8.1 Defining and Using Sequences and Series
For use with Exploration 8.1

Essential Question  How can you write a rule for the \( n \)th term of a sequence?

A sequence is an ordered list of numbers. There can be a limited number or an infinite number of terms of a sequence.

\[ a_1, a_2, a_3, a_4, \ldots, a_n, \ldots \]

Terms of a sequence

Here is an example.

\[ 1, 4, 7, 10, \ldots, 3n - 2, \ldots \]

1 EXPLORATION: Writing Rules for Sequences

Work with a partner. Match each sequence with its graph on the next page. The horizontal axes represent \( n \), the position of each term in the sequence. Then write a rule for the \( n \)th term of the sequence, and use the rule to find \( a_{10} \).

\begin{align*}
\text{a.} & \quad 1, 2.5, 4, 5.5, 7, \ldots \\
\text{b.} & \quad 8, 6.5, 5, 3.5, 2, \ldots \\
\text{c.} & \quad \frac{1}{4}, \frac{4}{4}, \frac{9}{4}, \frac{16}{4}, \frac{25}{4}, \ldots \\
\text{d.} & \quad \frac{25}{4}, \frac{16}{4}, \frac{9}{4}, \frac{4}{4}, \frac{1}{4}, \ldots \\
\text{e.} & \quad \frac{1}{2}, 1, 2, 4, 8, \ldots \\
\text{f.} & \quad 8, 4, 2, 1, \frac{1}{2}, \ldots 
\end{align*}
Communicate Your Answer

2. How can you write a rule for the $n$th term of a sequence?

3. What do you notice about the relationship between the terms in (a) an arithmetic sequence and (b) a geometric sequence? Justify your answers.
8.1 Notetaking with Vocabulary

For use after Lesson 8.1

In your own words, write the meaning of each vocabulary term.

sequence

terms of a sequence

series

summation notation

sigma notation

Core Concepts

Sequences

A sequence is an ordered list of numbers. A finite sequence is a function that has a limited number of terms and whose domain is the finite set \{1, 2, 3, \ldots, n\}. The values in the range are called the terms of the sequence.

Domain: \[1 \ 2 \ 3 \ 4 \ \ldots \ n\] Relative position of each term

\[\downarrow \ \downarrow \ \downarrow \ \downarrow \ \downarrow \] 

Range: \[a_1 \ a_2 \ a_3 \ a_4 \ \ldots \ a_n\] Terms of the sequence

An infinite sequence is a function that continues without stopping and whose domain is the set of positive integers. Here are examples of a finite sequence and an infinite sequence.

Finite sequence: \[2, 4, 6, 8\] \hspace{1cm} Infinite sequence: \[2, 4, 6, 8, \ldots\]

A sequence can be specified by an equation, or rule. For example, both sequences above can be described by the rule \[a_n = 2n\] or \[f(n) = 2n\].

Notes:
8.1 Notetaking with Vocabulary (continued)

Series and Summation Notation

When the terms of a sequence are added together, the resulting expression is a series. A series can be finite or infinite.

**Finite series:** \(2 + 4 + 6 + 8\)

**Infinite series:** \(2 + 4 + 6 + 8 + \cdots\)

You can use summation notation to write a series. For example, the two series above can be written in summation notation as follows:

**Finite series:** \(\sum_{i=1}^{4} 2i\)

**Infinite series:** \(\sum_{i=1}^{\infty} 2i\)

For both series, the index of summation is \(i\) and the lower limit of summation is 1. The upper limit of summation is 4 for the finite series and \(\infty\) (infinity) for the infinite series. Summation notation is also called sigma notation because it uses the uppercase Greek letter \(\sigma\), written \(\Sigma\).

Notes:

Formulas for Special Series

**Sum of \(n\) terms of 1:** \(\sum_{i=1}^{n} 1 = n\)

**Sum of first \(n\) positive integers:** \(\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}\)

**Sum of squares of first \(n\) positive integers:** \(\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}\)

Notes:
Extra Practice

In Exercises 1 and 2, write the first six terms of the sequence.

1. \(a_n = n^3 - 1\)
2. \(f(n) = (-2)^{n-1}\)

In Exercises 3 and 4, describe the pattern, write the next term, and write a rule for the \(n\)th term of the sequence.

3. \(-3, -1, 1, 3, \ldots\)
4. \(\frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}, \ldots\)

5. Write the series \(-1 + 4 - 9 + 16 - 25 + \cdots\) using summation notation.

In Exercises 6 and 7, find the sum.

6. \(\sum_{n=2}^{5} \frac{n}{n - 1}\)
7. \(\sum_{i=1}^{18} i^2\)