8.3 Analyzing Geometric Sequences and Series
For use with Exploration 8.3

Essential Question  How can you recognize a geometric sequence from its graph?

In a geometric sequence, the ratio of any term to the previous term, called the common ratio, is constant. For example, in the geometric sequence 1, 2, 4, 8, . . . , the common ratio is 2.

EXPLORATION: Recognizing Graphs of Geometric Sequences

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Determine whether each graph shows a geometric sequence. If it does, then write a rule for the \( n \)th term of the sequence and use a spreadsheet to find the sum of the first 20 terms. What do you notice about the graph of a geometric sequence?

a.  

b.  

c.  

d.  

Work with a partner. You can write the \( n \)th term of a geometric sequence with first term \( a_1 \) and common ratio \( r \) as

\[
a_n = a_1 r^{n-1}.
\]

So, you can write the sum \( S_n \) of the first \( n \) terms of a geometric sequence as

\[
S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1}.
\]

Rewrite this formula by finding the difference \( S_n - rS_n \) and solve for \( S_n \). Then verify your rewritten formula by finding the sums of the first 20 terms of the geometric sequences in Exploration 1. Compare your answers to those you obtained using a spreadsheet.

Communicate Your Answer

3. How can you recognize a geometric sequence from its graph?

4. Find the sum of the terms of each geometric sequence.
   a. 1, 2, 4, 8, \ldots, 8192
   b. 0.1, 0.01, 0.001, 0.0001, \ldots, 10^{-10}
8.3 Notetaking with Vocabulary
For use after Lesson 8.3

In your own words, write the meaning of each vocabulary term.

geometric sequence

common ratio

geometric series

Core Concepts

Rule for a Geometric Sequence

Algebra

The \( n \)th term of a geometric sequence with first term \( a_1 \) and common ratio \( r \) is given by:

\[
a_n = a_1 r^{n-1}
\]

Example

The \( n \)th term of a geometric sequence with a first term of 2 and a common ratio of 3 is given by:

\[
a_n = 2(3)^{n-1}
\]

Notes:

The Sum of a Finite Geometric Series

The sum of the first \( n \) terms of a geometric series with common ratio \( r \neq 1 \) is

\[
S_n = a_1 \left(\frac{1 - r^n}{1 - r}\right)
\]

Notes:
8.3 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–4, tell whether the sequence is geometric. Explain your reasoning.

1. 4, 12, 36, 108, 324, ...
2. 45, 40, 35, 30, 25, ...
3. 1.3, 7.8, 46.8, 280.8, 1684.8, ...
4. \(\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \ldots\)

In Exercises 5–8, write a rule for the \(n\)th term of the sequence. Then find \(a_6\).

5. 6, 18, 54, 162, ...
6. 3, –6, 12, –24, ...
7. \(\frac{5}{2}, \frac{25}{4}, \frac{125}{8}, \ldots\)
8. –2.4, –16.8, –117.6, –823.2, …
9. Write a rule for the \( n \)th term where \( a_8 = 384 \) and \( r = 2 \).
   Then graph the first six terms of the sequence.

In Exercises 10 and 11, write a rule for the \( n \)th term of the geometric sequence.

10. \( a_3 = 54, a_6 = 1458 \)

11. \( a_2 = -2, a_5 = \frac{2}{125} \)

12. Find the sum \( \sum_{i=0}^{10} \left( \frac{3}{2} \right)^{i-1} \).