Name_____ Date _____

8.4 Finding Sums of Infinite Geometric Series

For use with Exploration 8.4

Essential Question How can you find the sum of an infinite geometric series?

1 EXPLORATION: Finding Sums of Infinite Geometric Series

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Enter each geometric series in a spreadsheet. Then use the spreadsheet to determine whether the infinite geometric series has a finite sum. If it does, find the sum. Explain your reasoning. (The figure shows a partially completed spreadsheet for part (a).)

a. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

b. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$

c. $1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \frac{81}{16} + \cdots$

d. $1 + \frac{5}{4} + \frac{25}{16} + \frac{125}{64} + \frac{625}{256} + \cdots$

e. $1 + \frac{4}{5} + \frac{16}{25} + \frac{64}{125} + \frac{256}{625} + \cdots$

f. $1 + \frac{9}{10} + \frac{81}{100} + \frac{729}{1000} + \frac{6561}{10{,}000} + \cdots$

	A	B
1	1	1
2	2	0.5
3	3	0.25
4	4	0.125
5	5	0.0625
6	6	0.03125
7	7	
8	8	
9	9	
10	10	
11	11	
12	12	
13	13	
14	14	
15	15	
16	**Sum**	

2 EXPLORATION: Writing a Conjecture

Work with a partner. Look back at the infinite geometric series in Exploration 1. Write a conjecture about how you can determine whether the infinite geometric series

$$a_1 + a_1r + a_1r^2 + a_1r^3 + \cdots$$

has a finite sum.

3 EXPLORATION: Writing a Formula

Work with a partner. In Lesson 8.3, you learned that the sum of the first n terms of a geometric series with first term a_1 and common ratio $r \neq 1$ is

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right).$$

When an infinite geometric series has a finite sum, what happens to r^n as n increases? Explain your reasoning. Write a formula to find the sum of an infinite geometric series. Then verify your formula by checking the sums you obtained in Exploration 1.

Communicate Your Answer

4. How can you find the sum of an infinite geometric series?

5. Find the sum of each infinite geometric series, if it exists.

a. $1 + 0.1 + 0.01 + 0.001 + 0.0001 + \cdots$

b. $2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \frac{32}{81} + \cdots$

Name_____ Date _____

8.4 Notetaking with Vocabulary

For use after Lesson 8.4

In your own words, write the meaning of each vocabulary term.

partial sum

Core Concepts

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, then the series has no sum.

Notes:

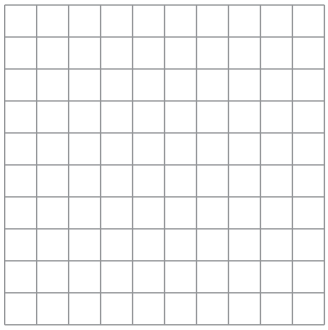
Name _____ Date _____

8.4 Notetaking with Vocabulary (continued)

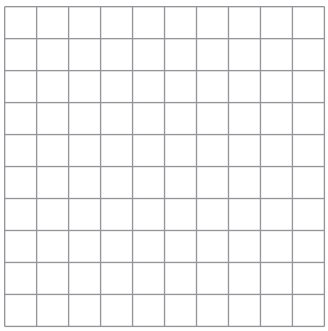
Extra Practice

In Exercises 1 and 2, consider the infinite geometric series. Find and graph the partial sums S_n for n = 1, 2, 3, 4, and 5. Then describe what happens to S_n as n increases.

1. $\frac{1}{4} + \frac{1}{6} + \frac{1}{9} + \frac{2}{27} + \frac{4}{81} + \cdots$



2. $3 + \frac{3}{5} + \frac{3}{25} + \frac{3}{125} + \frac{3}{625} + \cdots$



Name_____ Date _____

8.4 Notetaking with Vocabulary (continued)

In Exercises 3–6, find the sum of the infinite geometric series, if it exists.

3. $\sum_{n=1}^{\infty} 6\left(\frac{3}{5}\right)^{n-1}$

4. $\sum_{i=1}^{\infty} \frac{10}{3}\left(\frac{5}{2}\right)^{i-1}$

5. $5 + \frac{5}{3} + \frac{5}{9} + \frac{5}{27} + \cdots$

6. $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$

7. A child pushes a tumbler toy and lets it swing freely. On the first swing, the toy travels 30 centimeters. On each successive swing, the toy travels 75% of the distance of the previous swing. What is the total distance the toy swings?

8. Write 0.121212… as a fraction in simplest form.