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8.3 Analyzing Geometric Sequences and Series

For use with Exploration 8.3

Essential Question

How can you recognize a geometric sequence from its graph?

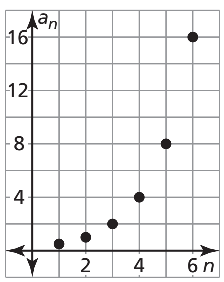
In a **geometric sequence**, the ratio of any term to the previous term, called the *common ratio*, is constant. For example, in the geometric sequence 1, 2, 4, 8, . . . , the common ratio is 2.

1 EXPLORATION: Recognizing Graphs of Geometric Sequences

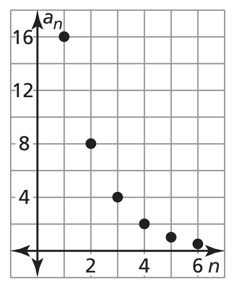
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Determine whether each graph shows a geometric sequence. If it does, then write a rule for the *n*th term of the sequence and use a spreadsheet to find the sum of the first 20 terms. What do you notice about the graph of a geometric sequence?

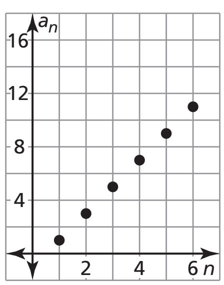
a.



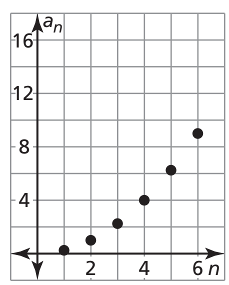
b.



c.



d.



2 EXPLORATION: Finding the Sum of a Geometric Sequence

Work with a partner. You can write the nth term of a geometric sequence with first term a_1 and common ratio r as

$$a_n = a_1 r^{n-1}.$$

So, you can write the sum S_n of the first n terms of a geometric sequence as

$$S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1}.$$

Rewrite this formula by finding the difference $S_n - rS_n$ and solve for S_n. Then verify your rewritten formula by finding the sums of the first 20 terms of the geometric sequences in Exploration 1. Compare your answers to those you obtained using a spreadsheet.

Communicate Your Answer

3. How can you recognize a geometric sequence from its graph?

4. Find the sum of the terms of each geometric sequence.

a. $1, 2, 4, 8, \ldots, 8192$

b. $0.1, 0.01, 0.001, 0.0001, \ldots, 10^{-10}$

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8.3 Notetaking with Vocabulary

For use after Lesson 8.3

In your own words, write the meaning of each vocabulary term.

geometric sequence

common ratio

geometric series

Core Concepts

Rule for a Geometric Sequence

Algebra The nth term of a geometric sequence with first term a_1 and common ratio r is given by:

$$a_n = a_1 r^{n-1}$$

Example The nth term of a geometric sequence with a first term of 2 and a common ratio of 3 is given by:

$$a_n = 2(3)^{n-1}$$

Notes:

The Sum of a Finite Geometric Series

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right).$$

Notes:

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Extra Practice

In Exercises 1–4, tell whether the sequence is geometric. Explain your reasoning.

1. $4, 12, 36, 108, 324, \ldots$

2. $45, 40, 35, 30, 25, \ldots$

3. $1.3, 7.8, 46.8, 280.8, 1684.8, \ldots$

4. $\frac{3}{2}, -\frac{3}{4}, \frac{3}{8}, -\frac{3}{16}, \frac{3}{32}, \ldots$

In Exercises 5–8, write a rule for the *n*th term of the sequence. Then find a_6.

5. $6, 18, 54, 162, \ldots$

6. $3, -6, 12, -24, \ldots$

7. $1, \frac{5}{2}, \frac{25}{4}, \frac{125}{8}, \ldots$

8. $-2.4, -16.8, -117.6, -823.2, \ldots$

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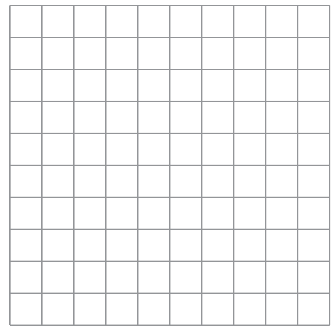
8.3 Notetaking with Vocabulary (continued)

9. Write a rule for the nth term where $a_8 = 384$ and $r = 2$. Then graph the first six terms of the sequence.



In Exercises 10 and 11, write a rule for the *n*th term of the geometric sequence.

10. $a_3 = 54, a_6 = 1458$

11. $a_2 = -2, a_5 = \dfrac{2}{125}$

12. Find the sum $\displaystyle\sum_{i=0}^{10} 3\left(\frac{3}{2}\right)^{i-1}$.