Write an equation for the \(n\)th term of the arithmetic sequence. Then find \(a_{20}\).

1. 5, 11, 17, 23, ...
2. 22, 34, 46, 58, ...
3. -13, 0, 13, 26, ...
4. -4.5, -4.0, -3.5, -3.0, ...
5. 40, 25, 10, -5, ...
6. \(-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\)

Solve the literal equation for \(x\).

7. \(3x - 9y = 12\)
8. \(16y - 4x = 40\)
9. \(6x + 5 = 30y - 7\)
10. \(2x - y = 11x - 18\)
11. \(10y = 2x + 3zx + 1\)
12. \(14z = 2x + 4xy\)
Essential Question  When is a conditional statement true or false?

A conditional statement, symbolized by $p \rightarrow q$, can be written as an “if-then statement” in which $p$ is the hypothesis and $q$ is the conclusion. Here is an example.

If a polygon is a triangle, then the sum of its angle measures is $180^\circ$.

Work with a partner. A hypothesis can either be true or false. The same is true of a conclusion. For a conditional statement to be true, the hypothesis and conclusion do not necessarily both have to be true. Determine whether each conditional statement is true or false. Justify your answer.

a. If yesterday was Wednesday, then today is Thursday.

b. If an angle is acute, then it has a measure of $30^\circ$.

c. If a month has 30 days, then it is June.

d. If an even number is not divisible by 2, then 9 is a perfect cube.

Work with a partner. Use the points in the coordinate plane to determine whether each statement is true or false. Justify your answer.

a. $\triangle ABC$ is a right triangle.
2.1 Conditional Statements (continued)

2 EXPLORATION: Determining Whether a Statement Is True or False (continued)

b. $\triangle BDC$ is an equilateral triangle.

c. $\triangle BDC$ is an isosceles triangle.

d. Quadrilateral $ABCD$ is a trapezoid.

e. Quadrilateral $ABCD$ is a parallelogram.

3 EXPLORATION: Determining Whether a Statement Is True or False

Work with a partner. Determine whether each conditional statement is true or false. Justify your answer.

a. If $\triangle ADC$ is a right triangle, then the Pythagorean Theorem is valid for $\triangle ADC$.

b. If $\angle A$ and $\angle B$ are complementary, then the sum of their measures is $180^\circ$.

c. If figure $ABCD$ is a quadrilateral, then the sum of its angle measures is $180^\circ$.

d. If points $A$, $B$, and $C$ are collinear, then they lie on the same line.

e. If $\overline{AB}$ and $\overline{BD}$ intersect at a point, then they form two pairs of vertical angles.

Communicate Your Answer

4. When is a conditional statement true or false?

5. Write one true conditional statement and one false conditional statement that are different from those given in Exploration 3. Justify your answer.
In your own words, write the meaning of each vocabulary term.

conditional statement

if-then form

hypothesis

conclusion

negation

converse

inverse

contrapositive

equivalent statements

perpendicular lines

biconditional statement

truth value

truth table
Core Concepts

Conditional Statement

A conditional statement is a logical statement that has two parts, a hypothesis $p$ and a conclusion $q$. When a conditional statement is written in if-then form, the “if” part contains the hypothesis and the “then” part contains the conclusion.

Words: If $p$, then $q$.  
Symbols: $p \rightarrow q$ (read as “$p$ implies $q$”)

Notes:

Negation

The negation of a statement is the opposite of the original statement. To write the negation of a statement $p$, you write the symbol for negation ($\neg$) before the letter. So, “not $p$” is written $\neg p$.

Words: not $p$.  
Symbols: $\neg p$

Notes:

Related Conditionals

Consider the conditional statement below.

Words: If $p$, then $q$.  
Symbols: $p \rightarrow q$

Converse
To write the converse of a conditional statement, exchange the hypothesis and the conclusion.

Words: If $q$, then $p$.  
Symbols: $q \rightarrow p$

Inverse
To write the inverse of a conditional statement, negate both the hypothesis and the conclusion.

Words: If not $p$, then not $q$.  
Symbols: $\neg p \rightarrow \neg q$
2.1 Notetaking with Vocabulary (continued)

Related Conditionals (continued)

Contrapositive  To write the contrapositive of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

**Words**: If not $q$, then not $p$.  
**Symbols**: $\neg q \rightarrow \neg p$

A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called equivalent statements.

**Notes:**

---

Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single biconditional statement. A biconditional statement is a statement that contains the phrase “if and only if.”

**Words**: $p$ if and only if $q$  
**Symbols**: $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

**Notes:**
2.1 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1 and 2, rewrite the conditional statement in if-then form.

1. \(13x - 5 = -18\), because \(x = -1\).

2. The sum of the measures of interior angles of a triangle is 180°.

3. Let \(p\) be “Quadrilateral \(ABCD\) is a rectangle” and let \(q\) be “the sum of the angle measures is 360°.” Write the conditional statement \(p \rightarrow q\), the converse \(q \rightarrow p\), the inverse \(\sim p \rightarrow \sim q\), and the contrapositive \(\sim q \rightarrow \sim p\) in words. Then decide whether each statement is true or false.

In Exercises 4–6, decide whether the statement about the diagram is true. Explain your answer using the definitions you have learned.

4. \(\overline{LM}\) bisects \(\overline{JK}\)

5. \(\angle JRP\) and \(\angle PRL\) are complementary.

6. \(\angle MRQ \cong \angle PRL\)
2.2 Inductive and Deductive Reasoning
For use with Exploration 2.2

Essential Question  How can you use reasoning to solve problems?

A conjecture is an unproven statement based on observations.

1 EXPLORATION: Writing a Conjecture

Work with a partner. Write a conjecture about the pattern. Then use your conjecture to draw the 10th object in the pattern.

a. 1 2 3 4 5 6 7

b. c.  

2 EXPLORATION: Using a Venn Diagram

Work with a partner. Use the Venn diagram to determine whether the statement is true or false. Justify your answer. Assume that no region of the Venn diagram is empty.

a. If an item has Property B, then it has Property A.

b. If an item has Property A, then it has Property B.
2.2 Inductive and Deductive Reasoning (continued)

2 EXPLORATION: Using a Venn Diagram (continued)

- c. If an item has Property A, then it has Property C.

- d. Some items that have Property A do not have Property B.

- e. If an item has Property C, then it does not have Property B.

- f. Some items have both Properties A and C.

- g. Some items have both Properties B and C.

3 EXPLORATION: Reasoning and Venn Diagrams

Work with a partner. Draw a Venn diagram that shows the relationship between different types of quadrilaterals: squares, rectangles, parallelograms, trapezoids, rhombuses, and kites. Then write several conditional statements that are shown in your diagram, such as “If a quadrilateral is a square, then it is a rectangle.”

Communicate Your Answer

4. How can you use reasoning to solve problems?

5. Give an example of how you used reasoning to solve a real-life problem.
2.2 Notetaking with Vocabulary
For use after Lesson 2.2

In your own words, write the meaning of each vocabulary term.

conjecture

inductive reasoning

counterexample

deductive reasoning

Core Concepts

Inductive Reasoning

A conjecture is an unproven statement that is based on observations. You use inductive reasoning when you find a pattern in specific cases and then write a conjecture for the general case.

Notes:

Counterexample

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one counterexample. A counterexample is a specific case for which the conjecture is false.

Notes:
2.2 Notetaking with Vocabulary (continued)

Deductive Reasoning

Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from inductive reasoning, which uses specific examples and patterns to form a conjecture.

Laws of Logic

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Law of Syllogism

If hypothesis \( p \), then conclusion \( q \).

If hypothesis \( q \), then conclusion \( r \).

If hypothesis \( p \), then conclusion \( r \).  

then this statement is true.

Notes:

Extra Practice

In Exercises 1–4, describe the pattern. Then write or draw the next two numbers, letters, or figures.

1. 20, 19, 17, 14, 10, ...

2. 2, −3, 5, −7, 11, ...

3. C, E, G, I, K, ...

4. ![Diagram of figures]
In Exercises 5 and 6, make and test a conjecture about the given quantity.

5. the sum of two negative integers

6. the product of three consecutive nonzero integers

In Exercises 7 and 8, find a counterexample to show that the conjecture is false.

7. If \( n \) is a rational number, then \( n^2 \) is always less than \( n \).

8. Line \( k \) intersects plane \( P \) at point \( Q \) on the plane. Plane \( P \) is perpendicular to line \( k \).

In Exercises 9 and 10, use the Law of Detachment to determine what you can conclude from the given information, if possible.

9. If a triangle has equal side lengths, then each interior angle measure is 60°. \( \triangle ABC \) has equal side lengths.

10. If a quadrilateral is a rhombus, then it has two pairs of opposite sides that are parallel. Quadrilateral \( PQRS \) has two pairs of opposite sides that are parallel.

In Exercises 11 and 12, use the Law of Syllogism to write a new conditional statement that follows from the pair of true statements, if possible.

11. If it does not rain, then I will walk to school.
   If I walk to school, then I will wear my walking shoes.

12. If \( x > 1 \), then \( 3x > 3 \).
   If \( 3x > 3 \), then \( (3x)^2 > 9 \).
Essential Question  In a diagram, what can be assumed and what needs to be labeled?

1 EXPLORATION: Looking at a Diagram

Work with a partner. On a piece of paper, draw two perpendicular lines. Label them \( \overline{AB} \) and \( \overline{CD} \). Look at the diagram from different angles. Do the lines appear perpendicular regardless of the angle at which you look at them? Describe all the angles at which you can look at the lines and have them appear perpendicular.

2 EXPLORATION: Interpreting a Diagram

Work with a partner. When you draw a diagram, you are communicating with others. It is important that you include sufficient information in the diagram. Use the diagram to determine which of the following statements you can assume to be true. Explain your reasoning.

a. All the points shown are coplanar.

b. Points \( D, G, \) and \( I \) are collinear.

c. Points \( A, C, \) and \( H \) are collinear.

d. \( \overline{EG} \) and \( \overline{AH} \) are perpendicular.
2.3 Postulates and Diagrams (continued)

2 EXPLORATION: Interpreting a Diagram (continued)

e. \( \angle BCA \) and \( \angle ACD \) are a linear pair.

f. \( \overrightarrow{AF} \) and \( \overrightarrow{BD} \) are perpendicular.

g. \( \overrightarrow{EG} \) and \( \overrightarrow{BD} \) are parallel.

h. \( \overrightarrow{AF} \) and \( \overrightarrow{BD} \) are coplanar.

i. \( \overrightarrow{EG} \) and \( \overrightarrow{BD} \) do not intersect.

j. \( \overrightarrow{AF} \) and \( \overrightarrow{BD} \) intersect.

k. \( \overrightarrow{EG} \) and \( \overrightarrow{BD} \) are perpendicular.

l. \( \angle ACD \) and \( \angle BCF \) are vertical angles.

m. \( \overrightarrow{AC} \) and \( \overrightarrow{FH} \) are the same line.

Communicate Your Answer

3. In a diagram, what can be assumed and what needs to be labeled?

4. Use the diagram in Exploration 2 to write two statements you can assume to be true and two statements you cannot assume to be true. Your statements should be different from those given in Exploration 2. Explain your reasoning.
In your own words, write the meaning of each vocabulary term.

line perpendicular to a plane

**Postulates**

**Point, Line, and Plane Postulates**

<table>
<thead>
<tr>
<th>Postulate</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.1 Two Point Postulate</strong></td>
<td><img src="#" alt="Diagram" /> Through points $A$ and $B$, there is exactly one line $\ell$. Line $\ell$ contains at least two points.</td>
</tr>
<tr>
<td><strong>2.2 Line-Point Postulate</strong></td>
<td>A line contains at least two points.</td>
</tr>
<tr>
<td><strong>2.3 Line Intersection Postulate</strong></td>
<td>The intersection of line $m$ and line $n$ is point $C$.</td>
</tr>
<tr>
<td><strong>2.4 Three Point Postulate</strong></td>
<td><img src="#" alt="Diagram" /> Through points $D$, $E$, and $F$, there is exactly one plane, plane $R$. Plane $R$ contains at least three noncollinear points.</td>
</tr>
<tr>
<td><strong>2.5 Plane-Point Postulate</strong></td>
<td>A plane contains at least three noncollinear points</td>
</tr>
</tbody>
</table>

**Notes:**

[Diagram: Line perpendicular to a plane]
2.3 Notetaking with Vocabulary (continued)

Point, Line, and Plane Postulates (continued)

<table>
<thead>
<tr>
<th>Postulate</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2.6 Plane-Line Postulate</strong></td>
<td>Points $D$ and $E$ lie in plane $R$, so</td>
</tr>
<tr>
<td>If two points lie in a plane, then</td>
<td>line containing them lies in the plane.</td>
</tr>
<tr>
<td>$\overline{DE}$ lies in plane $R$.</td>
<td></td>
</tr>
<tr>
<td><strong>2.7 Plane Intersection Postulate</strong></td>
<td>The intersection of plane $S$ and plane $T$ is</td>
</tr>
<tr>
<td>If two planes intersect, then their</td>
<td>line $\ell$.</td>
</tr>
<tr>
<td>intersection is a line.</td>
<td></td>
</tr>
</tbody>
</table>

Notes:

Extra Practice

In Exercises 1 and 2, state the postulate illustrated by the diagram.

1. If $B$ then $C$.

2. If $M$ then $L$. 
2.3 Notetaking with Vocabulary (continued)

In Exercises 3–6, use the diagram to write an example of the postulate.

3. Two Point Postulate (Postulate 2.1)

4. Line-Point Postulate (Postulate 2.2)

5. Plane-Point Postulate (Postulate 2.5)

6. Plane Intersection Postulate (Postulate 2.7)

In Exercises 7 and 8, sketch a diagram of the description.

7. \( RS \) bisecting \( KL \) at point \( R \)

8. \( AB \) in plane \( U \) intersecting \( CD \) at point \( E \), and point \( C \) not on plane \( U \)

In Exercises 9–14, use the diagram to determine whether you can assume the statement.

9. Planes \( A \) and \( B \) intersect at \( EF \).

10. Points \( C \) and \( D \) are collinear.

11. \( HJ \) and \( ID \) are perpendicular.

12. \( GD \) is a bisector of \( EF \) at point \( D \).

13. \( HH \equiv HG \)

14. \( \angle HJD \) and \( \angle HDJ \) are complementary angles.
### Essential Question
How can algebraic properties help you solve an equation?

#### EXPLORATION: Justifying Steps in a Solution

**Work with a partner.** In previous courses, you studied different properties, such as the properties of equality and the Distributive, Commutative, and Associative Properties. Write the property that justifies each of the following solution steps.

<table>
<thead>
<tr>
<th>Algebraic Step</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2(x + 3) - 5 = 5x + 4$</td>
<td>Write given equation.</td>
</tr>
<tr>
<td>$2x + 6 - 5 = 5x + 4$</td>
<td></td>
</tr>
<tr>
<td>$2x + 1 = 5x + 4$</td>
<td></td>
</tr>
<tr>
<td>$2x - 2x + 1 = 5x - 2x + 4$</td>
<td></td>
</tr>
<tr>
<td>$1 = 3x + 4$</td>
<td></td>
</tr>
<tr>
<td>$1 - 4 = 3x + 4 - 4$</td>
<td></td>
</tr>
<tr>
<td>$-3 = 3x$</td>
<td></td>
</tr>
<tr>
<td>$\frac{-3}{3} = \frac{3x}{3}$</td>
<td></td>
</tr>
<tr>
<td>$-1 = x$</td>
<td></td>
</tr>
<tr>
<td>$x = -1$</td>
<td></td>
</tr>
</tbody>
</table>
### 2.4 Algebraic Reasoning (continued)

#### 2. Work with a partner.

The symbols ◆ and ★ represent addition and multiplication (not necessarily in that order). Determine which symbol represents which operation. Justify your answer. Then state each algebraic property being illustrated.

<table>
<thead>
<tr>
<th>Example of Property</th>
<th>Name of Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ◆ 6 = 6 ◆ 5</td>
<td></td>
</tr>
<tr>
<td>5 ★ 6 = 6 ★ 5</td>
<td></td>
</tr>
<tr>
<td>4 ◆ (5 ◆ 6) = (4 ◆ 5) ◆ 6</td>
<td></td>
</tr>
<tr>
<td>4 ★ (5 ★ 6) = (4 ★ 5) ★ 6</td>
<td></td>
</tr>
<tr>
<td>0 ◆ 5 = 0</td>
<td></td>
</tr>
<tr>
<td>0 ★ 5 = 5</td>
<td></td>
</tr>
<tr>
<td>1 ◆ 5 = 5</td>
<td></td>
</tr>
<tr>
<td>4 ◆ (5 ◆ 6) = 4 ◆ 5 ★ 4 ◆ 6</td>
<td></td>
</tr>
</tbody>
</table>

#### Communicate Your Answer

3. How can algebraic properties help you solve an equation?

4. Solve \(3(x + 1) - 1 = -13\). Justify each step.
2.4 Notetaking with Vocabulary
For use after Lesson 2.4

In your own words, write the meaning of each vocabulary term.

equation

solve an equation

formula

Core Concepts

Algebraic Properties of Equality

Let $a$, $b$, and $c$ be real numbers.

Addition Property of Equality  
If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality  
If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality  
If $a = b$, then $a \cdot c = b \cdot c$, $c \neq 0$.

Division Property of Equality  
If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.

Substitution Property of Equality  
If $a = b$, then $a$ can be substituted for $b$ (or $b$ for $a$) in any equation or expression.

Notes:
2.4 Notetaking with Vocabulary (continued)

Distributive Property
Let $a$, $b$, and $c$ be real numbers.

Sum: $a(b + c) = ab + ac$

Difference: $a(b - c) = ab - ac$

Notes:

Reflexive, Symmetric, and Transitive Properties of Equality

<table>
<thead>
<tr>
<th>Property</th>
<th>Real Numbers</th>
<th>Segment Lengths</th>
<th>Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive Property</td>
<td>$a = a$</td>
<td>$AB = AB$</td>
<td>$m\angle A = m\angle A$</td>
</tr>
<tr>
<td>Symmetric Property</td>
<td>If $a = b$, then $b = a.$</td>
<td>If $AB = CD$, then $CD = AB.$</td>
<td>If $m\angle A = m\angle B$, then $m\angle B = m\angle A.$</td>
</tr>
<tr>
<td>Transitive Property</td>
<td>If $a = b$ and $b = c$, then $a = c.$</td>
<td>If $AB = CD$ and $CD = EF$, then $AB = EF.$</td>
<td>If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C.$</td>
</tr>
</tbody>
</table>

Notes:
2.4 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–3, solve the equation. Justify each step.

1. \[3x - 7 = 5x - 19\]

2. \[20 - 2(3x - 1) = x - 6\]

3. \[-5(2u + 10) = 2(u - 7)\]

In Exercises 4 and 5, solve the equation for the given variable. Justify each step.

4. \[9x + 2y = 5; y\]

5. \[\frac{1}{15}s - \frac{2}{3}t = -2; s\]

6. The formula for the surface area \(S\) of a cone is \(S = \pi r^2 + \pi rs\), where \(r\) is the radius and \(s\) is the slant height. Solve the formula for \(s\). Justify each step. Then find the slant height of the cone when the surface area is 113 square feet and the radius is 4 feet. Approximate to the nearest tenth.
2.5 Proving Statements about Segments and Angles
For use with Exploration 2.5

Essential Question  How can you prove a mathematical statement?

A proof is a logical argument that uses deductive reasoning to show that a statement is true.

1 EXPLORATION: Writing Reasons in a Proof

Work with a partner. Four steps of a proof are shown. Write the reasons for each statement.

Given  \( AC = AB + AB \)

Prove  \( AB = BC \)

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AC = AB + AB )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB + BC = AC )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( AB + AB = AB + BC )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( AB = BC )</td>
<td>4.</td>
</tr>
</tbody>
</table>

2 EXPLORATION: Writing Steps in a Proof

Work with a partner. Six steps of a proof are shown. Complete the statements that correspond to each reason.

Given  \( m\angle 1 = m\angle 3 \)

Prove  \( m\angle EBA = m\angle CBD \)
### 2.5 Proving Statements about Segments and Angles (continued)

**EXPLORATION:** Writing Steps in a Proof (continued)

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ______________________________</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle EBA = m\angle 2 + m\angle 3 )</td>
<td>2. Angle Addition Postulate (Post. 1.4)</td>
</tr>
<tr>
<td>3. ( m\angle EBA = m\angle 2 + m\angle 1 )</td>
<td>3. Substitution Property of Equality</td>
</tr>
<tr>
<td>4. ( m\angle EBA = \text{____________} )</td>
<td>4. Commutative Property of Addition</td>
</tr>
<tr>
<td>5. ( m\angle 1 + m\angle 2 = \text{____________} )</td>
<td>5. Angle Addition Postulate (Post. 1.4)</td>
</tr>
<tr>
<td>6. ______________________________</td>
<td>6. Transitive Property of Equality</td>
</tr>
</tbody>
</table>

### Communicate Your Answer

3. How can you prove a mathematical statement?

4. Use the given information and the figure to write a proof for the statement.

**Given**  
- \( B \) is the midpoint of \( AC \).  
- \( C \) is the midpoint of \( BD \).  

**Prove**  
\( AB = CD \)
2.5 Notetaking with Vocabulary
For use after Lesson 2.5

In your own words, write the meaning of each vocabulary term.

proof

two-column proof

theorem

Theorems

Theorem 2.1 Properties of Segment Congruence
Segment congruence is reflexive, symmetric, and transitive.

**Reflexive**
For any segment $AB$, $\overline{AB} \cong \overline{AB}$.

**Symmetric**
If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

**Transitive**
If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Theorem 2.2 Properties of Angle Congruence
Angle congruence is reflexive, symmetric, and transitive.

**Reflexive**
For any angle $\angle A$, $\angle A \cong \angle A$.

**Symmetric**
If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

**Transitive**
If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Notes:
2.5 Notetaking with Vocabulary (continued)

Core Concepts

Writing a Two-Column Proof

In a proof, you make one statement at a time until you reach the conclusion. Because you make statements based on facts, you are using deductive reasoning. Usually the first statement-and-reason pair you write is given information.

Proof of the Symmetric Property of Angle Congruence

Given $\angle 1 \cong \angle 2$  Prove $\angle 2 \cong \angle 1$

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \cong \angle 2$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle 1 = m\angle 2$</td>
<td>2. Definition of congruent angles</td>
</tr>
<tr>
<td>3. $m\angle 2 = m\angle 1$</td>
<td>3. Symmetric Property of Equality</td>
</tr>
<tr>
<td>4. $\angle 2 \cong \angle 1$</td>
<td>4. Definition of congruent angles</td>
</tr>
</tbody>
</table>

The number of statements will vary.

Remember to give a reason for the last statement.

Extra Practice

In Exercises 1 and 2, complete the proof.

1. Given $\overline{AB}$ and $\overline{CD}$ bisect each other at point $M$ and $BM \cong CM$.

   Prove $AB = AM + DM$

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $BM \cong CM$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $CM \cong DM$</td>
<td>2. __________________</td>
</tr>
<tr>
<td>3. $BM \cong DM$</td>
<td>3. __________________</td>
</tr>
<tr>
<td>4. $BM = DM$</td>
<td>4. __________________</td>
</tr>
<tr>
<td>5. __________________</td>
<td>5. Segment Addition Postulate (Post. 1.2)</td>
</tr>
<tr>
<td>6. $AB = AM + DM$</td>
<td>6. __________________</td>
</tr>
</tbody>
</table>
2. Given \( \angle AEB \) is a complement of \( \angle BEC \).
   Prove \( m\angle AED = 90^\circ \)

<table>
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<tr>
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<th>REASONS</th>
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</thead>
<tbody>
<tr>
<td>1. ( \angle AEB ) is a complement of ( \angle BEC ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. _______________________________</td>
<td>2. Definition of complementary angles</td>
</tr>
<tr>
<td>3. ( m\angle AEC = m\angle AEB + m\angle BEC )</td>
<td>3. ________________________________</td>
</tr>
<tr>
<td>4. ( m\angle AEC = 90^\circ )</td>
<td>4. ________________________________</td>
</tr>
<tr>
<td>5. ( m\angle AED + m\angle AEC = 180^\circ )</td>
<td>5. Definition of supplementary angles</td>
</tr>
<tr>
<td>6. _______________________________</td>
<td>6. Substitution Property of Equality</td>
</tr>
<tr>
<td>7. ( m\angle AED = 90^\circ )</td>
<td>7. ________________________________</td>
</tr>
</tbody>
</table>

In Exercises 3 and 4, name the property that the statement illustrates.

3. If \( \angle RST \cong \angle TSU \) and \( \angle TSU \cong \angle VWX \), then \( \angle RST \cong \angle VWX \).

4. If \( \overline{GH} \cong \overline{JK} \), then \( \overline{JK} \cong \overline{GH} \).

5. Write a two-column proof.

   Given \( M \) is the midpoint of \( \overline{RT} \).
   Prove \( MT = RS + SM \)

<table>
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</table>
Essential Question: How can you use a flowchart to prove a mathematical statement?

Work with a partner. Match each reason with the correct step in the flowchart.

Given: \( AC = AB + AB \)
Prove: \( AB = BC \)

\[
\begin{align*}
AC &= AB + AB \\
AB + BC &= AC \\
AB + AB &= AB + BC \\
AB &= BC
\end{align*}
\]

A. Segment Addition Postulate (Post. 1.2)  
B. Given  
C. Transitive Property of Equality  
D. Subtraction Property of Equality
2.6 Proving Geometric Relationships (continued)

**EXPLORATION:** Matching Reasons in a Flowchart Proof

Work with a partner. Match each reason with the correct step in the flowchart.

Given \( m\angle 1 = m\angle 3 \)

Prove \( m\angle EBA = m\angle CBD \)

\[
\begin{align*}
& m\angle 1 = m\angle 3 \\
\Rightarrow & m\angle EBA = m\angle 2 + m\angle 3 \\
\Rightarrow & m\angle EBA = m\angle 2 + m\angle 1 \\
\Rightarrow & m\angle EBA = m\angle 1 + m\angle 2 \\
\Rightarrow & m\angle 1 + m\angle 2 = m\angle CBD \\
\Rightarrow & m\angle EBA = m\angle CBD
\end{align*}
\]

A. Angle Addition Postulate (Post. 1.4)  
B. Transitive Property of Equality  
C. Substitution Property of Equality  
D. Angle Addition Postulate (Post. 1.4)  
E. Given  
F. Commutative Property of Addition

**Communicate Your Answer**

3. How can you use a flowchart to prove a mathematical statement?

4. Compare the flowchart proofs above with the two-column proofs in the Section 2.5 Explorations. Explain the advantages and disadvantages of each.
2.6 Notetaking with Vocabulary
For use after Lesson 2.6

In your own words, write the meaning of each vocabulary term.

flowchart proof, or flow proof

paragraph proof

Theorems and Postulates

Theorem 2.3 Right Angles Congruence Theorem
All right angles are congruent.

Notes:

Theorem 2.4 Congruent Supplements Theorem
If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

If $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 2$ are supplementary, then $\angle 1 \cong \angle 3$.

Notes:
2.6 Notetaking with Vocabulary (continued)

**Theorem 2.5  Congruent Complements Theorem**

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

If \( \angle 4 \) and \( \angle 5 \) are complementary and \( \angle 6 \) and \( \angle 5 \) are complementary, then \( \angle 4 \cong \angle 6 \).

**Notes:**

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**Postulate 2.8  Linear Pair Postulate**

If two angles form a linear pair, then they are supplementary.

\( \angle 1 \) and \( \angle 2 \) form a linear pair, so \( \angle 1 \) and \( \angle 2 \) are supplementary and \( m\angle 1 + m\angle 2 = 180^\circ \).

**Notes:**

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**Theorem 2.6  Vertical Angles Congruence Theorem**

Vertical angles are congruent.

**Notes:**

\( \angle 1 \cong \angle 3, \quad \angle 2 \cong \angle 4 \)
2.6 Notetaking with Vocabulary (continued)

Extra Practice

1. Complete the flowchart proof. Then write a two-column proof.

Given \( \angle ACB \) and \( \angle ACD \) are supplementary.
\( \angle EGF \) and \( \angle ACD \) are supplementary.

Prove \( \angle ACB \cong \angle EGF \)

Two-Column Proof

<table>
<thead>
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<tr>
<td>( \angle ACB ) and ( \angle ACD ) are supplementary. ( \text{Given} )</td>
<td>( m \angle ACB + m \angle ACD = 180^\circ ) ( \text{Transitive Property of Equality} )</td>
</tr>
<tr>
<td>( \angle EGF ) and ( \angle ACD ) are supplementary. ( \text{Given} )</td>
<td>( m \angle EGF + m \angle ACD = 180^\circ ) ( \text{Subtraction Property of Equality} )</td>
</tr>
<tr>
<td>( m \angle ACB = m \angle EGF )</td>
<td>( \angle ACB \cong \angle EGF )</td>
</tr>
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