

9.7

Using Trigonometric Identities

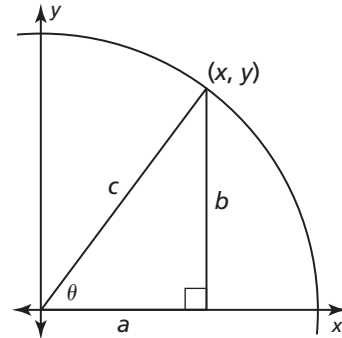
For use with Exploration 9.7

Essential Question How can you verify a trigonometric identity?

1 EXPLORATION: Writing a Trigonometric Identity

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. In the figure, the point (x, y) is on a circle of radius c with center at the origin.



- a. Write an equation that relates a , b , and c .
- b. Write expressions for the sine and cosine ratios of angle θ .
- c. Use the results from parts (a) and (b) to find the sum of $\sin^2 \theta$ and $\cos^2 \theta$. What do you observe?
- d. Complete the table to verify that the identity you wrote in part (c) is valid for angles (of your choice) in each of the four quadrants.

	θ	$\sin^2 \theta$	$\cos^2 \theta$	$\sin^2 \theta + \cos^2 \theta$
QI				
QII				
QIII				
QIV				

9.7 Using Trigonometric Identities (continued)**2 EXPLORATION: Writing Other Trigonometric Identities**

Work with a partner. The trigonometric identity you derived in Exploration 1 is called a Pythagorean identity. There are two other Pythagorean identities. To derive them, recall the four relationships:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \qquad \csc \theta = \frac{1}{\sin \theta}$$

- a. Divide each side of the Pythagorean identity you derived in Exploration 1 by $\cos^2 \theta$ and simplify. What do you observe?

- b. Divide each side of the Pythagorean identity you derived in Exploration 1 by $\sin^2 \theta$ and simplify. What do you observe?

Communicate Your Answer

3. How can you verify a trigonometric identity?

4. Is $\sin \theta = \cos \theta$ a trigonometric identity? Explain your reasoning.

5. Give some examples of trigonometric identities that are different than those in Explorations 1 and 2.

9.7**Notetaking with Vocabulary**

For use after Lesson 9.7

In your own words, write the meaning of each vocabulary term.

trigonometric identity

Core Concepts**Fundamental Trigonometric Identities****Reciprocal Identities**

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

Negative Angle Identities

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

Notes:

9.7 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–4, find the values of the other five trigonometric functions of θ .

1. $\sin \theta = \frac{1}{5}, \frac{\pi}{2} < \theta < \pi$

2. $\cos \theta = -\frac{4}{5}, \pi < \theta < \frac{3\pi}{2}$

3. $\cot \theta = \frac{4}{7}, 0 < \theta < \frac{\pi}{2}$

4. $\sec \theta = \frac{\sqrt{13}}{3}, \frac{3\pi}{2} < \theta < 2\pi$

In Exercises 5–8, simplify the expression.

5. $-\frac{\tan \theta}{\sec \theta}$

6. $\cos\left(\frac{\pi}{2} - \theta\right)(1 - \cos^2 \theta)$

9.7 Notetaking with Vocabulary (continued)

7.
$$\frac{2 \sec^2 x - 2 \tan^2 x}{\tan(-x) \cos(-x)}$$

8.
$$\frac{-\sin\left(\frac{\pi}{2} - \theta\right)}{\sec(-\theta)} - \sin^2 \theta$$

In Exercises 9 and 10, verify the identity.

9.
$$\frac{1 - \cos^2 \theta}{\sec^2 \theta} - \sin^2 \theta = -\sin^4 \theta$$

10.
$$\csc x + \cot x = \frac{\sin x}{1 - \cos x}$$