

**Chapter**  
**11****Maintaining Mathematical Proficiency**

Find the mean, median, and mode of the data set. Then determine which measure of center best represents the data. Explain.

1. 26, 24, 55, 21, 32, 26

2. 63, 66, 61, 70, 69, 67, 63, 65

3. 40, 37, 21, 43, 37, 41, 43, 25, 37

Find and interpret the standard deviation of the data set.

4. 18, 11, 15, 20, 16

5. 78, 71, 68, 75, 46, 66

**11.1****Using Normal Distributions**

For use with Exploration 11.1

**Essential Question** In a normal distribution, about what percent of the data lies within one, two, and three standard deviations of the mean?

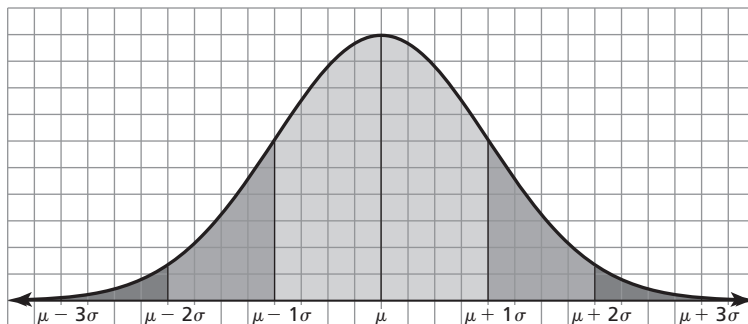
Recall that the standard deviation  $\sigma$  of a numerical data set is given by

$$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_n - \mu)^2}{n}}$$

where  $n$  is the number of values in the data set and  $\mu$  is the mean of the data set.

**1 EXPLORATION: Analyzing a Normal Distribution**

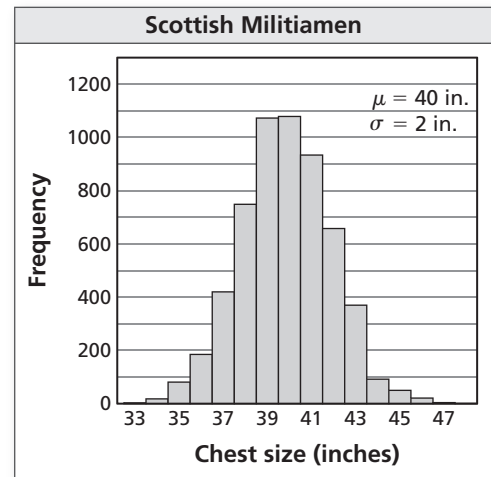
**Work with a partner.** In many naturally occurring data sets, the histogram of the data is bell-shaped. In statistics, such data sets are said to have a *normal distribution*. For the normal distribution shown below, estimate the percent of the data that lies within one, two, and three standard deviations of the mean. Each square on the grid represents 1%.



**11.1 Using Normal Distributions (continued)****2 EXPLORATION: Analyzing a Data Set**

**Work with a partner.** A famous data set was collected in Scotland in the mid-1800s. It contains the chest sizes (in inches) of 5738 men in the Scottish Militia. Do the data fit a normal distribution? Explain.

Chest size	Number of men
33	3
34	18
35	81
36	185
37	420
38	749
39	1073
40	1079
41	934
42	658
43	370
44	92
45	50
46	21
47	4
48	1

**Communicate Your Answer**

- In a normal distribution, about what percent of the data lies within one, two, and three standard deviations of the mean?
- Use the Internet or some other reference to find another data set that is normally distributed. Display your data in a histogram.

# 11.1

## Notetaking with Vocabulary

For use after Lesson 11.1

In your own words, write the meaning of each vocabulary term.

normal distribution

normal curve

standard normal distribution

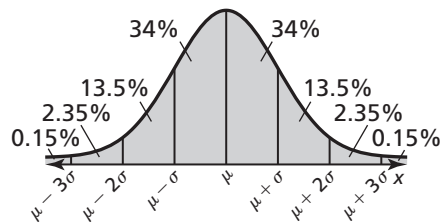
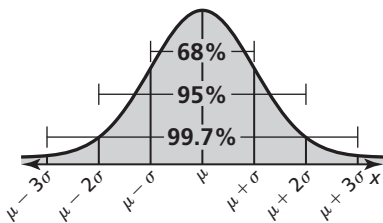
z-score

### Core Concepts

#### Areas Under a Normal Curve

A normal distribution with mean  $\mu$  and standard deviation  $\sigma$  has these properties.

- The total area under the related normal curve is 1.
- About 68% of the area lies within 1 standard deviation of the mean.
- About 95% of the area lies within 2 standard deviations of the mean.
- About 99.7% of the area lies within 3 standard deviations of the mean.



Notes:

**11.1** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–6, a normal distribution has mean  $\mu$  and standard deviation  $\sigma$ . Find the indicated probability for a randomly selected  $x$ -value from the distribution.

1.  $P(x \leq \mu - 2\sigma)$

2.  $P(x \geq \mu - 3\sigma)$

3.  $P(x \leq \mu + 2\sigma)$

4.  $P(x \geq \mu + 3\sigma)$

5.  $P(\mu - \sigma \leq x \leq \mu + 3\sigma)$

6.  $P(\mu - 2\sigma \leq x \leq \mu + \sigma)$

7. The scores for a math course test are normally distributed with a mean of 61 and a standard deviation of 11. The test scores range from 0 to 100.

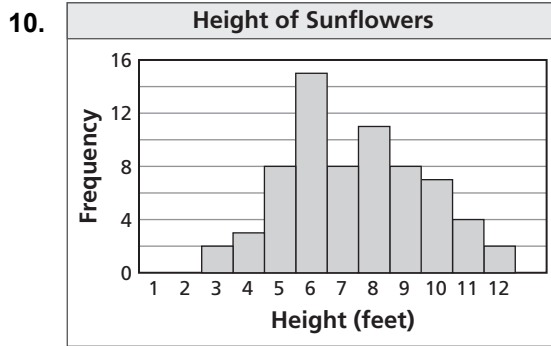
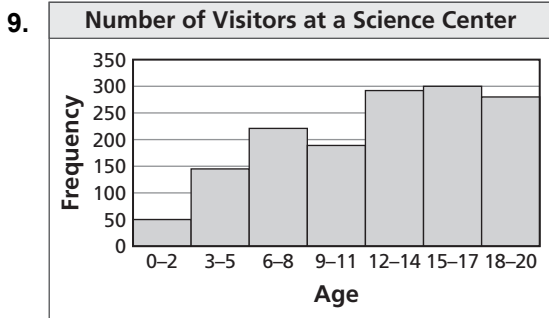
a. About what percent of the students taking the test have scores between 72 and 83?

b. About what percent of the students taking the test have scores less than 50?

**11.1** Notetaking with Vocabulary (continued)

8. The temperatures of a city are normally distributed over the course of a year. The mean temperature is  $55.2^{\circ}\text{F}$  and the standard deviation is  $6.3^{\circ}\text{F}$ . A day is randomly chosen.
- What is the probability that the chosen day is  $45^{\circ}\text{F}$  or cooler?
  - What is the probability that the chosen day is cooler than  $32.5^{\circ}\text{F}$ ?
  - What is the probability that the chosen day is between  $32.5^{\circ}\text{F}$  and  $45^{\circ}\text{F}$ ?
  - What is the probability that the chosen day is  $60^{\circ}\text{F}$  or warmer?

In Exercises 9 and 10, determine whether the histogram has a normal distribution.



**11.2****Populations, Samples, and Hypotheses**

For use with Exploration 11.2

**Essential Question** How can you test theoretical probability using sample data?

**1 EXPLORATION: Using Sample Data**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- When two six-sided dice are rolled, what is the theoretical probability that you roll the same number on both dice?
- Conduct an experiment to check your answer in part (a). What sample size did you use? Explain your reasoning.
- Use the dice rolling simulator at *BigIdeasMath.com* to complete the table. Do your experimental data check the theoretical probability you found in part (a)? Explain. What happens as you increase the sample size?

Number of Rolls	Number of Times Same Number Appears	Experimental Probability
100		
500		
1000		
5000		
10,000		

**2 EXPLORATION: Using Sample Data**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- When three six-sided dice are rolled, what is the theoretical probability that you roll the same number on all three dice?

**11.2 Populations, Samples, and Hypotheses (continued)****2 EXPLORATION:** Using Sample Data (continued)

- b. Compare the theoretical probability you found in part (a) with the theoretical probability you found in Exploration 1(a).

- c. Conduct an experiment to check your answer in part (a). How does adding a die affect the sample size that you use? Explain your reasoning.



- d. Use the dice rolling simulator at *BigIdeasMath.com* to check your answer to part (a). What happens as you increase the sample size?

**Communicate Your Answer**

3. How can you test theoretical probability using sample data?
4. Conduct an experiment to determine the probability of rolling a sum of 7 when two six-sided dice are rolled. Then find the theoretical probability and compare your answers.



# 11.2

## Notetaking with Vocabulary

For use after Lesson 11.2

In your own words, write the meaning of each vocabulary term.

population

sample

parameter

statistic

hypothesis

**Notes:**

**11.2** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–3, identify the population and sample. Describe the sample.

1. In a city, a survey of 3257 adults ages 18 and over found that 2605 of them own a tablet.
2. To find out the consumers' response towards a new flavor of sports drink, a company surveys 1000 athletes who drink sports drinks and finds that 726 of them like the new flavor.
3. In a school district, a survey of 1500 high school students found that 824 of them have a part time job in the summer.

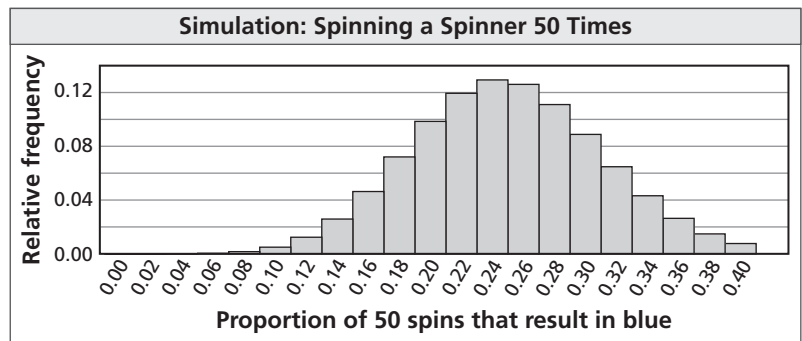
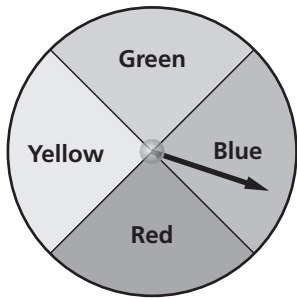
In Exercises 4–7, determine whether the numerical value is a parameter or a statistic.

Explain your reasoning.

4. Eighty-two percent of the residents in one neighborhood in a town voted to approve building a bike lane through town.

**11.2** Notetaking with Vocabulary (continued)

5. In a science class, 25% of the students wear glasses.
  
6. In a recent year, the median household income in the United States was about \$52,000.
  
7. A survey of some visitors to a museum found that 84% thought the new planetarium was very exciting.
  
8. You spin the spinner five times and every time the spinner lands on blue. You suspect the spinner favors blue. The maker of the spinner claims that the spinner does not favor any color. You simulate spinning the spinner 50 times by repeatedly drawing 200 random samples of size 50. The histogram shows the results. Use the histogram to determine what you should conclude when you spin the actual spinner 50 times and the spinner lands on blue (a) 12 times and (b) 19 times.



**11.3****Collecting Data**

For use with Exploration 11.3

**Essential Question** What are some considerations when undertaking a statistical study?

**1 EXPLORATION: Analyzing Sampling Techniques**

**Work with a partner.** Determine whether each sample is representative of the population. Explain your reasoning.

- a. To determine the number of hours people exercise during a week, researchers use random-digit dialing and call 1500 people.
  
  
  
  
  
  
  
  
  
  
- b. To determine how many text messages high school students send in a week, researchers post a survey on a website and receive 750 responses.
  
  
  
  
  
  
  
  
  
  
- c. To determine how much money college students spend on clothes each semester, a researcher surveys 450 college students as they leave the university library.
  
  
  
  
  
  
  
  
  
  
- d. To determine the quality of service customers receive, an airline sends an e-mail survey to each customer after the completion of a flight.

**2 EXPLORATION: Analyzing Survey Questions**

**Work with a partner.** Determine whether each survey question is biased. Explain your reasoning. If so, suggest an unbiased rewording of the question.

- a. Does eating nutritious, whole-grain foods improve your health?

**11.3 Collecting Data (continued)****2 EXPLORATION: Analyzing Survey Questions (continued)**

- b. Do you ever attempt the dangerous activity of texting while driving?
  
- c. How many hours do you sleep each night?
  
- d. How can the mayor of your city improve his or her public image?

**3 EXPLORATION: Analyzing Survey Randomness and Truthfulness**

**Work with a partner.** Discuss each potential problem in obtaining a random survey of a population. Include suggestions for overcoming the problem.

- a. The people selected might not be a random sample of the population.
  
- b. The people selected might not be willing to participate in the survey.
  
- c. The people selected might not be truthful when answering the question.
  
- d. The people selected might not understand the survey question.

**Communicate Your Answer**

- 4. What are some considerations when undertaking a statistical study?
  
- 5. Find a real-life example of a biased survey question. Then suggest an unbiased rewording of the question.

# 11.3

## Notetaking with Vocabulary

For use after Lesson 11.3

In your own words, write the meaning of each vocabulary term.

random sample

self-selected sample

systematic sample

stratified sample

cluster sample

convenience sample

bias

unbiased sample

biased sample

experiment

observational study

survey

**11.3** Notetaking with Vocabulary (continued)

simulation

biased question

**Core Concepts**

**Types of Samples**

For a **self-selected sample**, members of a population can volunteer to be in the sample.



For a **systematic sample**, a rule is used to select members of a population. For instance, selecting every other person.



For a **stratified sample**, a population is divided into smaller groups that share a similar characteristic. A sample is then randomly selected from each group.



For a **cluster sample**, a population is divided into groups, called *clusters*. All of the members in one or more of the clusters are selected.



For a **convenience sample**, only members of a population who are easy to reach are selected.

**11.3** Notetaking with Vocabulary (continued)**Methods of Collecting Data**

An **experiment** imposes a treatment on individuals in order to collect data on their response to the treatment. The treatment may be a medical treatment, or it can be any action that might affect a variable in the experiment, such as adding methanol to gasoline and then measuring its effect on fuel efficiency.

An **observational study** observes individuals and measures variables without controlling the individuals or their environment. This type of study is used when it is difficult to control or isolate the variable being studied, or when it may be unethical to subject people to a certain treatment or to withhold it from them.

A **survey** is an investigation of one or more characteristics of a population. In a survey, every member of a sample is asked one or more questions.

A **simulation** uses a model to reproduce the conditions of a situation or process so that the simulated outcomes closely match the real-world outcomes. Simulations allow you to study situations that are impractical or dangerous to create in real life.

**Notes:**



**11.3** Notetaking with Vocabulary (continued)**Extra Practice**

**In Exercises 1–3, identify the type of sample described.**

1. A restaurant owner wants to know whether the customers are satisfied with the service. Every fifth customer who exits the restaurant is surveyed.
2. An electronic manufacturer wants to know the customers' responses towards a newly released media player. Emails are sent to customers who recently purchased the device to participate in an online survey at their convenience.
3. A survey is conducted in a state to find out how many households own more than one vehicle. Households are divided into north, east, south, and west regions of the state, and a sample is randomly surveyed from each region.

**In Exercises 4 and 5, identify the type of sample and explain why the sample is biased.**

4. A manager of a company wants to determine whether the employees are satisfied with the lounge room. The manager surveys the employees who are in the lounge room during lunch break.
5. A news station asks its viewers to participate in an online poll about the presidential candidates.

**In Exercises 6 and 7, identify the method of data collection the situation describes.**

6. A researcher records whether shoppers at a grocery store buy magazines at the checkout aisles while waiting in line to check out.
7. A meteorologist uses a computer model to track the trajectory of a hurricane.

# 11.4

## Experimental Design

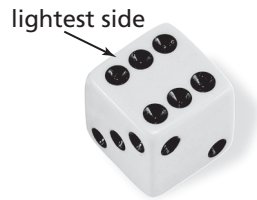
For use with Exploration 11.4

**Essential Question** How can you use an experiment to test a conjecture?

**1 EXPLORATION:** Using an Experiment

**Work with a partner.** Standard white playing dice are manufactured with black dots that are indentations, as shown. So, the side with six indentations is the lightest side and the side with one indentation is the heaviest side.

You make a conjecture that when you roll a standard playing die, the number 6 will come up more often because it is the lightest side, and the number 1 will come up least often because it is the heaviest side. To test your conjecture, roll a standard playing die 25 times. Record the results in the table. Does the experiment confirm your conjecture? Explain your reasoning.



<b>Number</b>						
<b>Rolls</b>						

**11.4** Experimental Design (continued)**2** **EXPLORATION:** Analyzing an Experiment

**Work with a partner.** To overcome the imbalance of standard playing dice, one of the authors of this book invented and patented 12-sided dice, on which each number from 1 through 6 appears twice (on opposing sides). See *BigIdeasMath.com*.



As part of the patent process, a standard playing die was rolled 27,090 times. The results are shown below.

Number	1	2	3	4	5	6
Rolls	4293	4524	4492	4397	4623	4761

What can you conclude from the results of this experiment? Explain your reasoning.

**Communicate Your Answer**

- How can you use an experiment to test a conjecture?
- Exploration 2 shows the results of rolling a standard playing die 27,090 times to test the conjecture in Exploration 1. Why do you think the number of trials was so large?
- Make a conjecture about the outcomes of rolling the 12-sided die in Exploration 2. Then design an experiment that could be used to test your conjecture. Be sure that your experiment is practical to complete and includes enough trials to give meaningful results.

# 11.4

## Notetaking with Vocabulary

For use after Lesson 11.4

In your own words, write the meaning of each vocabulary term.

controlled experiment

control group

treatment group

randomization

randomized comparative experiment

placebo

replication

### Core Concepts

#### Comparative Studies and Causality

- A rigorous randomized comparative experiment, by eliminating sources of variation other than the controlled variable, can make valid cause-and-effect conclusions possible.
- An observational study can identify *correlation* between variables, but not *causality*. Variables, other than what is being measured, may be affecting the results.

Notes:

**11.4** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1 and 2, determine whether the study is a randomized comparative experiment. If it is, describe the treatment, the treatment group, and the control group. If it is not, explain why not and discuss whether the conclusions drawn from the study are valid.

1.

<i>Baby DVDs</i>
<b>Baby DVDs Improves Language Ability</b>  To test whether baby DVDs that highlight words and introduce music and art can improve language ability, parents with babies 0–24 months were given the choice of whether to let their babies watch the DVDs. Fifty babies who watched the DVDs were observed for a year as well as 50 other babies who did not watch the DVDs. At the end of the year, babies who watched the DVDs scored higher in a language development test.

2.

<i>Type 1 Diabetes</i>
<b>New Drug Improves Blood Glucose Control</b>  In a clinical trial, 100 Type 1 diabetic patients volunteered to take a new drug. Fifty percent of the patients received the drug and the other fifty percent received a placebo. After one year, the patients who received the drug had better blood glucose control while the placebo group experienced no significant change.

**11.4 Notetaking with Vocabulary (continued)**

**In Exercises 3 and 4, explain whether the research topic is best investigated through an experiment or an observational study. Then describe the design of the experiment or observational study.**

3. A criminologist wants to know whether social factors are the cause of the criminal behavior.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
4. A pharmaceutical company wants to know whether the new medication on heart disease has a side effect on individuals.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
5. A company wants to test the effectiveness of a new moisturizing cream designed to help improve skin complexion. Identify a potential problem, if any, with each experimental design. Then describe how you can improve it.
  - a. The company randomly selects ten individuals. Five subjects are given the new moisturizing cream and the other five are given a placebo. After eight weeks, each subject is evaluated and it is determined that the five subjects who have been using the cream have improved skin complexion.
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  - b. The company randomly selects a large group of individuals. Half of the individuals are given the new moisturizing cream and the other half of the individuals may use their own existing moisturizers or none at all. After eight weeks, each subject is evaluated and it is determined that a significant large number of subjects who received the moisturizing cream have improved skin complexion.

# 11.5

## Making Inferences from Sample Surveys

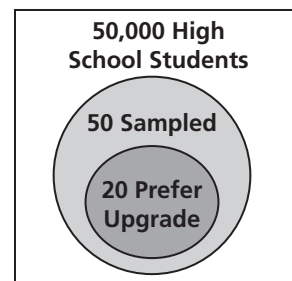
For use with Exploration 11.5

**Essential Question** How can you use a sample survey to infer a conclusion about a population?

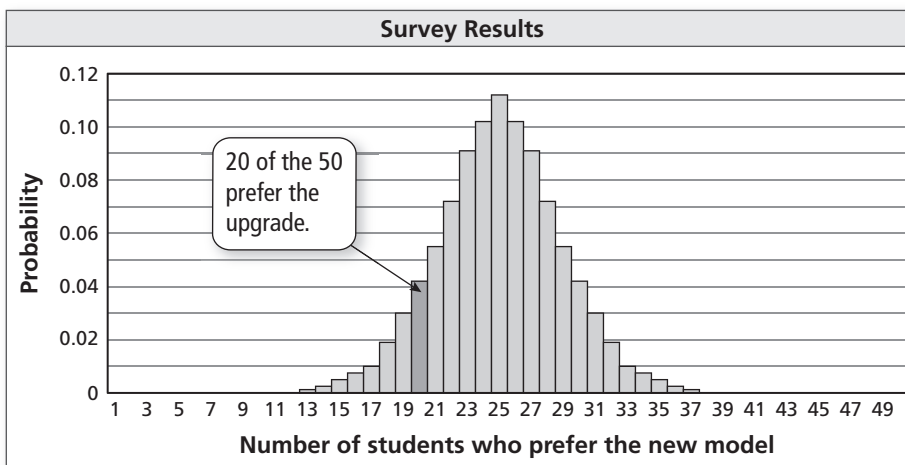
### 1 EXPLORATION: Making an Inference from a Sample

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** You conduct a study to determine what percent of the high school students in your city would prefer an upgraded model of their current cell phone. Based on your intuition and talking with a few acquaintances, you think that 50% of high school students would prefer an upgrade. You survey 50 randomly chosen high school students and find that 20 of them prefer an upgraded model.



- Based on your sample survey, what percent of the high school students in your city would prefer an upgraded model? Explain your reasoning.
- In spite of your sample survey, is it still possible that 50% of the high school students in your city prefer an upgraded model? Explain your reasoning.
- To investigate the likelihood that you could have selected a sample of 50 from a population in which 50% of the population does prefer an upgraded model, you create a binomial distribution as shown below. From the distribution, estimate the probability that exactly 20 students surveyed prefer an upgraded model. Is this event likely to occur? Explain your reasoning.



## 11.5 Making Inferences from Sample Surveys (continued)

### 1 EXPLORATION: Making an Inference from a Sample (continued)

- d. When making inferences from sample surveys, the sample must be random. In the situation described on the previous page, describe how you could design and conduct a survey using a random sample of 50 high school students who live in a large city.

### Communicate Your Answer

2. How can you use a sample survey to infer a conclusion about a population?
3. In Exploration 1(c), what is the probability that exactly 25 students you survey prefer an upgraded model?



**11.5****Notetaking with Vocabulary**

For use after Lesson 11.5

In your own words, write the meaning of each vocabulary term.

descriptive statistics

inferential statistics

margin of error

**Core Concepts****Margin of Error Formula**

When a random sample of size  $n$  is taken from a large population, the margin of error is approximated by

$$\text{Margin of error} = \pm \frac{1}{\sqrt{n}}.$$

This means that if the percent of the sample responding a certain way is  $p$  (expressed as a decimal), then the percent of the population who would respond the same way is likely to be between  $p - \frac{1}{\sqrt{n}}$  and  $p + \frac{1}{\sqrt{n}}$ .

**Notes:**

**11.5** Notetaking with Vocabulary (continued)**Extra Practice**

1. The numbers of minutes spent each day on a social networking website by a random sample of people between the ages of 18 and 64 are shown in the table. Estimate the population mean  $\mu$ .

Number of Minutes				
175	15	190	180	45
100	210	240	190	60
102	165	253	192	102
12	180	189	193	230
300	185	190	395	186
183	200	165	195	409

2. Use the data in Exercise 1 to answer each question.
- Estimate the population proportion  $\rho$  of social network users between the ages of 18 and 64 who spend more than 120 minutes each day on a social networking website.
  - Estimate the population proportion  $\rho$  of social network users between the ages of 18 and 64 who spend fewer than 60 minutes each day on a social networking website.

**11.5** Notetaking with Vocabulary (continued)

3. Two candidates, A and B, are running for the student council president position. The table shows the results from four surveys of randomly selected students in the school. The students are asked whether they will vote for candidate A. The results are shown in the table.

Sample Size	Number of Votes for Candidate A	Percent of Votes for Candidate A
10	6	60%
20	11	55%
50	20	40%
150	64	42.7%

- a. Based on the results of the first two surveys, do you think Candidate A will win the election? Explain.
- b. Based on the results in the table, do you think Candidate A will win the election? Explain.
4. A national polling company claims that 39% of Americans rate the overall quality of the environment in the nation as “good.” You survey a random sample of 50 people. What can you conclude about the accuracy of the claim that the population proportion is 0.39 when 19 Americans say the quality of the environment is good?
5. In a survey of 2680 people in the U.S., 60% said that their diet is somewhat healthy.
- a. What is the margin of error for the survey?
- b. Give an interval that is likely to contain the exact percent of all people in the U.S. who think their diet is somewhat healthy.

**11.6****Making Inferences from Experiments**

For use with Exploration 11.6

**Essential Question** How can you test a hypothesis about an experiment?

**1 EXPLORATION: Resampling Data**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

**Work with a partner.** A randomized comparative experiment tests whether water with dissolved calcium affects the yields of yellow squash plants. The table shows the results.

Yield (kilograms)	
Control Group	Treatment Group
1.0	1.1
1.2	1.3
1.5	1.4
0.9	1.2
1.1	1.0
1.4	1.7
0.8	1.8
0.9	1.1
1.3	1.1
1.6	1.8

- Find the mean yield of the control group and the mean yield of the treatment group. Then find the difference of the two means. Record the results.
- Write each yield measurement from the table on an equal-sized piece of paper. Place the pieces of paper in a bag, shake, and randomly choose 10 pieces of paper. Call this the “control” group, and call the 10 pieces in the bag the “treatment” group. Then repeat part (a) and return the pieces to the bag. Perform this resampling experiment five times.

- How does the difference in the means of the control and treatment groups compare with the differences resulting from chance?

**11.6 Making Inferences from Experiments (continued)****2 EXPLORATION: Evaluating Results**

**Work as a class.** To conclude that the treatment is responsible for the difference in yield, you need strong evidence to reject the hypothesis:

*Water dissolved in calcium has no effect on the yields of yellow squash plants.*

To evaluate this hypothesis, compare the experimental difference of means with the resampling differences.

- Collect all the resampling differences of means found in Exploration 1(b) for the whole class and display these values in a histogram.
- Draw a vertical line on your class histogram to represent the experimental difference of means found in Exploration 1(a).
- Where on the histogram should the experimental difference of means lie to give evidence for rejecting the hypothesis?
- Is your class able to reject the hypothesis? Explain your reasoning.

**Communicate Your Answer**

- How can you test a hypothesis about an experiment?
- The randomized comparative experiment described in Exploration 1 is replicated and the results are shown in the table. Repeat Explorations 1 and 2 using this data set. Explain any differences in your answers.

	Yield (kilograms)									
Control Group	0.9	0.9	1.4	0.6	1.0	1.1	0.7	0.6	1.2	1.3
Treatment Group	1.0	1.2	1.2	1.3	1.0	1.8	1.7	1.2	1.0	1.9

# 11.6

## Notetaking with Vocabulary

For use after Lesson 11.6

In your own words, write the meaning of each vocabulary term.

randomized comparative experiment

control group

treatment group

mean

dot plot

outlier

simulation

hypothesis

**Notes:**

**11.6** Notetaking with Vocabulary (continued)**Extra Practice**

1. A randomized comparative experiment tests whether students who are given weekly quizzes do better on the comprehensive final exam. The control group has 10 students and the treatment group, which receives weekly quizzes, has 10 students. The table shows the results.

	Final Exam Scores (out of 100 points)									
Control Group	82	55	76	92	76	76	82	58	69	79
Treatment Group	92	90	88	73	88	63	94	81	81	77

- a. Find the mean score of the control group.
- b. Find the mean score of the treatment group.
- c. Find the experimental difference of the means.
- d. Display the data in a double dot plot.
- e. What can you conclude?

**11.6** Notetaking with Vocabulary (continued)

2. Resample the data in Exercise 1 using a simulation. Use the means of the new control and treatment groups to calculate the difference of the means.

	Final Exam Scores (out of 100 points)									
<b>New Control Group</b>										
<b>New Treatment Group</b>										

3. To analyze the hypothesis below, use the histogram which shows the results from 200 resamplings of the data in Exercise 1.

*Weekly Quizzes have no effect on final exam scores.*

Compare the experimental difference in Exercise 1 with the resampling differences. What can you conclude about the hypothesis? Do weekly quizzes have an effect on final exam scores?

