \angle A \cong \angle D, \angle C \cong \angle F, \) and \( BC \cong EF, \) then \( \triangle ABC \cong \triangle DEF. \)

Notes:
Extra Practice

In Exercises 1–4, decide whether enough information is given to prove that the triangles are congruent. If so, state the theorem you would use.

1. \( \triangle GHK, \triangle JKH \)

2. \( \triangle ABC, \triangle DEC \)

3. \( \triangle JKL, \triangle MLK \)

4. \( \triangle RST, \triangle UVW \)

In Exercises 5 and 6, decide whether you can use the given information to prove that \( \triangle LMN \cong \triangle PQR \). Explain your reasoning.

5. \( \angle M \cong \angle Q, \angle N \cong \angle R, \overline{NL} \cong \overline{RP} \)

6. \( \angle L \cong \angle R, \angle M \cong \angle Q, \overline{LM} \cong \overline{PQ} \)
7. Prove that the triangles are congruent using the ASA Congruence Theorem (Theorem 5.10).

**Given** \( AC \) bisects \( \angle DAB \) and \( \angle DCB \).

**Prove** \( \triangle ABC \cong \triangle ADC \)

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8. Prove that the triangles are congruent using the AAS Congruence Theorem (Theorem 5.11).

**Given** \( O \) is the center of the circle and \( \angle N \cong \angle P \).

**Prove** \( \triangle MNO \cong \triangle PQO \)

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