Chapter 1

Maintaining Mathematical Proficiency

Simplify the expression.

1. \(|-3 + (-1)| =

2. \(|10 - 11| =

3. \(|-6 + 8| =

4. \(|9 - (-1)| =

5. \(|-12 - (-8)| =

6. \(|-15 - 7| =

7. \(|-12 + 3| =

8. \(|5 + (-15)| =

9. \(|1 - 12| =

Find the area of the triangle.

10. \(\text{Area} = \frac{1}{2} \times 15 \, \text{cm} \times 16 \, \text{cm}

11. \(\text{Area} = \frac{1}{2} \times 6 \, \text{ft} \times 12 \, \text{ft}

12. \(\text{Area} = \frac{1}{2} \times 9 \, \text{km} \times 31 \, \text{km}
1.1 Points, Lines, and Planes
For use with Exploration 1.1

**Essential Question**  How can you use dynamic geometry software to visualize geometric concepts?

### 1 EXPLORATION: Using Dynamic Geometry Software

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

**Work with a partner.** Use dynamic geometry software to draw several points. Also, draw some lines, line segments, and rays. What is the difference between a line, a line segment, and a ray?

![Sample drawing](image)

### 2 EXPLORATION: Intersections of Lines and Planes

**Work with a partner.**

**a.** Describe and sketch the ways in which two lines can intersect or not intersect. Give examples of each using the lines formed by the walls, floor, and ceiling in your classroom.

![Intersections of lines and planes](image)
b. Describe and sketch the ways in which a line and a plane can intersect or not intersect. Give examples of each using the walls, floor, and ceiling in your classroom.

c. Describe and sketch the ways in which two planes can intersect or not intersect. Give examples of each using the walls, floor, and ceiling in your classroom.

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use dynamic geometry software to explore geometry. Use the software to find a term or concept that is unfamiliar to you. Then use the capabilities of the software to determine the meaning of the term or concept.

Communicate Your Answer

4. How can you use dynamic geometry software to visualize geometric concepts?
In your own words, write the meaning of each vocabulary term.

undefined terms

point

line

plane

collinear points

coplanar points

defined terms

line segment, or segment

endpoints

ray

opposite rays

intersection
1.1 Notetaking with Vocabulary (continued)

**Core Concepts**

**Undefined Terms: Point, Line, and Plane**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point</strong></td>
<td>A point has no dimension. A dot represents a point.</td>
<td><img src="image1" alt="Point Diagram" /></td>
</tr>
<tr>
<td><strong>Line</strong></td>
<td>A line has one dimension. It is represented by a line with two arrowheads, but it extends without end. Through any two points, there is exactly one line. You can use any two points on a line to name it.</td>
<td><img src="image2" alt="Line Diagram" /></td>
</tr>
<tr>
<td><strong>Plane</strong></td>
<td>A plane has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end. Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.</td>
<td><img src="image3" alt="Plane Diagram" /></td>
</tr>
</tbody>
</table>

**Notes:**

**Defined Terms: Segment and Ray**

The definitions below use line $AB$ (written as $\overline{AB}$) and points $A$ and $B$.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Segment</strong></td>
<td>The line segment $AB$, or segment $\overline{AB}$ (written as $\overline{AB}$) consists of the endpoints $A$ and $B$ and all points on $\overline{AB}$ that are between $A$ and $B$. Note that $\overline{AB}$ can also be named $\overline{BA}$.</td>
<td><img src="image4" alt="Segment Diagram" /></td>
</tr>
<tr>
<td><strong>Ray</strong></td>
<td>The ray $AB$ (written as $\overline{AB}$) consists of the endpoint $A$ and all points on $\overline{AB}$ that lie on the same side of $A$ as $B$. Note that $\overline{AB}$ and $\overline{BA}$ are different rays.</td>
<td><img src="image5" alt="Ray Diagram" /></td>
</tr>
</tbody>
</table>

**Opposite Rays**

If point $C$ lies on $\overline{AB}$ between $A$ and $B$, then $\overline{CA}$ and $\overline{CB}$ are opposite rays.

**Notes:**
Extra Practice

In Exercises 1–4, use the diagram.

1. Give two other names for $\overline{CD}$.

2. Give another name for plane $M$.

3. Name three points that are collinear. Then name a fourth point that is not collinear with these three points.

4. Name a point that is not coplanar with points $A, C, E$.

In Exercises 5–8, use the diagram.

5. What is another name for $\overline{PQ}$?

6. What is another name for $\overline{RS}$?

7. Name all rays with endpoint $T$. Which of these rays are opposite rays?

8. On the diagram, draw planes $M$ and $N$ that intersect at line $k$.

In Exercises 9 and 10, sketch the figure described.

9. $\overline{AB}$ and $\overline{BC}$

10. line $k$ in plane $M$
1.2 Measuring and Constructing Segments
For use with Exploration 1.2

Essential Question  How can you measure and construct a line segment?

1 EXPLORATION: Measuring Line Segments Using Nonstandard Units

Work with a partner.

a. Draw a line segment that has a length of 6 inches.

b. Use a standard-sized paper clip to measure the length of the line segment. Explain how you measured the line segment in “paper clips.”

b. Use a standard-sized paper clip to measure the length of the line segment. Explain how you measured the line segment in “paper clips.”

1 paper clip = ___ in.

1 in. = ___ paper clip

c. Write conversion factors from paper clips to inches and vice versa.

1 paper clip = ___ in.

1 in. = ___ paper clip

d. A straightedge is a tool that you can use to draw a straight line. An example of a straightedge is a ruler. Use only a pencil, straightedge, paper clip, and paper to draw another line segment that is 6 inches long. Explain your process.
1.2 Measuring and Constructing Segments (continued)

2 EXPLORATION: Measuring Line Segments Using Nonstandard Units

Work with a partner.

a. Fold a 3-inch by 5-inch index card on one of its diagonals.

b. Use the Pythagorean Theorem to algebraically determine the length of the diagonal in inches. Use a ruler to check your answer.

3 EXPLORATION: Measuring Heights Using Nonstandard Units

Work with a partner. Consider a unit of length that is equal to the length of the diagonal you found in Exploration 2. Call this length “1 diag.” How tall are you in diags? Explain how you obtained your answer.

Communicate Your Answer

4. How can you measure and construct a line segment?
1.2 Notetaking with Vocabulary
For use after Lesson 1.2

In your own words, write the meaning of each vocabulary term.

postulate

axiom

coordinate

distance

construction

congruent segments

between

Postulate 1.1  Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point.

The distance between points $A$ and $B$, written as $AB$, is the absolute value of the difference of the coordinates of $A$ and $B$.

Notes:
Core Concepts

Congruent Segments

Line segments that have the same length are called **congruent segments**. You can say “the length of $\overline{AB}$ is equal to the length of $\overline{CD}$,” or you can say “$\overline{AB}$ is congruent to $\overline{CD}$.” The symbol $\cong$ means “is congruent to.”

![Diagram showing congruent segments]

Lengths are equal. Segments are congruent.

- $\overline{AB} = \overline{CD}$
- $\overline{AB} \cong \overline{CD}$

Notes:

Postulate 1.2  Segment Addition Postulate

If $B$ is between $A$ and $C$, then $AB + BC = AC$.

If $AB + BC = AC$, then $B$ is between $A$ and $C$.

Notes:
Extra Practice

In Exercises 1–3, plot the points in the coordinate plane. Then determine whether \( \overline{AB} \) and \( \overline{CD} \) are congruent.

1. \( A(-5, 5), B(-2, 5) \)  
   \( C(2, -4), D(-1, -4) \)

2. \( A(4, 0), B(4, 3) \)
   \( C(-4, -4), D(-4, 1) \)

3. \( A(-1, 5), B(5, 5) \)  
   \( C(1, 3), D(1, -3) \)

In Exercises 4–6, find \( VW \).

4. \( V \)  
   \( W \)  
   \( U \)

5. \( U \)  
   \( V \)  
   \( W \)

6. \( U \)  
   \( V \)  
   \( W \)

7. A bookstore and a movie theater are 6 kilometers apart along the same street. A florist is located between the bookstore and the theater on the same street. The florist is 2.5 kilometers from the theater. How far is the florist from the bookstore?
1.3 Using Midpoint and Distance Formulas
For use with Exploration 1.3

**Essential Question**  How can you find the midpoint and length of a line segment in a coordinate plane?

1 **EXPLORATION:** Finding the Midpoint of a Line Segment

**Work with a partner.** Use centimeter graph paper.

a. Graph \( \overline{AB} \), where the points \( A \) and \( B \) are as shown.

b. Explain how to bisect \( \overline{AB} \), that is, to divide \( \overline{AB} \) into two congruent line segments. Then bisect \( \overline{AB} \) and use the result to find the midpoint \( M \) of \( \overline{AB} \).

![Graph showing points A and B with midpoint M]

(c. What are the coordinates of the midpoint \( M \)?

\[ M \left( \frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right) \]

\[ M \left( \frac{3 + (-5)}{2}, \frac{4 + (-2)}{2} \right) \]

\[ M \left( -1, 1 \right) \]

d. Compare the \( x \)-coordinates of \( A, B, \) and \( M \). Compare the \( y \)-coordinates of \( A, B, \) and \( M \). How are the coordinates of the midpoint \( M \) related to the coordinates of \( A \) and \( B \)?
1.3 Using Midpoint and Distance Formulas (continued)

2 EXPLORATION: Finding the Length of a Line Segment

Work with a partner. Use centimeter graph paper.

a. Add point $C$ to your graph as shown.

b. Use the Pythagorean Theorem to find the length of $AB$.

c. Use a centimeter ruler to verify the length you found in part (b).

d. Use the Pythagorean Theorem and point $M$ from Exploration 1 to find the lengths of $AM$ and $MB$. What can you conclude?

Communicate Your Answer

3. How can you find the midpoint and length of a line segment in a coordinate plane?

4. Find the coordinates of the midpoint $M$ and the length of the line segment whose endpoints are given.

   a. $D(-10, -4), E(14, 6)$
   
   b. $F(-4, 8), G(9, 0)$
In your own words, write the meaning of each vocabulary term.

midpoint

segment bisector

**Core Concepts**

**Midpoints and Segment Bisectors**

The **midpoint** of a segment is the point that divides the segment into two congruent segments.

![Diagram of midpoint](image)

\[ M \text{ is the midpoint of } \overline{AB}. \]
\[ \text{So, } AM \cong MB \text{ and } AM = MB. \]

A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.

![Diagram of segment bisector](image)

\[ CD \text{ is a segment bisector of } \overline{AB}. \]
\[ \text{So, } AM \cong MB \text{ and } AM = MB. \]
1.3 Notetaking with Vocabulary (continued)

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the $x$-coordinates and of the $y$-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint $M$ of $\overline{AB}$ has coordinates

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Notes:

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between $A$ and $B$ is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Notes:
Extra Practice

In Exercises 1–3, identify the segment bisector of $AB$. Then find $AB$.

1. $A \hspace{1cm} 13 \hspace{1cm} M \hspace{1cm} B$

2. $A \hspace{1cm} 8 \hspace{1cm} M \hspace{1cm} B$

3. $A \hspace{1cm} M \hspace{1cm} 26 \hspace{1cm} B$

In Exercises 4–6, identify the segment bisector of $EF$. Then find $EF$.

4. $E \hspace{1cm} 6x \hspace{1cm} M \hspace{1cm} 2x + 4 \hspace{1cm} F$

5. $E \hspace{1cm} M \hspace{1cm} m \hspace{1cm} F$

6. $E \hspace{1cm} 3x - 4 \hspace{1cm} M \hspace{1cm} 5x - 12 \hspace{1cm} F$

In Exercises 7–9, the endpoints of $PQ$ are given. Find the coordinates of the midpoint $M$.

7. $P(-4, 3)$ and $Q(0, 5)$  
8. $P(-2, 7)$ and $Q(10, -3)$  
9. $P(3, -15)$ and $Q(9, -3)$

In Exercises 10–12, the midpoint $M$ and one endpoint of $JK$ are given. Find the coordinates of the other endpoint.

10. $J(7, 2)$ and $M(1, -2)$  
11. $J(5, -2)$ and $M(0, -1)$  
12. $J(2, 16)$ and $M\left(-\frac{9}{2}, 7\right)$
1.4 Perimeter and Area in the Coordinate Plane
For use with Exploration 1.4

**Essential Question**  How can you find the perimeter and area of a polygon in a coordinate plane?

**EXPLORATION:** Finding the Perimeter and Area of a Quadrilateral

Work with a partner.

a. On the centimeter graph paper, draw quadrilateral \(ABCD\) in a coordinate plane.
   Label the points \(A(1, 4), B(-3, 1), C(0, -3),\) and \(D(4, 0)\).

b. Find the perimeter of quadrilateral \(ABCD\).

c. Are adjacent sides of quadrilateral \(ABCD\) perpendicular to each other? How can you tell?

d. What is the definition of a square? Is quadrilateral \(ABCD\) a square? Justify your answer. Find the area of quadrilateral \(ABCD\).
EXPLORATION: Finding the Area of a Polygon

Work with a partner.

a. Quadrilateral $ABCD$ is partitioned into four right triangles and one square, as shown. Find the coordinates of the vertices for the five smaller polygons.

b. Find the areas of the five smaller polygons.

Area of Triangle $BPA$:

Area of Triangle $AQD$:

Area of Triangle $DRC$:

Area of Triangle $CSB$:

Area of Square $PQRS$:

c. Is the sum of the areas of the five smaller polygons equal to the area of quadrilateral $ABCD$? Justify your answer.

Communicate Your Answer

3. How can you find the perimeter and area of a polygon in a coordinate plane?

4. Repeat Exploration 1 for quadrilateral $EFGH$, where the coordinates of the vertices are $E(-3, 6), F(-7, 3), G(-1, -5)$, and $H(3, -2)$. 
In your own words, write the meaning of each vocabulary term.

**Polygon**

**Side**

**Vertex**

**n-gon**

**Convex**

**Concave**

**Core Concepts**

**Polygons**

In geometry, a figure that lies in a plane is called a plane figure. Recall that a polygon is a closed plane figure formed by three or more line segments called sides. Each side intersects exactly two sides, one at each vertex, so that no two sides with a common vertex are collinear. You can name a polygon by listing the vertices in consecutive order.

**Notes:**
1.4 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–4, classify the polygon by the number of sides. Tell whether it is convex or concave.

1. 

2. 

3. 

4. 

In Exercises 5–8, find the perimeter and area of the polygon with the given vertices.

5. \(X(2, 4), Y(0, -2), Z(2, -2)\)

6. \(P(1, 3), Q(1, 1), R(-4, 2)\)

7. \(J(-4, 1), K(-4, -2), L(6, -2), M(6, 1)\)

8. \(D(5, -3), E(5, -6), F(2, -6), G(2, -3)\)
In Exercises 9–14, use the diagram.

9. Find the perimeter of $\triangle ABD$.

10. Find the perimeter of $\triangle BCD$.

11. Find the perimeter of quadrilateral $ABCD$.

12. Find the area of $\triangle ABD$.

13. Find the area of $\triangle BCD$.

14. Find the area of quadrilateral $ABCD$. 

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Name _________________________________________________________  Date ______

1.4 Notetaking with Vocabulary (continued)
1.5 Measuring and Constructing Angles
For use with Exploration 1.5

Essential Question  How can you measure and classify an angle?

1 EXPLORATION: Measuring and Classifying Angles

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Find the degree measure of each of the following angles. Classify each angle as acute, right, or obtuse.

[Diagram of protractor with angles labeled]

a. \( \angle AOB \)
b. \( \angle AOC \)
c. \( \angle BOC \)
d. \( \angle BOE \)
e. \( \angle COE \)
f. \( \angle COD \)
g. \( \angle BOD \)
h. \( \angle AOE \)
1.5 Measuring and Constructing Angles (continued)

2 exploration: Drawing a Regular Polygon

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. On a separate sheet of paper or an index card, use a ruler and protractor to draw the triangular pattern shown at the right.

b. Cut out the pattern and use it to draw three regular hexagons, as shown in your book.

c. The sum of the angle measures of a polygon with $n$ sides is equal to $180(n - 2)°$.
   Do the angle measures of your hexagons agree with this rule? Explain.

d. Partition your hexagons into smaller polygons, as shown in your book. For each hexagon, find the sum of the angle measures of the smaller polygons. Does each sum equal the sum of the angle measures of a hexagon? Explain.

Communicate Your Answer

3. How can you measure and classify an angle?
In your own words, write the meaning of each vocabulary term.

angle

vertex

sides of an angle

interior of an angle

exterior of an angle

measure of an angle

acute angle

right angle

obtuse angle

straight angle

congruent angles

angle bisector
Postulate 1.3  Protractor Postulate

Consider \( \overline{OB} \) and a point \( A \) on one side of \( \overline{OB} \). The rays of the form \( \overline{OA} \) can be matched one to one with the real numbers from 0 to 180.

The measure of \( \angle AOB \), which can be written as \( m\angle AOB \), is equal to the absolute value of the difference between the real numbers matched with \( \overline{OA} \) and \( \overline{OB} \) on a protractor.

Notes:

Core Concepts
Types of Angles

- **Acute Angle**: Measures greater than 0° and less than 90°
- **Right Angle**: Measures 90°
- **Obtuse Angle**: Measures greater than 90° and less than 180°
- **Straight Angle**: Measures 180°

Notes:

Postulate 1.4  Angle Addition Postulate

**Words** If \( P \) is the interior of \( \angle RST \), then the measure of \( \angle RST \) is equal to the sum of the measures of \( \angle SPR \) and \( \angle PST \).

**Symbols** If \( P \) is in the interior of \( \angle RST \), then

\[
m\angle RST = m\angle RSP + m\angle PST.
\]

Notes:
In Exercises 1–3, name three different angles in the diagram.

1. \( \angle GEF \)
2. \( \angle GHE \)
3. \( \angle HGF \)

In Exercises 4–9, find the indicated angle measure(s).

4. Find \( m\angle JKL \).

5. \( m\angle RSU = 91^\circ \).
   Find \( m\angle RST \).

6. \( \angle UWX \) is a straight angle.
   Find \( m\angle UWV \) and \( m\angle XWV \).

7. Find \( m\angle CAD \) and \( m\angle BAD \).

8. \( \overline{EG} \) bisects \( \angle DEF \).
   Find \( m\angle DEG \) and \( m\angle GEF \).

9. \( \overline{QR} \) bisects \( \angle PQS \).
   Find \( m\angle PQR \) and \( m\angle PQS \).
1.6 Describing Pairs of Angles
For use with Exploration 1.6

Essential Question How can you describe angle pair relationships and use these descriptions to find angle measures?

EXPLORATION: Finding Angle Measures

Work with a partner. The five-pointed star has a regular pentagon at its center.

a. What do you notice about the following angle pairs?

\( x^\circ \) and \( y^\circ \)

\( y^\circ \) and \( z^\circ \)

\( x^\circ \) and \( z^\circ \)

b. Find the values of the indicated variables. Do not use a protractor to measure the angles.

\( x = \)

\( y = \)

\( z = \)

\( w = \)

\( v = \)

Explain how you obtained each answer.
1.6 Describing Pairs of Angles (continued)

**EXPLORATION:** Finding Angle Measures

Work with a partner. A square is divided by its diagonals into four triangles.

a. What do you notice about the following angle pairs?
   - \(a^\circ\) and \(b^\circ\)
   - \(c^\circ\) and \(d^\circ\)
   - \(c^\circ\) and \(e^\circ\)

b. Find the values of the indicated variables. Do not use a protractor to measure the angles.
   - \(c = \)
   - \(d = \)
   - \(e = \)

Explain how you obtained each answer.

**Communicate Your Answer**

3. How can you describe angle pair relationships and use these descriptions to find angle measures?

4. What do you notice about the angle measures of complementary angles, supplementary angles, and vertical angles?
1.6 Notetaking with Vocabulary
For use after Lesson 1.6

In your own words, write the meaning of each vocabulary term.

complementary angles

supplementary angles

adjacent angles

linear pair

vertical angles

Core Concepts

Complementary and Supplementary Angles

complementary angles

Two positive angles whose measures have a sum of $90^\circ$. Each angle is the complement of the other.

supplementary angles

Two positive angles whose measures have a sum of $180^\circ$. Each angle is the supplement of the other.

Notes:
Adjacent Angles

Complementary angles and supplementary angles can be adjacent angles or nonadjacent angles. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.

\[ \angle 5 \text{ and } \angle 6 \text{ are adjacent angles} \]

\[ \angle 7 \text{ and } \angle 8 \text{ are nonadjacent angles.} \]

Notes:

Linear Pairs and Vertical Angles

Two adjacent angles are a **linear pair** when their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.

\[ \angle 1 \text{ and } \angle 2 \text{ are a linear pair.} \]

Two angles are **vertical angles** when their sides form two pairs of opposite rays.

\[ \angle 3 \text{ and } \angle 6 \text{ are vertical angles.} \]

\[ \angle 4 \text{ and } \angle 5 \text{ are vertical angles.} \]

Notes:
1.6 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1 and 2, use the figure.

1. Name the pair(s) of adjacent complementary angles.

2. Name the pair(s) of nonadjacent supplementary angles.

In Exercises 3 and 4, find the angle measure.

3. \( \angle A \) is a complement of \( \angle B \) and \( m\angle A = 36^\circ \). Find \( m\angle B \).

4. \( \angle C \) is a supplement of \( \angle D \) and \( m\angle D = 117^\circ \). Find \( m\angle C \).

In Exercises 5 and 6, find the measure of each angle.

5. \( (10x + 22)^\circ \)

6. \( (12x + 5)^\circ \)

In Exercises 7–9, use the figure.

7. Identify the linear pair(s) that include \( \angle 1 \).

8. Identify the vertical angles.

9. Are \( \angle 6 \) and \( \angle 7 \) a linear pair? Explain.