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## Congruence and Transformations

For use with Exploration 4.4

## Essential Question What conjectures can you make about a figure reflected in two lines?

1 EXPLORATION: Reflections in Parallel Lines
Go to BigIdeasMath.com for an interactive tool to investigate this exploration.
Work with a partner. Use dynamic geometry software to draw any scalene triangle and label it $\triangle A B C$.
a. Draw any line $\overleftrightarrow{D E}$. Reflect $\triangle A B C$ in $\overleftrightarrow{D E}$ to form $\triangle A^{\prime} B^{\prime} C^{\prime}$.
b. Draw a line parallel to $\stackrel{\rightharpoonup D E}{ }$. Reflect $\triangle A^{\prime} B^{\prime} C^{\prime}$ in the new line to form $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$.
c. Draw the line through point $A$ that is perpendicular to $\overleftrightarrow{D E}$. What do you notice?
d. Find the distance between points $A$ and $A^{\prime \prime}$. Find the distance between the two parallel lines. What do you notice?
e. Hide $\triangle A^{\prime} B^{\prime} C^{\prime}$. Is there a single transformation that maps $\triangle A B C$ to $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$. Explain.
f. Make conjectures based on your answers in parts (c)-(e). Test your conjectures by changing $\triangle A B C$ and the parallel lines.
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4.4 Congruence and Transformations (continued)

2 EXPLORATION: Reflections in Intersecting Lines
Go to BigIdeasMath.com for an interactive tool to investigate this exploration.
Work with a partner. Use dynamic geometry software to draw any scalene triangle and label it $\triangle A B C$.
a. Draw any line $\stackrel{\rightharpoonup D}{D E}$. Reflect $\triangle A B C$ in $\overleftrightarrow{D E}$ to form $\triangle A^{\prime} B^{\prime} C^{\prime}$.
b. Draw any line $\overrightarrow{D F}$ so that $\angle E D F$ is less than or equal to $90^{\circ}$. Reflect $\triangle A^{\prime} B^{\prime} C^{\prime}$ in $\overrightarrow{D F}$ to form $\triangle A^{\prime \prime} B^{\prime \prime} C^{\prime \prime}$.
c. Find the measure of $\angle E D F$. Rotate $\triangle A B C$ counterclockwise about point $D$ twice using the measure of $\angle E D F$.
d. Make a conjecture about a figure reflected in two intersecting lines. Test your conjecture by changing $\triangle A B C$ and the lines.

## Communicate Your Answer

3. What conjectures can you make about a figure reflected in two lines?
4. Point $Q$ is reflected in two parallel lines, $\overleftrightarrow{G H}$ and $\overleftrightarrow{J K}$, to form $Q^{\prime}$ and $Q^{\prime \prime}$. The distance from $\overrightarrow{G H}$ to $\widehat{J K}$ is 3.2 inches. What is the distance $Q Q^{\prime \prime}$ ?
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In your own words, write the meaning of each vocabulary term.
congruent figures
congruence transformation

## Theorems

## Theorem 4.2 Reflections in Parallel Lines Theorem

If lines $k$ and $m$ are parallel, then a reflection in line $k$ followed by a reflection in line $m$ is the same as a translation.

If $A^{\prime \prime}$ is the image of $A$, then

1. $A A^{\prime \prime}$ is perpendicular to $k$ and $m$, and
2. $\quad A A^{\prime \prime}=2 d$, where $d$ is the distance between $k$ and $m$.


Proof Ex. 31. p. 206

Notes:

## Theorem 4.3 Reflections in Intersecting Lines Theorem

If lines $k$ and $m$ intersect at point $P$, then a reflection in line $k$ followed by a reflection in line $m$ is the same as a rotation about point $P$.

The angle of rotation is $2 x^{\circ}$, where $x^{\circ}$ is the measure of the acute or right angle formed by lines $k$ and $m$.

Proof Ex. 31. p. 206


Notes:
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### 4.4 Notetaking with Vocabulary (continued)

## Extra Practice

1. Identify any congruent figures in the coordinate plane. Explain.

2. Describe a congruence transformation that maps $\triangle P Q R$ to $\triangle S T U$.

3. Describe a congruence transformation that maps polygon $A B C D$ to polygon $E F G H$.


Name $\qquad$

### 4.4 Notetaking with Vocabulary (continued)

In Exercises 4 and 5, determine whether the polygons with the given vertices are congruent. Use transformations to explain your reasoning.
4. $A(2,2), B(3,1), C(1,1)$ and
$D(2,-2), E(3,-1), F(1,-1)$

5. $G(3,3), H(2,1), I(6,2), J(6,3)$ and

$$
K(2,-1), L(-3,-3), M(2,-2), N(2,-1)
$$



In Exercises 6-9, $k \| m, \overline{U V}$ is reflected in line $k$, and $\overline{U^{\prime} V^{\prime}}$ is reflected in line $m$.
6. A translation maps $\overline{U V}$ onto which segment?
7. Which lines are perpendicular to $\overline{U U^{\prime \prime}}$ ?
8. Why is $V^{\prime \prime}$ the image of $V$ ? Explain your reasoning.

9. If the distance between $k$ and $m$ is 5 inches, what is the length of $\overline{V V^{\prime \prime}}$ ?
10. What is the angle of rotation that maps $A$ onto $A^{\prime \prime}$ ?


