Name____ Date __________

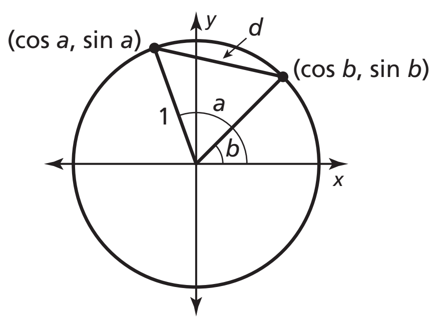
9.8 Using Sum and Difference Formulas
For use with Exploration 9.8

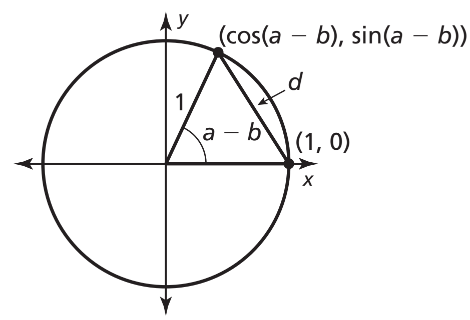
Essential Question How can you evaluate trigonometric functions of the sum or difference of two angles?

1 EXPLORATION: Deriving a Difference Formula

Work with a partner.

a. Explain why the two triangles shown are congruent.





b. Use the Distance Formula to write an expression for d in the first unit circle.

c. Use the Distance Formula to write an expression for d in the second unit circle.

d. Write an equation that relates the expressions in parts (b) and (c). Then simplify this equation to obtain a formula for $\cos(a - b)$.

2 EXPLORATION: Deriving a Sum Formula

Work with a partner. Use the difference formula you derived in Exploration 1 to write a formula for $\cos(a + b)$ in terms of sine and cosine of a and b. *Hint*: Use the fact that

$$\cos(a + b) = \cos[a - (-b)].$$

9.8 Using Sum and Difference Formulas (continued)

3 EXPLORATION: Deriving Difference and Sum Formulas

Work with a partner. Use the formulas you derived in Explorations 1 and 2 to write formulas for $\sin(a - b)$ and $\sin(a + b)$ in terms of sine and cosine of a and b. *Hint*: Use the cofunction identities

$$\sin\left(\frac{\pi}{2} - a\right) = \cos a \text{ and } \cos\left(\frac{\pi}{2} - a\right) = \sin a$$

and the fact that

$$\cos\left[\left(\frac{\pi}{2} - a\right) + b\right] = \sin(a - b) \text{ and } \sin(a + b) = \sin[a - (-b)].$$

Communicate Your Answer

4. How can you evaluate trigonometric functions of the sum or difference of two angles?

5. **a.** Find the exact values of sin 75° and cos 75° using sum formulas. Explain your reasoning.

b. Find the exact values of sin 75° and cos 75° using difference formulas. Compare your answers to those in part (a).

Name____ Date__________

9.8 Notetaking with Vocabulary

For use after Lesson 9.8

In your own words, write the meaning of each vocabulary term.

ratio

Core Concepts

Sum and Difference Formulas

Sum Formulas	Difference Formulas
$\sin(a + b) = \sin a \cos b + \cos a \sin b$	$\sin(a - b) = \sin a \cos b - \cos a \sin b$
$\cos(a + b) = \cos a \cos b - \sin a \sin b$	$\cos(a - b) = \cos a \cos b + \sin a \sin b$
$\tan(a + b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$	$\tan(a - b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

Notes:

Extra Practice

In Exercises 1–4, find the exact value of the expression.

1. $\sin(-75^\circ)$

2. $\tan 120^\circ$

3. $\cos\left(-\frac{7\pi}{12}\right)$

4. $\cos \frac{35\pi}{12}$

In Exercises 5–8, evaluate the expression given that $\sin a = -\frac{4}{5}$ with $\pi < a < \frac{3\pi}{2}$ and $\cos b = \frac{5}{13}$ with $0 < b < \frac{\pi}{2}$.

5. $\cos(a - b)$

6. $\sin(a + b)$

9.8 Notetaking with Vocabulary (continued)

7. $\tan(a + b)$

8. $\tan(a - b)$

In Exercises 9–12, simplify the expression.

9. $\sin\left(x + \frac{\pi}{2}\right)$

10. $\tan(x - 2\pi)$

11. $\cos(x - 2\pi)$

12. $\cos\left(x + \frac{5\pi}{2}\right)$

In Exercises 13 and 14, solve the equation for $0 \leq x \leq 2\pi$.

13. $\sin\left(x + \frac{3\pi}{2}\right) = 1$

14. $\sin\left(x - \frac{\pi}{2}\right) + \cos\left(x - \frac{\pi}{2}\right) = 0$