Essential Question  What are the characteristics of the graph of the tangent function?

1 EXPLORATION: Graphing the Tangent Function

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner.

a. Complete the table for \( y = \tan x \), where \( x \) is an angle measure in radians.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-\frac{\pi}{2})</th>
<th>(-\frac{\pi}{3})</th>
<th>(-\frac{\pi}{4})</th>
<th>(-\frac{\pi}{6})</th>
<th>0</th>
<th>(\frac{\pi}{6})</th>
<th>(\frac{\pi}{4})</th>
<th>(\frac{\pi}{3})</th>
<th>(\frac{\pi}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \tan x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>(\frac{2\pi}{3})</th>
<th>(\frac{3\pi}{4})</th>
<th>(\frac{5\pi}{6})</th>
<th>(\pi)</th>
<th>(\frac{7\pi}{6})</th>
<th>(\frac{5\pi}{4})</th>
<th>(\frac{4\pi}{3})</th>
<th>(\frac{3\pi}{2})</th>
<th>(\frac{5\pi}{3})</th>
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</tbody>
</table>

b. The graph of \( y = \tan x \) has vertical asymptotes at \( x \)-values where \( \tan x \) is undefined. Plot the points \((x, y)\) from part (a). Then use the asymptotes to sketch the graph of \( y = \tan x \).
c. For the graph of \( y = \tan x \), identify the asymptotes, the \( x \)-intercepts, and the intervals for which the function is increasing or decreasing over \( -\frac{\pi}{2} \leq x \leq \frac{3\pi}{2} \). Is the tangent function even, odd, or neither?

Communicate Your Answer

2. What are the characteristics of the graph of the tangent function?

3. Describe the asymptotes of the graph of \( y = \cot x \) on the interval \( -\frac{\pi}{2} < x < \frac{3\pi}{2} \).
In your own words, write the meaning of each vocabulary term.

asymptote

period

amplitude

x-intercept

transformations

Core Concepts

Characteristics of $y = \tan x$ and $y = \cot x$

The functions $y = \tan x$ and $y = \cot x$ have the following characteristics.

- The domain of $y = \tan x$ is all real numbers except odd multiples of $\frac{\pi}{2}$. At these $x$-values, the graph has vertical asymptotes.
- The domain of $y = \cot x$ is all real numbers except multiples of $\pi$. At these $x$-values, the graph has vertical asymptotes.
- The range of each function is all real numbers. So, the functions do not have maximum or minimum values, and the graphs do not have an amplitude.
- The period of each graph is $\pi$.
- The $x$-intercepts for $y = \tan x$ occur when $x = 0, \pm \pi, \pm 2\pi, \pm 3\pi, \ldots$
- The $x$-intercepts for $y = \cot x$ occur when $x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2}, \ldots$

Notes:
Period and Vertical Asymptotes of $y = a \tan bx$ and $y = a \cot bx$

The period and vertical asymptotes of the graphs of $y = a \tan bx$ and $y = a \cot bx$, where $a$ and $b$ are nonzero real numbers, are as follows.

- The period of the graph of each function is $\frac{\pi}{|b|}$.
- The vertical asymptotes for $y = a \tan bx$ are at odd multiples of $\frac{\pi}{2|b|}$.
- The vertical asymptotes for $y = a \cot bx$ are at multiples of $\frac{\pi}{|b|}$.

Notes:

Characteristics of $y = \sec x$ and $y = \csc x$

The functions $y = \sec x$ and $y = \csc x$ have the following characteristics.

- The domain of $y = \sec x$ is all real numbers except odd multiples of $\frac{\pi}{2}$. At these $x$-values, the graph has vertical asymptotes.
- The domain of $y = \csc x$ is all real numbers except multiples of $\pi$. At these $x$-values, the graph has vertical asymptotes.
- The range of each function is $y \leq -1$ and $y \geq 1$. So, the graphs do not have an amplitude.
- The period of each graph is $2\pi$.

Notes:
Extra Practice

In Exercises 1–6, graph one period of the function. Describe the graph of \( g \) as a transformation of the graph of its parent function.

1. \( g(x) = \tan 2x \)

2. \( g(x) = 2 \cot \frac{1}{2}x \)

3. \( g(x) = \frac{1}{4} \tan \frac{\pi}{4}x \)

4. \( g(x) = \frac{1}{2} \cot 3x \)

5. \( g(x) = 2 \sec 2x \)

6. \( g(x) = \csc 2\pi x \)