

**Chapter  
2****Maintaining Mathematical Proficiency**

Write an equation for the  $n$ th term of the arithmetic sequence. Then find  $a_{20}$ .

1. 5, 11, 17, 23, ...

2. 22, 34, 46, 58, ...

3. -13, 0, 13, 26, ...

4. -4.5, -4.0, -3.5, -3.0, ...

5. 40, 25, 10, -5, ...

6.  $-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$

Solve the literal equation for  $x$ .

7.  $3x - 9y = 12$

8.  $16y - 4x = 40$

9.  $6x + 5 = 30y - 7$

10.  $2x - y = 11x - 18$

11.  $10y = 2x + 3zx + 1$

12.  $14z = 2x + 4xy$

**2.1****Conditional Statements**

For use with Exploration 2.1

**Essential Question** When is a conditional statement true or false?

A conditional statement, symbolized by  $p \rightarrow q$ , can be written as an “if-then statement” in which  $p$  is the *hypothesis* and  $q$  is the *conclusion*. Here is an example.

If a polygon is a triangle, then the sum of its angle measures is  $180^\circ$ .  
 hypothesis,  $p$  conclusion,  $q$

**1 EXPLORATION: Determining Whether a Statement Is True or False**

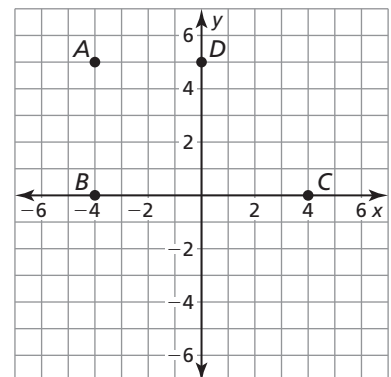
**Work with a partner.** A hypothesis can either be true or false. The same is true of a conclusion. For a conditional statement to be true, the hypothesis and conclusion do not necessarily both have to be true. Determine whether each conditional statement is true or false. Justify your answer.

- a. If yesterday was Wednesday, then today is Thursday.
- b. If an angle is acute, then it has a measure of  $30^\circ$ .
- c. If a month has 30 days, then it is June.
- d. If an even number is not divisible by 2, then 9 is a perfect cube.

**2 EXPLORATION: Determining Whether a Statement Is True or False?**

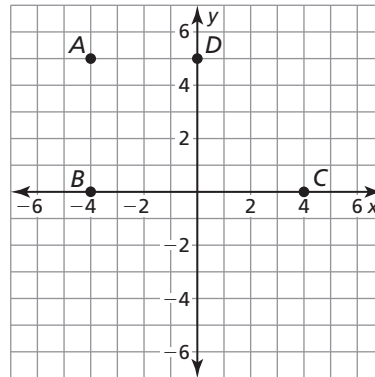
**Work with a partner.** Use the points in the coordinate plane to determine whether each statement is true or false. Justify your answer.

- a.  $\triangle ABC$  is a right triangle.



**2.1 Conditional Statements (continued)****2 EXPLORATION:** Determining Whether a Statement Is True or False (continued)

- b.  $\triangle BDC$  is an equilateral triangle.
- c.  $\triangle BDC$  is an isosceles triangle.
- d. Quadrilateral  $ABCD$  is a trapezoid.
- e. Quadrilateral  $ABCD$  is a parallelogram.

**3 EXPLORATION:** Determining Whether a Statement Is True or False

**Work with a partner.** Determine whether each conditional statement is true or false. Justify your answer.

- a. If  $\triangle ADC$  is a right triangle, then the Pythagorean Theorem is valid for  $\triangle ADC$ .
- b. If  $\angle A$  and  $\angle B$  are complementary, then the sum of their measures is  $180^\circ$ .
- c. If figure  $ABCD$  is a quadrilateral, then the sum of its angle measures is  $180^\circ$ .
- d. If points  $A$ ,  $B$ , and  $C$  are collinear, then they lie on the same line.
- e. If  $\overline{AB}$  and  $\overline{BD}$  intersect at a point, then they form two pairs of vertical angles.

**Communicate Your Answer**

4. When is a conditional statement true or false?
5. Write one true conditional statement and one false conditional statement that are different from those given in Exploration 3. Justify your answer.

**2.1****Notetaking with Vocabulary**

For use after Lesson 2.1

In your own words, write the meaning of each vocabulary term.

conditional statement

if-then form

hypothesis

conclusion

negation

converse

inverse

contrapositive

equivalent statements

perpendicular lines

biconditional statement

truth value

truth table

**2.1** Notetaking with Vocabulary (continued)**Core Concepts****Conditional Statement**

A **conditional statement** is a logical statement that has two parts, a *hypothesis*  $p$  and a *conclusion*  $q$ . When a conditional statement is written in **if-then form**, the “if” part contains the **hypothesis** and the “then” part contains the **conclusion**.

**Words** If  $p$ , then  $q$ .      **Symbols**  $p \rightarrow q$  (read as “ $p$  implies  $q$ ”)

**Notes:**

**Negation**

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement  $p$ , you write the symbol for negation ( $\sim$ ) before the letter. So, “not  $p$ ” is written  $\sim p$ .

**Words** not  $p$       **Symbols**  $\sim p$

**Notes:**

**Related Conditionals**

Consider the conditional statement below.

**Words** If  $p$ , then  $q$ .      **Symbols**  $p \rightarrow q$

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**Converse** To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

**Words** If  $q$ , then  $p$ .      **Symbols**  $q \rightarrow p$

---

**Inverse** To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

**Words** If not  $p$ , then not  $q$ .      **Symbols**  $\sim p \rightarrow \sim q$

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**2.1** Notetaking with Vocabulary (continued)**Related Conditionals (continued)**

**Contrapositive** To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

**Words** If not  $q$ , then not  $p$ .      **Symbols**  $\sim q \rightarrow \sim p$

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A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

**Notes:****Biconditional Statement**

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase “if and only if.”

**Words**  $p$  if and only if  $q$       **Symbols**  $p \leftrightarrow q$

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Any definition can be written as a biconditional statement.

**Notes:**

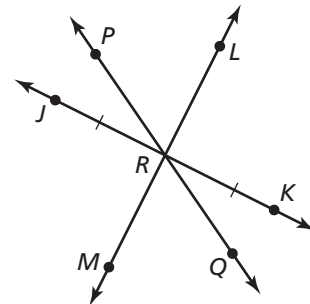
**2.1** Notetaking with Vocabulary (continued)

**Extra Practice**

In Exercises 1 and 2, rewrite the conditional statement in if-then form.

1.  $13x - 5 = -18$ , because  $x = -1$ .
  
2. The sum of the measures of interior angles of a triangle is  $180^\circ$ .
  
3. Let  $p$  be “Quadrilateral  $ABCD$  is a rectangle” and let  $q$  be “the sum of the angle measures is  $360^\circ$ .” Write the conditional statement  $p \rightarrow q$ , the converse  $q \rightarrow p$ , the inverse  $\sim p \rightarrow \sim q$ , and the contrapositive  $\sim q \rightarrow \sim p$  in words. Then decide whether each statement is true or false.

In Exercises 4–6, decide whether the statement about the diagram is true. Explain your answer using the definitions you have learned.



4.  $\overline{LM}$  bisects  $\overline{JK}$
  
5.  $\angle JRP$  and  $\angle PRL$  are complementary.
  
6.  $\angle MRQ \cong \angle PRL$

# 2.2

## Inductive and Deductive Reasoning

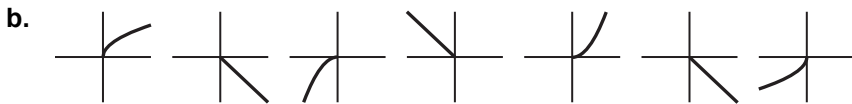
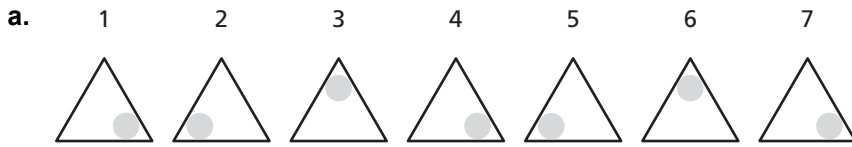
For use with Exploration 2.2

**Essential Question** How can you use reasoning to solve problems?

A **conjecture** is an unproven statement based on observations.

### 1 EXPLORATION: Writing a Conjecture

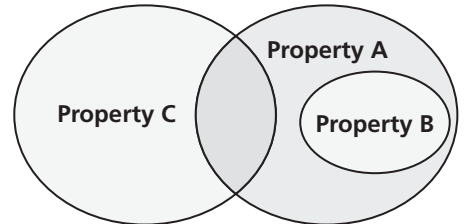
**Work with a partner.** Write a conjecture about the pattern. Then use your conjecture to draw the 10th object in the pattern.



### 2 EXPLORATION: Using a Venn Diagram

**Work with a partner.** Use the Venn diagram to determine whether the statement is true or false. Justify your answer. Assume that no region of the Venn diagram is empty.

- a. If an item has Property B, then it has Property A.
- b. If an item has Property A, then it has Property B.





**2.2 Inductive and Deductive Reasoning (continued)****2 EXPLORATION:** Using a Venn Diagram (continued)

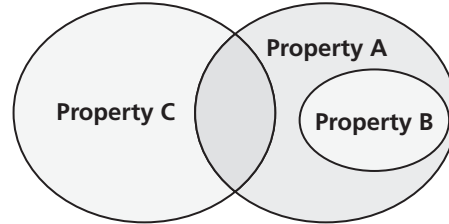
c. If an item has Property A, then it has Property C.

d. Some items that have Property A do not have Property B.

e. If an item has Property C, then it does not have Property B.

f. Some items have both Properties A and C.

g. Some items have both Properties B and C.

**3 EXPLORATION:** Reasoning and Venn Diagrams

**Work with a partner.** Draw a Venn diagram that shows the relationship between different types of quadrilaterals: squares, rectangles, parallelograms, trapezoids, rhombuses, and kites. Then write several conditional statements that are shown in your diagram, such as “If a quadrilateral is a square, then it is a rectangle.”

**Communicate Your Answer**

4. How can you use reasoning to solve problems?
5. Give an example of how you used reasoning to solve a real-life problem.

**2.2****Notetaking with Vocabulary**

For use after Lesson 2.2

In your own words, write the meaning of each vocabulary term.

conjecture

inductive reasoning

counterexample

deductive reasoning

**Core Concepts****Inductive Reasoning**

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

**Notes:****Counterexample**

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

**Notes:**

**2.2** Notetaking with Vocabulary (continued)

**Deductive Reasoning**

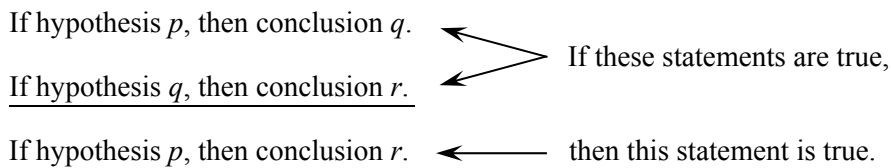
**Deductive reasoning** uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

**Laws of Logic**

**Law of Detachment**

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

**Law of Syllogism**



**Notes:**

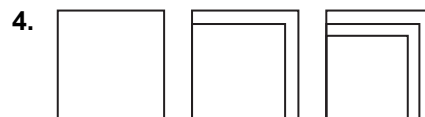
**Extra Practice**

In Exercises 1–4, describe the pattern. Then write or draw the next two numbers, letters, or figures.

1. 20, 19, 17, 14, 10, ...

2. 2, -3, 5, -7, 11, ...

3. C, E, G, I, K, ...



**2.2** Notetaking with Vocabulary (continued)

In Exercises 5 and 6, make and test a conjecture about the given quantity.

5. the sum of two negative integers
  
  
  
  
  
6. the product of three consecutive nonzero integers

In Exercises 7 and 8, find a counterexample to show that the conjecture is false.

7. If  $n$  is a rational number, then  $n^2$  is always less than  $n$ .
  
  
  
  
  
8. Line  $k$  intersects plane  $P$  at point  $Q$  on the plane. Plane  $P$  is perpendicular to line  $k$ .

In Exercises 9 and 10, use the Law of Detachment to determine what you can conclude from the given information, if possible.

9. If a triangle has equal side lengths, then each interior angle measure is  $60^\circ$ .  $\triangle ABC$  has equal side lengths.
  
  
  
  
  
10. If a quadrilateral is a rhombus, then it has two pairs of opposite sides that are parallel. Quadrilateral  $PQRS$  has two pairs of opposite sides that are parallel.

In Exercises 11 and 12, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements, if possible.

11. If it does not rain, then I will walk to school.  
If I walk to school, then I will wear my walking shoes.
  
  
  
  
  
12. If  $x > 1$ , then  $3x > 3$ .  
If  $3x > 3$ , then  $(3x)^2 > 9$ .

# 2.3

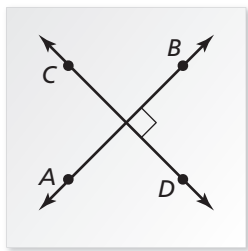
## Postulates and Diagrams

For use with Exploration 2.3

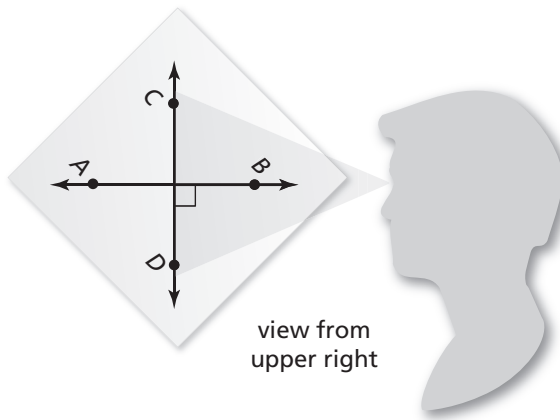
**Essential Question** In a diagram, what can be assumed and what needs to be labeled?

**1 EXPLORATION:** Looking at a Diagram

**Work with a partner.** On a piece of paper, draw two perpendicular lines. Label them  $\overline{AB}$  and  $\overline{CD}$ . Look at the diagram from different angles. Do the lines appear perpendicular regardless of the angle at which you look at them? Describe *all* the angles at which you can look at the lines and have them appear perpendicular.



view from above

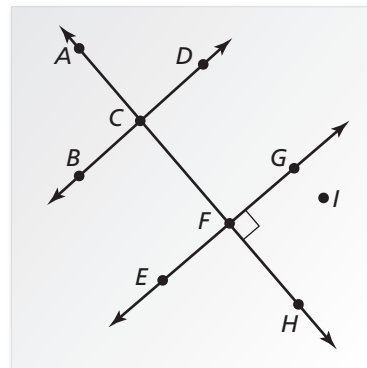


view from upper right

**2 EXPLORATION:** Interpreting a Diagram

**Work with a partner.** When you draw a diagram, you are communicating with others. It is important that you include sufficient information in the diagram. Use the diagram to determine which of the following statements you can assume to be true. Explain your reasoning.

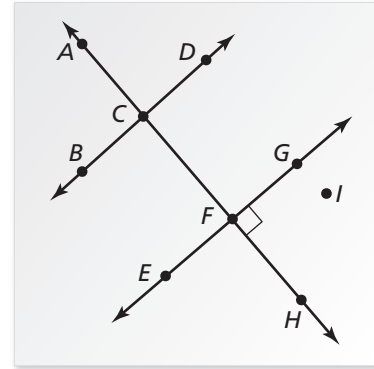
- a. All the points shown are coplanar.
- b. Points  $D$ ,  $G$ , and  $I$  are collinear.
- c. Points  $A$ ,  $C$ , and  $H$  are collinear.
- d.  $\overline{EG}$  and  $\overline{AH}$  are perpendicular.



**2.3 Postulates and Diagrams (continued)**

**2 EXPLORATION: Interpreting a Diagram (continued)**

- e.  $\angle BCA$  and  $\angle ACD$  are a linear pair.
- f.  $\overleftrightarrow{AF}$  and  $\overleftrightarrow{BD}$  are perpendicular.
- g.  $\overleftrightarrow{EG}$  and  $\overleftrightarrow{BD}$  are parallel.
- h.  $\overleftrightarrow{AF}$  and  $\overleftrightarrow{BD}$  are coplanar.
- i.  $\overleftrightarrow{EG}$  and  $\overleftrightarrow{BD}$  do not intersect.
- j.  $\overleftrightarrow{AF}$  and  $\overleftrightarrow{BD}$  intersect.
- k.  $\overleftrightarrow{EG}$  and  $\overleftrightarrow{BD}$  are perpendicular.
- l.  $\angle ACD$  and  $\angle BCF$  are vertical angles.
- m.  $\overleftrightarrow{AC}$  and  $\overleftrightarrow{FH}$  are the same line.



**Communicate Your Answer**

- 3. In a diagram, what can be assumed and what needs to be labeled?
- 4. Use the diagram in Exploration 2 to write two statements you can assume to be true and two statements you cannot assume to be true. Your statements should be different from those given in Exploration 2. Explain your reasoning.

# 2.3

## Notetaking with Vocabulary

For use after Lesson 2.3

In your own words, write the meaning of each vocabulary term.

line perpendicular to a plane

### Postulates

#### Point, Line, and Plane Postulates

##### Postulate

##### Example

##### 2.1 Two Point Postulate

Through any two points, there exists exactly one line.



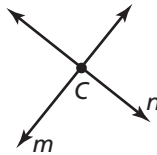
Through points  $A$  and  $B$ , there is exactly one line  $\ell$ . Line  $\ell$  contains at least two points.

##### 2.2 Line-Point Postulate

A line contains at least two points.

##### 2.3 Line Intersection Postulate

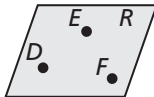
If two lines intersect, then their intersection is exactly one point.



The intersection of line  $m$  and line  $n$  is point  $C$ .

##### 2.4 Three Point Postulate

Through any three noncollinear points, there exists exactly one plane.



Through points  $D$ ,  $E$ , and  $F$ , there is exactly one plane, plane  $R$ . Plane  $R$  contains at least three noncollinear points.

##### 2.5 Plane-Point Postulate

A plane contains at least three noncollinear points

### Notes:

**2.3** Notetaking with Vocabulary (continued)

**Point, Line, and Plane Postulates (continued)**

**Postulate**

**Example**

**2.6 Plane-Line Postulate**

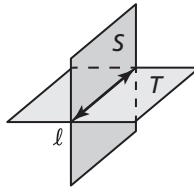
If two points lie in a plane, then the line containing them lies in the plane.



Points  $D$  and  $E$  lie in plane  $R$ , so  $\overline{DE}$  lies in plane  $R$ .

**2.7 Plane Intersection Postulate**

If two planes intersect, then their intersection is a line.



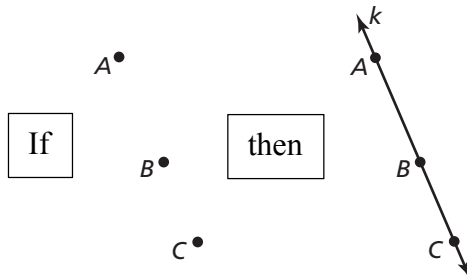
The intersection of plane  $S$  and plane  $T$  is line  $l$ .

**Notes:**

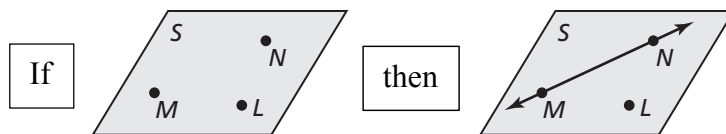
**Extra Practice**

In Exercises 1 and 2, state the postulate illustrated by the diagram.

1.



2.

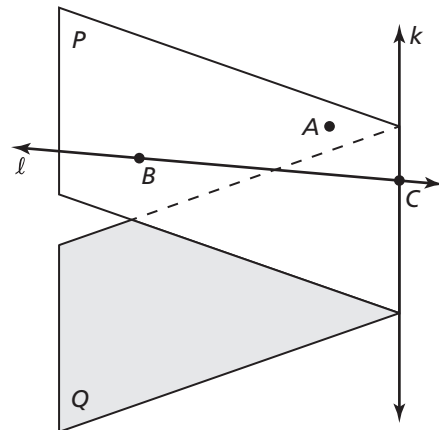




**2.3** Notetaking with Vocabulary (continued)

In Exercises 3–6, use the diagram to write an example of the postulate.

3. Two Point Postulate (Postulate 2.1)
4. Line-Point Postulate (Postulate 2.2)
5. Plane-Point Postulate (Postulate 2.5)
6. Plane Intersection Postulate (Postulate 2.7)

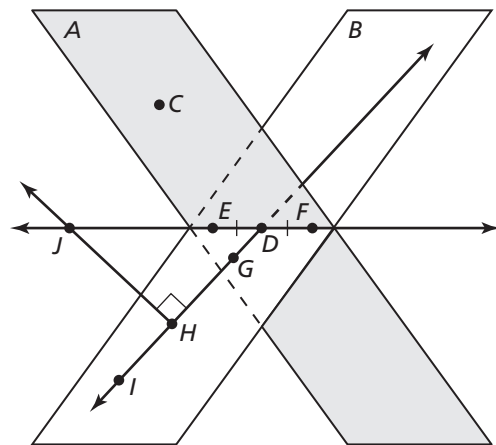


In Exercises 7 and 8, sketch a diagram of the description.

7.  $\overline{RS}$  bisecting  $\overline{KL}$  at point  $R$
8.  $\overline{AB}$  in plane  $U$  intersecting  $\overline{CD}$  at point  $E$ , and point  $C$  not on plane  $U$

In Exercises 9–14, use the diagram to determine whether you can assume the statement.

9. Planes  $A$  and  $B$  intersect at  $\overline{EF}$ .
10. Points  $C$  and  $D$  are collinear.
11.  $\overline{HJ}$  and  $\overline{ID}$  are perpendicular.
12.  $\overline{GD}$  is a bisector of  $\overline{EF}$  at point  $D$ .
13.  $\overline{IH} \cong \overline{HG}$
14.  $\angle HJD$  and  $\angle HDJ$  are complementary angles.



**2.4****Algebraic Reasoning**

For use with Exploration 2.4

**Essential Question** How can algebraic properties help you solve an equation?

**1 EXPLORATION: Justifying Steps in a Solution**

**Work with a partner.** In previous courses, you studied different properties, such as the properties of equality and the Distributive, Commutative, and Associative Properties. Write the property that justifies each of the following solution steps.

Algebraic Step	Justification
$2(x + 3) - 5 = 5x + 4$	Write given equation.
$2x + 6 - 5 = 5x + 4$	_____
$2x + 1 = 5x + 4$	_____
$2x - 2x + 1 = 5x - 2x + 4$	_____
$1 = 3x + 4$	_____
$1 - 4 = 3x + 4 - 4$	_____
$-3 = 3x$	_____
$\frac{-3}{3} = \frac{3x}{3}$	_____
$-1 = x$	_____
$x = -1$	_____

**2.4 Algebraic Reasoning (continued)****2 EXPLORATION: Stating Algebraic Properties**

**Work with a partner.** The symbols  $\blacklozenge$  and  $\bullet$  represent addition and multiplication (not necessarily in that order). Determine which symbol represents which operation. Justify your answer. Then state each algebraic property being illustrated.

**Example of Property****Name of Property**

$5 \blacklozenge 6 = 6 \blacklozenge 5$

\_\_\_\_\_

$5 \bullet 6 = 6 \bullet 5$

\_\_\_\_\_

$4 \blacklozenge (5 \blacklozenge 6) = (4 \blacklozenge 5) \blacklozenge 6$

\_\_\_\_\_

$4 \bullet (5 \bullet 6) = (4 \bullet 5) \bullet 6$

\_\_\_\_\_

$0 \blacklozenge 5 = 0$

\_\_\_\_\_

$0 \bullet 5 = 5$

\_\_\_\_\_

$1 \blacklozenge 5 = 5$

\_\_\_\_\_

$4 \blacklozenge (5 \bullet 6) = 4 \blacklozenge 5 \bullet 4 \blacklozenge 6$

\_\_\_\_\_

**Communicate Your Answer**

- How can algebraic properties help you solve an equation?
- Solve  $3(x + 1) - 1 = -13$ . Justify each step.

**2.4****Notetaking with Vocabulary**

For use after Lesson 2.4

In your own words, write the meaning of each vocabulary term.

equation

solve an equation

formula

**Core Concepts****Algebraic Properties of Equality**

Let  $a$ ,  $b$ , and  $c$  be real numbers.

**Addition Property of Equality**      If  $a = b$ , then  $a + c = b + c$ .

**Subtraction Property of Equality**      If  $a = b$ , then  $a - c = b - c$ .

**Multiplication Property of Equality**      If  $a = b$ , then  $a \cdot c = b \cdot c$ ,  $c \neq 0$ .

**Division Property of Equality**      If  $a = b$ , then  $\frac{a}{c} = \frac{b}{c}$ ,  $c \neq 0$ .

**Substitution Property of Equality**      If  $a = b$ , then  $a$  can be substituted for  $b$  (or  $b$  for  $a$ ) in any equation or expression.

**Notes:**

**2.4** Notetaking with Vocabulary (continued)**Distributive Property**Let  $a$ ,  $b$ , and  $c$  be real numbers.

**Sum**  $a(b + c) = ab + ac$

**Difference**  $a(b - c) = ab - ac$

**Notes:****Reflexive, Symmetric, and Transitive Properties of Equality**

	<b>Real Numbers</b>	<b>Segment Lengths</b>	<b>Angle Measures</b>
<b>Reflexive Property</b>	$a = a$	$AB = AB$	$m\angle A = m\angle A$
<b>Symmetric Property</b>	If $a = b$ , then $b = a$ .	If $AB = CD$ , then $CD = AB$ .	If $m\angle A = m\angle B$ , then $m\angle B = m\angle A$ .
<b>Transitive Property</b>	If $a = b$ and $b = c$ , then $a = c$ .	If $AB = CD$ and $CD = EF$ , then $AB = EF$ .	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$ , then $m\angle A = m\angle C$ .

**Notes:**

**2.4** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–3, solve the equation. Justify each step.

1.  $3x - 7 = 5x - 19$

2.  $20 - 2(3x - 1) = x - 6$

3.  $-5(2u + 10) = 2(u - 7)$

In Exercises 4 and 5, solve the equation for the given variable. Justify each step.

4.  $9x + 2y = 5; y$

5.  $\frac{1}{15}s - \frac{2}{3}t = -2; s$

6. The formula for the surface area  $S$  of a cone is  $S = \pi r^2 + \pi rs$ , where  $r$  is the radius and  $s$  is the slant height. Solve the formula for  $s$ . Justify each step. Then find the slant height of the cone when the surface area is 113 square feet and the radius is 4 feet. Approximate to the nearest tenth.

# 2.5

## Proving Statements about Segments and Angles

For use with Exploration 2.5

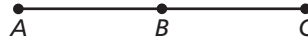
### Essential Question How can you prove a mathematical statement?

A **proof** is a logical argument that uses deductive reasoning to show that a statement is true.

**1 EXPLORATION: Writing Reasons in a Proof**

**Work with a partner.** Four steps of a proof are shown. Write the reasons for each statement.

**Given**  $AC = AB + AB$



**Prove**  $AB = BC$

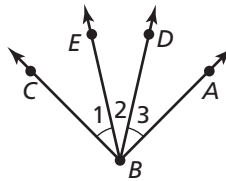
STATEMENTS	REASONS
1. $AC = AB + AB$	1. Given
2. $AB + BC = AC$	2. _____
3. $AB + AB = AB + BC$	3. _____
4. $AB = BC$	4. _____

**2 EXPLORATION: Writing Steps in a Proof**

**Work with a partner.** Six steps of a proof are shown. Complete the statements that correspond to each reason.

**Given**  $m\angle 1 = m\angle 3$

**Prove**  $m\angle EBA = m\angle CBD$



**2.5 Proving Statements about Segments and Angles (continued)**

**2 EXPLORATION: Writing Steps in a Proof (continued)**

STATEMENTS	REASONS
1. _____	1. Given
2. $m\angle EBA = m\angle 2 + m\angle 3$	2. Angle Addition Postulate (Post. 1.4)
3. $m\angle EBA = m\angle 2 + m\angle 1$	3. Substitution Property of Equality
4. $m\angle EBA =$ _____	4. Commutative Property of Addition
5. $m\angle 1 + m\angle 2 =$ _____	5. Angle Addition Postulate (Post. 1.4)
6. _____	6. Transitive Property of Equality

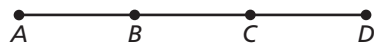
**Communicate Your Answer**

3. How can you prove a mathematical statement?

4. Use the given information and the figure to write a proof for the statement.

**Given**  $B$  is the midpoint of  $\overline{AC}$ .

$C$  is the midpoint of  $\overline{BD}$ .



**Prove**  $AB = CD$



**2.5****Notetaking with Vocabulary**

For use after Lesson 2.5

In your own words, write the meaning of each vocabulary term.

proof

two-column proof

theorem

**Theorems****Theorem 2.1 Properties of Segment Congruence**

Segment congruence is reflexive, symmetric, and transitive.

**Reflexive**For any segment  $AB$ ,  $\overline{AB} \cong \overline{AB}$ .**Symmetric**If  $\overline{AB} \cong \overline{CD}$ , then  $\overline{CD} \cong \overline{AB}$ .**Transitive**If  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{EF}$ , then  $\overline{AB} \cong \overline{EF}$ .**Theorem 2.2 Properties of Angle Congruence**

Angle congruence is reflexive, symmetric, and transitive.

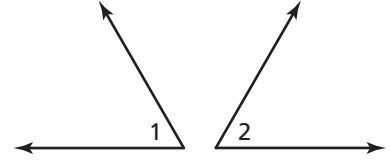
**Reflexive**For any angle  $A$ ,  $\angle A \cong \angle A$ .**Symmetric**If  $\angle A \cong \angle B$ , then  $\angle B \cong \angle A$ .**Transitive**If  $\angle A \cong \angle B$  and  $\angle B \cong \angle C$ , then  $\angle A \cong \angle C$ .**Notes:**

**2.5 Notetaking with Vocabulary (continued)**

**Core Concepts**

**Writing a Two-Column Proof**

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.



Copy or draw diagrams and label given information to help develop proofs. Do not mark or label the information in the Prove statement on the diagram.

**Proof of the Symmetric Property of Angle Congruence**

**Given**  $\angle 1 \cong \angle 2$       **Prove**  $\angle 2 \cong \angle 1$

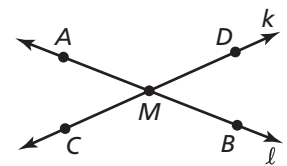
	STATEMENTS	REASONS	
statements based on facts that you know or on conclusions from deductive reasoning	<ol style="list-style-type: none"> <li>1. <math>\angle 1 \cong \angle 2</math></li> <li>2. <math>m\angle 1 = m\angle 2</math></li> <li>3. <math>m\angle 2 = m\angle 1</math></li> <li>4. <math>\angle 2 \cong \angle 1</math></li> </ol>	<ol style="list-style-type: none"> <li>1. Given</li> <li>2. Definition of congruent angles</li> <li>3. Symmetric Property of Equality</li> <li>4. Definition of congruent angles</li> </ol>	definitions, postulates, or proven theorems that allow you to state the corresponding statement
	The number of statements will vary.	Remember to give a reason for the last statement.	

**Notes:**

**Extra Practice**

In Exercises 1 and 2, complete the proof.

1. **Given**  $\overline{AB}$  and  $\overline{CD}$  bisect each other at point  $M$  and  $\overline{BM} \cong \overline{CM}$ .  
**Prove**  $AB = AM + DM$

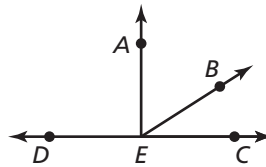


STATEMENTS	REASONS
1. $\overline{BM} \cong \overline{CM}$	1. Given
2. $\overline{CM} \cong \overline{DM}$	2. _____
3. $\overline{BM} \cong \overline{DM}$	3. _____
4. $BM = DM$	4. _____
5. _____	5. Segment Addition Postulate (Post. 1.2)
6. $AB = AM + DM$	6. _____

**2.5** Notetaking with Vocabulary (continued)

2. **Given**  $\angle AEB$  is a complement of  $\angle BEC$ .

**Prove**  $m\angle AED = 90^\circ$



STATEMENTS	REASONS
1. $\angle AEB$ is a complement of $\angle BEC$ .	1. Given
2. _____	2. Definition of complementary angles
3. $m\angle AEC = m\angle AEB + m\angle BEC$	3. _____
4. $m\angle AEC = 90^\circ$	4. _____
5. $m\angle AED + m\angle AEC = 180^\circ$	5. Definition of supplementary angles
6. _____	6. Substitution Property of Equality
7. $m\angle AED = 90^\circ$	7. _____

In Exercises 3 and 4, name the property that the statement illustrates.

3. If  $\angle RST \cong \angle TSU$  and  $\angle TSU \cong \angle VWX$ , then  $\angle RST \cong \angle VWX$ .

4. If  $\overline{GH} \cong \overline{JK}$ , then  $\overline{JK} \cong \overline{GH}$ .

5. Write a two-column proof.

**Given**  $M$  is the midpoint of  $\overline{RT}$ .

**Prove**  $MT = RS + SM$



STATEMENTS	REASONS

# 2.6

## Proving Geometric Relationships

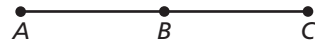
For use with Exploration 2.6

**Essential Question** How can you use a flowchart to prove a mathematical statement?

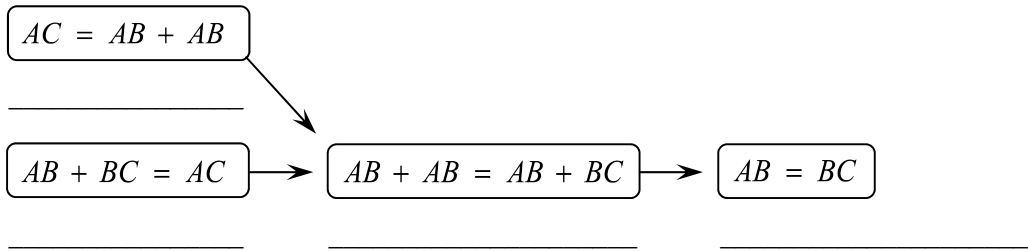
### 1 EXPLORATION: Matching Reasons in a Flowchart Proof

**Work with a partner.** Match each reason with the correct step in the flowchart.

**Given**  $AC = AB + AB$



**Prove**  $AB = BC$



- A. Segment Addition Postulate (Post. 1.2)
- B. Given
- C. Transitive Property of Equality
- D. Subtraction Property of Equality

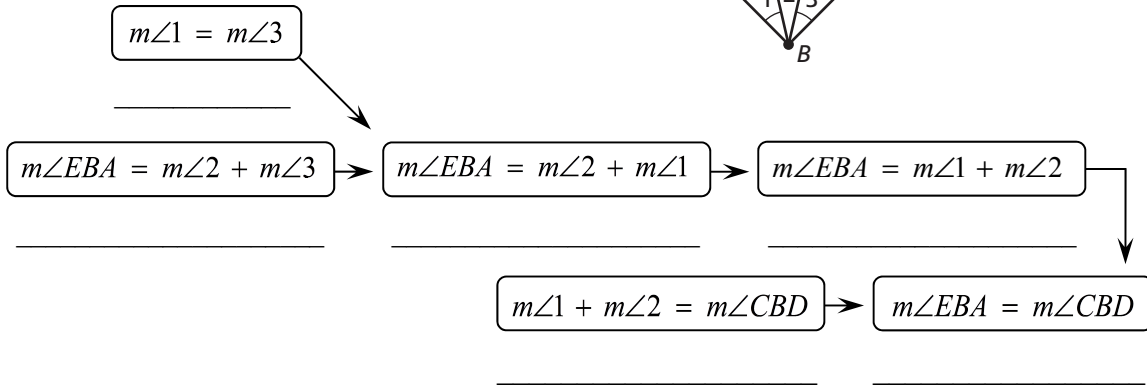
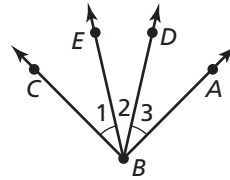
**2.6 Proving Geometric Relationships (continued)**

**2 EXPLORATION: Matching Reasons in a Flowchart Proof**

Work with a partner. Match each reason with the correct step in the flowchart.

Given  $m\angle 1 = m\angle 3$

Prove  $m\angle EBA = m\angle CBD$



- |   |   |
|---|---|
| A. Angle Addition Postulate (Post. 1.4) | B. Transitive Property of Equality      |
| C. Substitution Property of Equality    | D. Angle Addition Postulate (Post. 1.4) |
| E. Given                                | F. Commutative Property of Addition     |

**Communicate Your Answer**

- How can you use a flowchart to prove a mathematical statement?
- Compare the flowchart proofs above with the two-column proofs in the Section 2.5 Explorations. Explain the advantages and disadvantages of each.

**2.6****Notetaking with Vocabulary**

For use after Lesson 2.6

In your own words, write the meaning of each vocabulary term.

flowchart proof, or flow proof

paragraph proof

**Theorems and Postulates****Theorem 2.3 Right Angles Congruence Theorem**

All right angles are congruent.

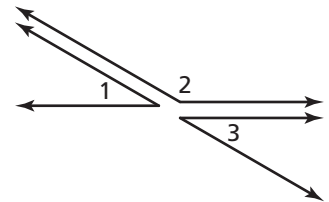
**Notes:**

**Theorem 2.4 Congruent Supplements Theorem**

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If  $\angle 1$  and  $\angle 2$  are supplementary and  $\angle 3$  and  $\angle 2$  are supplementary, then  $\angle 1 \cong \angle 3$ .

**Notes:**

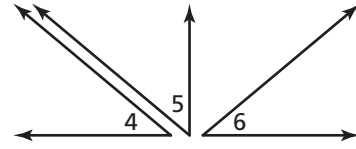


**2.6 Notetaking with Vocabulary (continued)**

**Theorem 2.5 Congruent Complements Theorem**

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If  $\angle 4$  and  $\angle 5$  are complementary and  $\angle 6$  and  $\angle 5$  are complementary, then  $\angle 4 \cong \angle 6$ .

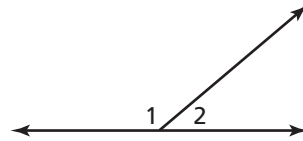


**Notes:**

**Postulate 2.8 Linear Pair Postulate**

If two angles form a linear pair, then they are supplementary.

$\angle 1$  and  $\angle 2$  form a linear pair, so  $\angle 1$  and  $\angle 2$  are supplementary and  $m\angle 1 + m\angle 2 = 180^\circ$ .

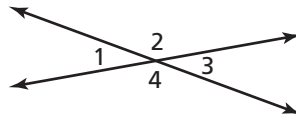


**Notes:**

**Theorem 2.6 Vertical Angles Congruence Theorem**

Vertical angles are congruent.

**Notes:**



$$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$$

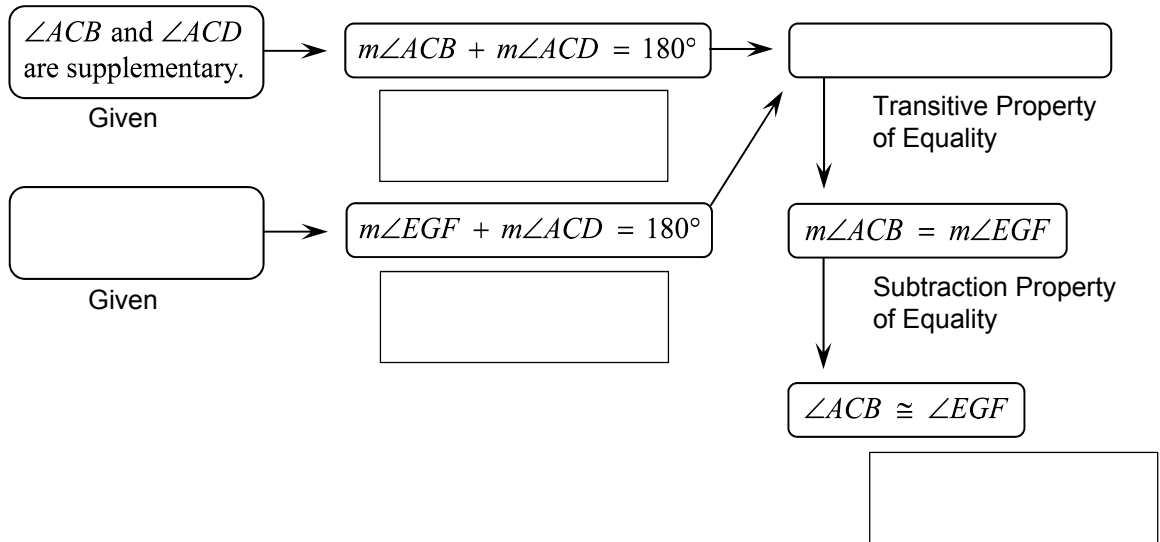
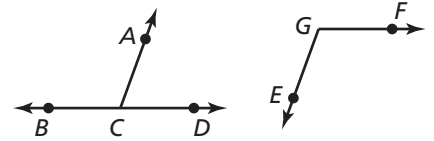
**2.6** Notetaking with Vocabulary (continued)

**Extra Practice**

1. Complete the flowchart proof. Then write a two-column proof.

**Given**  $\angle ACB$  and  $\angle ACD$  are supplementary.  
 $\angle EGF$  and  $\angle ACD$  are supplementary.

**Prove**  $\angle ACB \cong \angle EGF$



Two-Column Proof

STATEMENTS	REASONS