

9.4

Graphing Sine and Cosine Functions

For use with Exploration 9.4

Essential Question What are the characteristics of the graphs of the sine and cosine functions?

1 EXPLORATION: Graphing the Sine Function

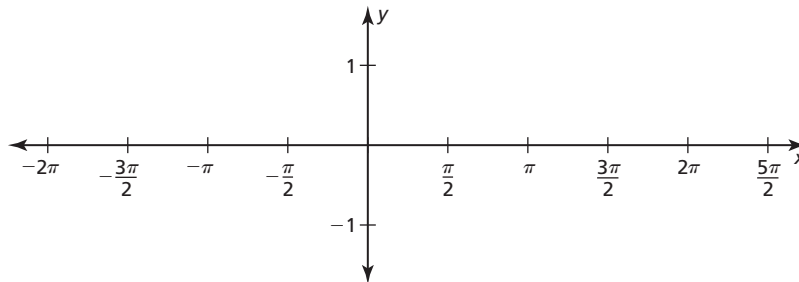
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- a. Complete the table for $y = \sin x$, where x is an angle measure in radians.

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0
$y = \sin x$									
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$
$y = \sin x$									

- b. Plot the points (x, y) from part (a). Draw a smooth curve through the points to sketch the graph of $y = \sin x$.



- c. Use the graph to identify the x -intercepts, the x -values where the local maximums and minimums occur, and the intervals for which the function is increasing or decreasing over $-2\pi \leq x \leq 2\pi$. Is the sine function *even*, *odd*, or *neither*?

9.4 Graphing Sine and Cosine Functions (continued)**2 EXPLORATION:** Graphing the Cosine Function

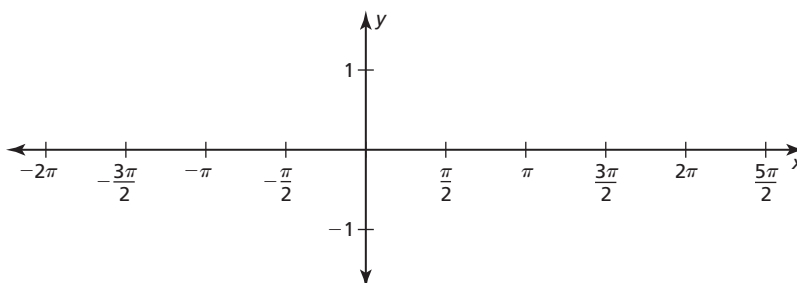
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- a. Complete the table for $y = \cos x$ using the same values of x as those used in Exploration 1.

x	-2π	$-\frac{7\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{4}$	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0
$y = \cos x$									
x	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$
$y = \cos x$									

- b. Plot the points (x, y) from part (a) and sketch the graph of $y = \cos x$



- c. Use the graph to identify the x -intercepts, the x -values where the local maximums and minimums occur, and the intervals for which the function is increasing or decreasing over $-2\pi \leq x \leq 2\pi$. Is the cosine function *even*, *odd*, or *neither*?

Communicate Your Answer

3. What are the characteristics of the graphs of the sine and cosine functions?
4. Describe the end behavior of the graph of $y = \sin x$.

9.4**Notetaking with Vocabulary**

For use after Lesson 9.4

In your own words, write the meaning of each vocabulary term.

amplitude

periodic function

cycle

period

phase shift

midline

Core Concepts**Characteristics of $y = \sin x$ and $y = \cos x$**

- The domain of each function is all real numbers.
- The range of each function is $-1 \leq y \leq 1$. So, the minimum value of each function is -1 and the maximum value is 1 .
- The **amplitude** of the graph of each function is one-half of the difference of the maximum value and the minimum value, or $\frac{1}{2}[1 - (-1)] = 1$.
- Each function is **periodic**, which means that its graph has a repeating pattern. The shortest repeating portion of the graph is called a **cycle**. The horizontal length of each cycle is called the **period**. The graph of each function has a period of 2π .
- The x -intercepts for $y = \sin x$ occur when $x = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots$
- The x -intercepts for $y = \cos x$ occur when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \dots$

9.4 Notetaking with Vocabulary (continued)**Amplitude and Period**

The amplitude and period of the graphs of $y = a \sin bx$ and $y = a \cos bx$, where a and b are nonzero real numbers, are as follows:

$$\text{Amplitude} = |a| \qquad \text{Period} = \frac{2\pi}{|b|}$$

Notes:**Graphing $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$**

To graph $y = a \sin b(x - h) + k$ or $y = a \cos b(x - h) + k$ where $a > 0$ and $b > 0$, follow these steps:

Step 1 Identify the amplitude a , the period $\frac{2\pi}{b}$, the horizontal shift h , and the vertical shift k of the graph.

Step 2 Draw the horizontal line $y = k$, called the **midline** of the graph.

Step 3 Find the five key points by translating the key points of $y = a \sin bx$ or $y = a \cos bx$ horizontally h units and vertically k units.

Step 4 Draw the graph through the five translated key points.

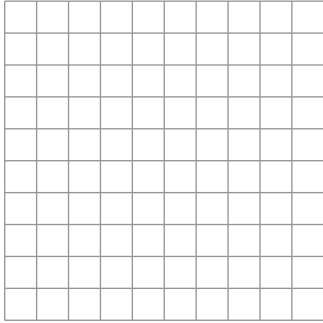
Notes:

9.4 Notetaking with Vocabulary (continued)

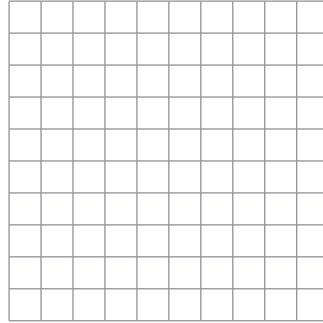
Extra Practice

In Exercises 1–4, identify the amplitude and period of the function. Then graph the function and describe the graph of g as a transformation of the graph of its parent function.

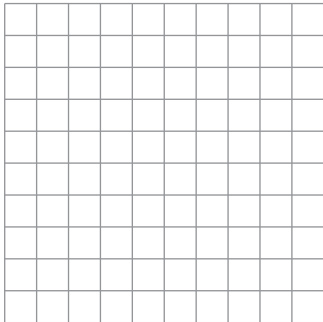
1. $g(x) = \sin 2x$



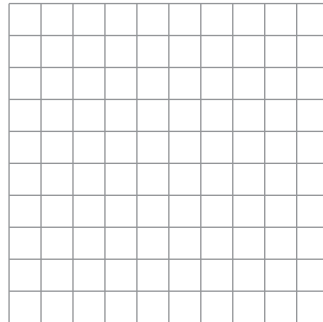
2. $g(x) = \frac{1}{3} \cos 2x$



3. $g(x) = 4 \sin 2\pi x$

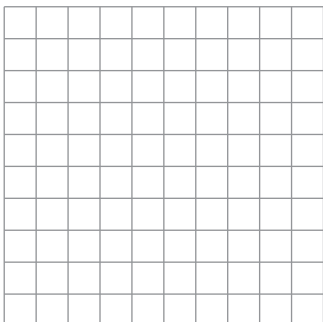


4. $g(x) = \frac{1}{2} \cos 3\pi x$



In Exercises 5 and 6, graph the function.

5. $g(x) = \sin \frac{1}{2}(x - \pi) + 1$



6. $g(x) = \cos \left(x + \frac{\pi}{2} \right) - 3$

