Name ____ Date __________

9.3 Trigonometric Functions of Any Angle

For use with Exploration 9.3

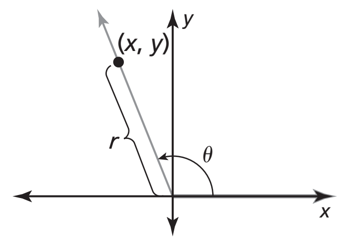
Essential Question How can you use the unit circle to define the trigonometric functions of any angle?

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$. The six trigonometric functions of θ are defined as shown.

$\sin\theta = \dfrac{y}{r}$ $\qquad$ $\csc\theta = \dfrac{r}{y},\ y \neq 0$

$\cos\theta = \dfrac{x}{r}$ $\qquad$ $\sec\theta = \dfrac{r}{x},\ x \neq 0$

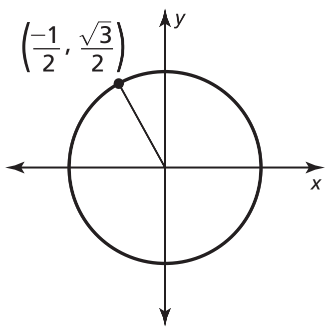
$\tan\theta = \dfrac{y}{x},\ x \neq 0$ $\qquad$ $\cot\theta = \dfrac{x}{y},\ y \neq 0$



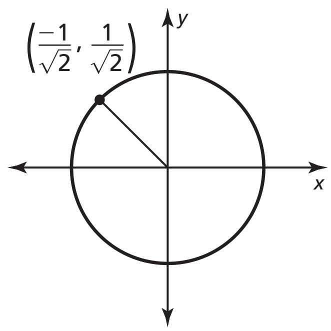
1 EXPLORATION: Writing Trigonometric Functions

Work with a partner. Find the sine, cosine, and tangent of the angle θ in standard position whose terminal side intersects the unit circle at the point (x, y) shown.

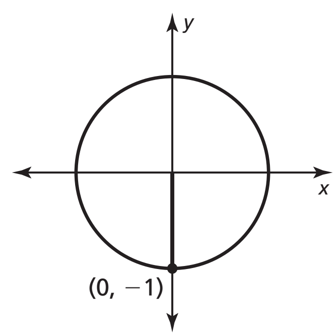
a.



b.



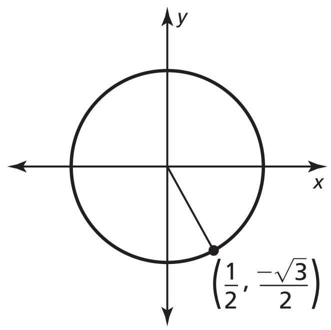
c.



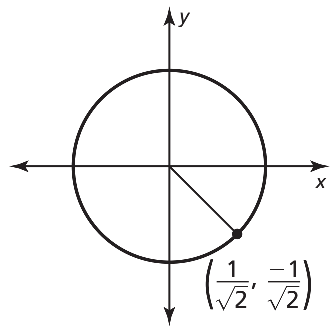
Name______________________________ Date__________

1 EXPLORATION: Writing Trigonometric Functions (continued)

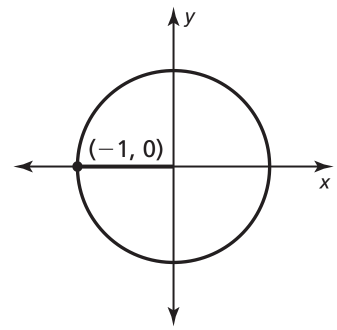
d.



e.



f.



Communicate Your Answer

2. How can you use the unit circle to define the trigonometric functions of any angle?

3. For which angles are each function undefined? Explain your reasoning.

a. tangent

b. cotangent

c. secant

d. cosecant

Name ____ Date __________

9.3 Notetaking with Vocabulary

For use after Lesson 9.3

In your own words, write the meaning of each vocabulary term.

unit circle

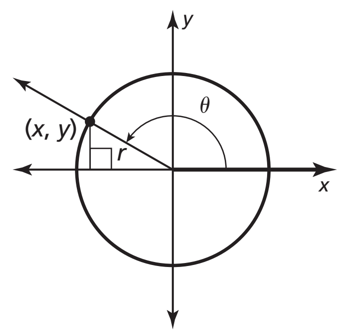
quadrantal angle

reference angle

Core Concepts

General Definitions of Trigonometric Functions

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as shown.



$\sin\theta = \frac{y}{r}$ $\quad\csc\theta = \frac{r}{y}, y \neq 0$

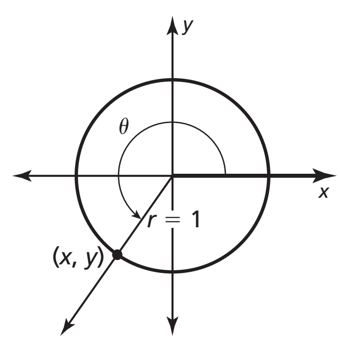
$\cos\theta = \frac{x}{r}$ $\quad\sec\theta = \frac{r}{x}, x \neq 0$

$\tan\theta = \frac{y}{x}, x \neq 0$ $\quad\cot\theta = \frac{x}{y}, y \neq 0$

These functions are sometimes called *circular functions*.

The Unit Circle

The circle $x^2 + y^2 = 1$, which has center (0, 0) and radius 1, is called the **unit circle**. The values of $\sin\theta$ and $\cos\theta$ are simply the y-coordinate and x-coordinate, respectively, of the point where the terminal side of θ intersects the unit circle.



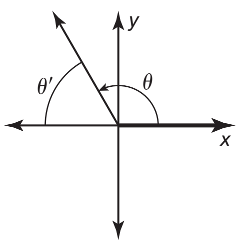
$\sin\theta = \frac{y}{r} = \frac{y}{1} = y \qquad \cos\theta = \frac{x}{r} = \frac{x}{1} = x$

Notes:

Reference Angle Relationships

Let θ be an angle in standard position. The **reference angle** for θ is the acute angle θ' formed by the terminal side of θ and the x-axis. The relationship between θ and θ' is shown below for nonquadrantal angles θ such that $90° < \theta < 360°$ or, in radians, $\frac{\pi}{2} < \theta < 2\pi$.

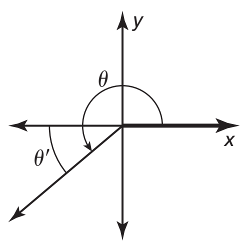
Quadrant II



Degrees: $\theta' = 180° - \theta$

Radians: $\theta' = \pi - \theta$

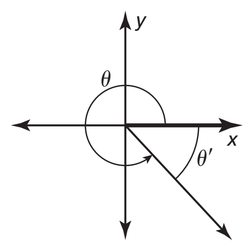
Quadrant III



Degrees: $\theta' = \theta - 180°$

Radians: $\theta' = \theta - \pi$

Quadrant IV



Degrees: $\theta' = 360° - \theta$

Radians: $\theta' = 2\pi - \theta$

Notes:

Evaluating Trigonometric Functions

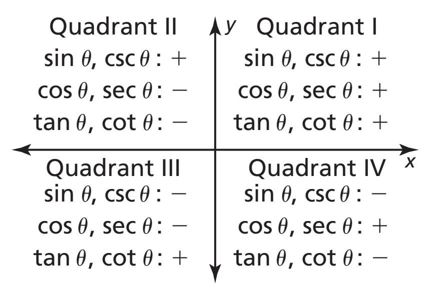
Use these steps to evaluate a trigonometric function for any angle θ:

Step 1 Find the reference angle θ'.

Step 2 Evaluate the trigonometric function for θ'.

Step 3 Determine the sign of the trigonometric function value from the quadrant in which θ lies.

Signs of Function Values



Notes:

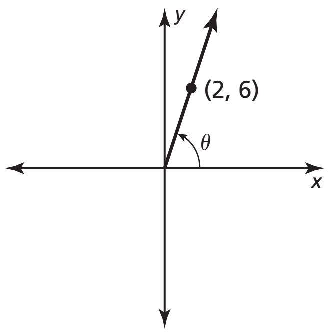
Name ______________________________ Date __________

9.3 Notetaking with Vocabulary (continued)

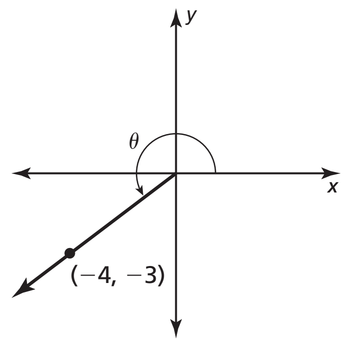
Extra Practice

In Exercises 1 and 2, evaluate the six trigonometric functions of θ.

1.



2.



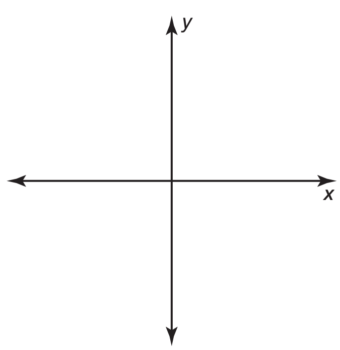
In Exercises 3 and 4, use the unit circle to evaluate the six trigonometric functions of θ.

3. $\theta = -90^\circ$

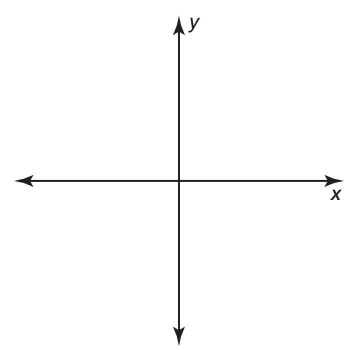
4. $\theta = 4\pi$

In Exercises 5 and 6, sketch the angle. Then find its reference angle.

5. -310°



6. $\dfrac{27\pi}{10}$



7. Evaluate the function $\csc 150^\circ$ without using a calculator.