Name____ Date__________

8.2 Analyzing Arithmetic Sequences and Series

For use with Exploration 8.2

Essential Question How can you recognize an arithmetic sequence from its graph?

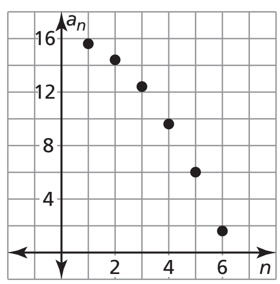
In an **arithmetic sequence**, the difference of consecutive terms, called the *common difference*, is constant. For example, in the arithmetic sequence 1, 4, 7, 10, . . . , the common difference is 3.

1 EXPLORATION: Recognizing Graphs of Arithmetic Sequences

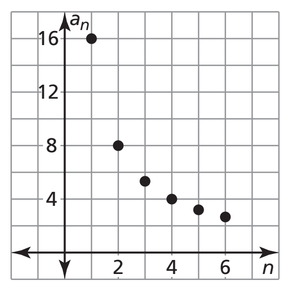
Go to *BigIdeasMath.co*m for an interactive tool to investigate this exploration.

Work with a partner. Determine whether each graph shows an arithmetic sequence. If it does, then write a rule for the *n*th term of the sequence, and use a spreadsheet to find the sum of the first 20 terms. What do you notice about the graph of an arithmetic sequence?

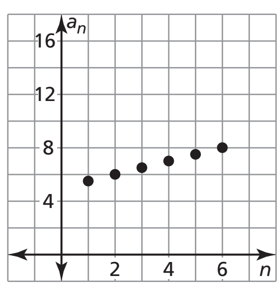
a.



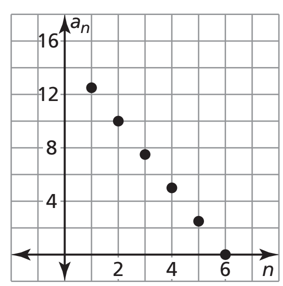
b.



c.



d.



2 EXPLORATION: Finding the Sum of an Arithmetic Sequence

Work with a partner. A teacher of German mathematician Carl Friedrich Gauss (1777–1855) asked him to find the sum of all the whole numbers from 1 through 100. To the astonishment of his teacher, Gauss came up with the answer after only a few moments. Here is what Gauss did:

$$\begin{array}{r} 1 + \quad 2 + \quad 3 + \cdots + 100 \\ 100 + \ 99 + \ 98 + \cdots + \quad 1 \\ \hline 101 + 101 + 101 + \cdots + 101 \end{array} \qquad \frac{100 \times 101}{2} = 5050$$

Explain Gauss's thought process. Then write a formula for the sum S_n of the first n terms of an arithmetic sequence. Verify your formula by finding the sums of the first 20 terms of the arithmetic sequences in Exploration 1. Compare your answers to those you obtained using a spreadsheet.

Communicate Your Answer

3. How can you recognize an arithmetic sequence from its graph?

4. Find the sum of the terms of each arithmetic sequence.

a. 1, 4, 7, 10, . . . , 301

b. 1, 2, 3, 4, . . . , 1000

c. 2, 4, 6, 8, . . . , 800

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8.2 Notetaking with Vocabulary
For use after Lesson 8.2

In your own words, write the meaning of each vocabulary term.

arithmetic sequence

common difference

arithmetic series

Core Concepts

Rule for an Arithmetic Sequence

Algebra The nth term of an arithmetic sequence with first term a_1 and common difference d is given by:

$$a_n = a_1 + (n - 1)d$$

Example The nth term of an arithmetic sequence with a first term of 3 and a common difference of 2 is given by:

$$a_n = 3 + (n - 1)2, \text{ or } a_n = 2n + 1$$

Notes:

The Sum of a Finite Arithmetic Series

The sum of the first n terms of an arithmetic series is $S_n = n\left(\frac{a_1 + a_n}{2}\right)$.

In words, S_n is the mean of the first and nth terms, multiplied by the number of terms.

Notes:

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8.2 **Notetaking with Vocabulary** (continued)

Extra Practice

In Exercises 1–4, tell whether the sequence is arithmetic. Explain your reasoning.

1. $1, 4, 7, 12, 17, \ldots$

2. $26, 23, 20, 17, 14, \ldots$

3. $0.3, 0.5, 0.7, 0.9, 1.1, \ldots$

4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$

In Exercises 5–8, write a rule for the *n*th term of the sequence. Then find a_{20}.

5. $3, 9, 15, 21, \ldots$

6. $8, 3, -2, -7, \ldots$

7. $-1, -\frac{1}{2}, 0, \frac{1}{2}, \ldots$

8. $0.7, 0.2, -0.3, -0.8, \ldots$

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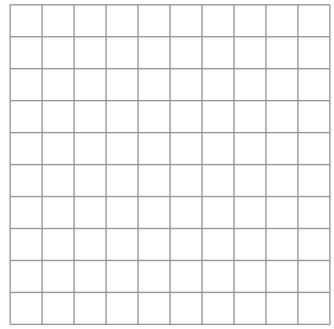
8.2 Notetaking with Vocabulary (continued)

9. Write a rule for the *n*th term of the sequence where $a_{12} = -13$ and $d = -2$. Then graph the first six terms of the sequence.



In Exercises 10 and 11, write a rule for the *n*th term of the sequence.

10. $a_8 = 59, a_{13} = 99$

11. $a_{18} = -5, a_{27} = -8$

12. Find the sum $\sum_{i=1}^{22} (5 - 2i)$.