5.3 Proving Triangle Congruence by SAS
For use with Exploration 5.3

Essential Question  What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

1 EXPLORATION: Drawing Triangles

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Construct a $40^\circ$ angle with its vertex at the origin. Label the vertex $A$.

b. Locate the point where one ray of the angle intersects the smaller circle and label this point $B$. Locate the point where the other ray of the angle intersects the larger circle and label this point $C$. Then draw $\triangle ABC$.

c. Find $BC$, $m\angle B$, and $m\angle C$.

d. Repeat parts (a)–(c) several times, redrawing the angle in different positions. Keep track of your results by completing the table on the next page. What can you conclude?
2. What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

3. How would you prove your conclusion in Exploration 1(d)?
In your own words, write the meaning of each vocabulary term.

congruent figures

rigid motion

Theorems

Theorem 5.5 Side-Angle-Side (SAS) Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

Notes:
5.3 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1 and 2, write a proof.

1. Given \( \overline{BD} \perp \overline{AC}, \overline{AD} \cong \overline{CD} \)
   
   Prove \( \triangle ABD \cong \triangle CBD \)

   **STATEMENTS**
   
   **REASONS**

   \[ \overline{BD} \perp \overline{AC} \]
   
   \[ \overline{AD} \cong \overline{CD} \]

   \[ \angle ABD \cong \angle CBD \]
   
   \[ \triangle ABD \cong \triangle CBD \]

2. Given \( \overline{JN} \cong \overline{MN}, \overline{NK} \cong \overline{NL} \)
   
   Prove \( \triangle JNK \cong \triangle MNL \)

   **STATEMENTS**
   
   **REASONS**

   \[ \overline{JN} \cong \overline{MN} \]
   
   \[ \overline{NK} \cong \overline{NL} \]

   \[ \triangle JNK \cong \triangle MNL \]
5.3 Notetaking with Vocabulary (continued)

In Exercises 3 and 4, use the given information to name two triangles that are congruent. Explain your reasoning.

3. $\angle EPF \cong \angle GPH$, and $P$ is the center of the circle.

4. $ABCDEF$ is a regular hexagon.

5. A quilt is made of triangles. You know $\overline{PS} \parallel \overline{QR}$ and $\overline{PS} \cong \overline{QR}$. Use the SAS Congruence Theorem (Theorem 5.5) to show that $\triangle PQR \cong \triangle RSP$. 