Name____ Date__________

Chapter 7 Maintaining Mathematical Proficiency

Evaluate.

1. $\frac{2}{3} + \frac{2}{3}$

2. $\frac{1}{5} + \frac{1}{4}$

3. $-\frac{5}{6} + \frac{3}{4}$

4. $\frac{9}{11} - \frac{2}{11}$

5. $\frac{1}{5} - \frac{7}{10}$

6. $\frac{5}{8} - \frac{1}{6}$

7. $-\frac{3}{8} + \frac{2}{9} - \frac{1}{2}$

8. $\frac{3}{4} - \left(-\frac{1}{8}\right)$

9. $\frac{13}{18} + \frac{2}{9} - \frac{1}{2}$

Simplify.

10. $\frac{\frac{2}{3}}{\frac{8}{15}}$

11. $\frac{\frac{1}{6}}{-\frac{2}{3}}$

12. $\frac{\frac{3}{4}}{12}$

13. $\frac{1}{\frac{1}{5} + \frac{2}{5}}$

14. $\frac{2}{\frac{4}{9} - \frac{2}{3}}$

15. $\frac{\frac{1}{2} + \frac{1}{5}}{\frac{7}{10} - \frac{2}{5}}$

Name ____ Date __________

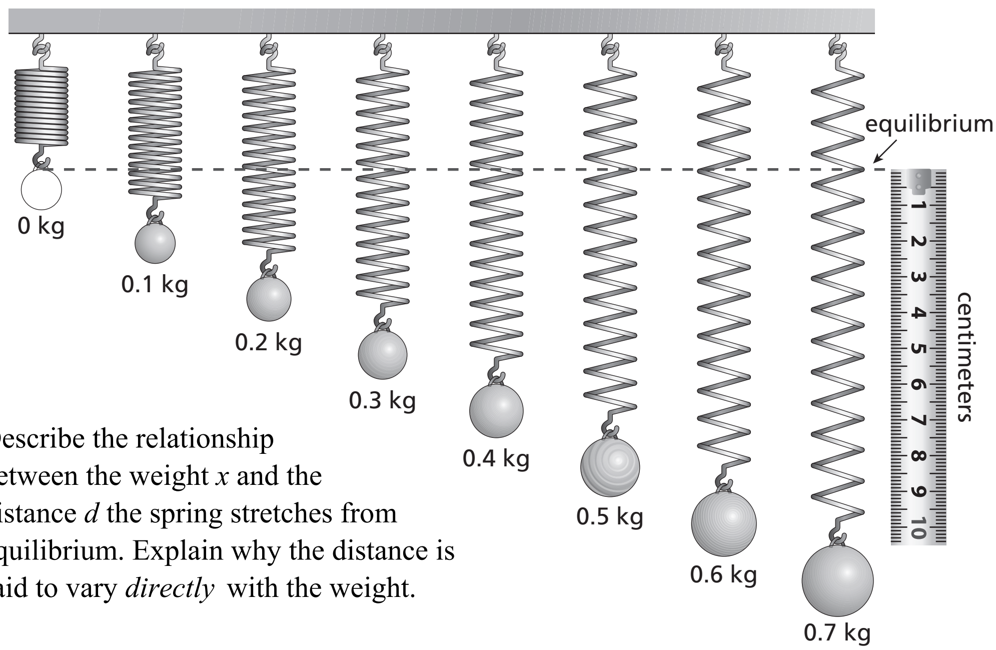
7.1 Inverse Variation
For use with Exploration 7.1

Essential Question How can you recognize when two quantities vary directly or inversely?

1 EXPLORATION: Recognizing Direct Variation

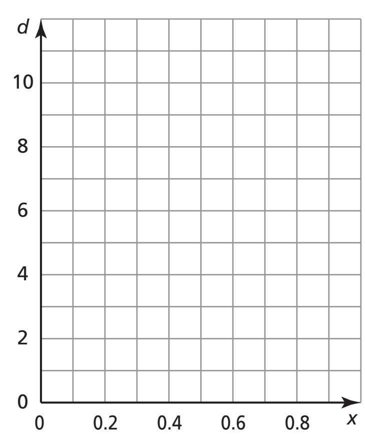
Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. You hang different weights from the same spring.



a. Describe the relationship between the weight x and the distance d the spring stretches from equilibrium. Explain why the distance is said to vary *directly* with the weight.

b. Estimate the values of d from the figure. Then draw a scatter plot of the data. What are the characteristics of the graph?



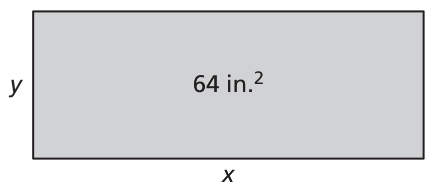
c. Write an equation that represents d as a function of x.

d. In physics, the relationship between d and x is described by *Hooke's Law*. How would you describe Hooke's Law?

2 EXPLORATION: Recognizing Inverse Variation

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. The table shows the length x (in inches) and the width y (in inches) of a rectangle. The area of each rectangle is 64 square inches.

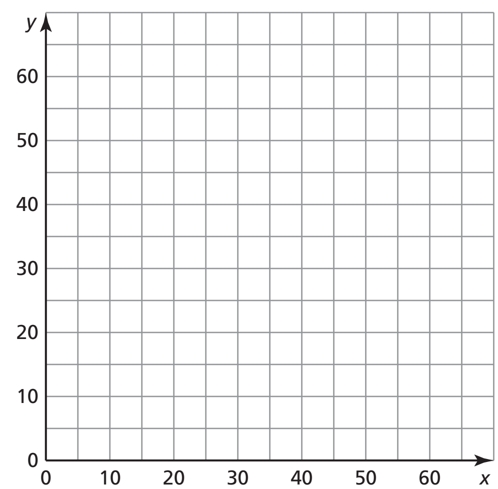


x	y
1	
2	
4	
8	
16	
32	
64	

a. Complete the table.

b. Describe the relationship between x and y. Explain why y is said to vary *inversely* with x.

c. Draw a scatter plot of the data. What are the characteristics of the graph?



d. Write an equation that represents y as a function of x.

Communicate Your Answer

3. How can you recognize when two quantities vary directly or inversely?

4. Does the flapping rate of the wings of a bird vary directly or inversely with the length of its wings? Explain your reasoning.

Name ____ Date __________

7.1 Notetaking with Vocabulary
For use after Lesson 7.1

In your own words, write the meaning of each vocabulary term.

inverse variation

constant of variation

Core Concepts

Inverse Variation

Two variables x and y show **inverse variation** when they are related as follows:

$$y = \frac{a}{x}, \ a \neq 0$$

The constant a is the **constant of variation,** and y is said to *vary inversely* with x.

Notes:

Extra Practice

In Exercises 1–9, tell whether *x* and *y* show *direct variation*, *inverse variation*, or *neither*.

1. $3xy = 1$

2. $\frac{5}{x} = y$

3. $x + 11 = y$

4. $x + y = -2$

5. $\frac{4}{5}x = y$

6. $x - 8y = 1$

7. $\frac{x}{7} = y$

8. $6xy = 0$

9. $\frac{y}{9x} = 1$

In Exercises 10–12, tell whether *x* and *y* show *direct variation*, *inverse variation*, or *neither*.

10.

x	2	4	6	8	10
y	4	16	36	64	100

11.

x	1	5	8	20	50
y	5	1	0.625	0.25	0.1

12.

x	2	5	8.4	12	15
y	0.5	1.25	2.1	3	3.75

Name ____ Date __________

7.1 Notetaking with Vocabulary (continued)

In Exercises 13–16, the variables *x* and *y* vary inversely. Use the given values to write an equation relating *x* and *y*. Then find *y* when $x = 5$.

13. $x = 2,\ y = 2$

14. $x = 6,\ y = 3$

15. $x = 20,\ y = \frac{7}{20}$

16. $x = \frac{10}{9},\ y = \frac{3}{2}$

17. When temperature is held constant, the volume V of a gas is inversely proportional to the pressure P of the gas on its container. A pressure of 32 pounds per square inch results in a volume of 20 cubic feet. What is the pressure if the volume becomes 10 cubic feet?

18. The time t (in days) that it takes to harvest a field varies inversely with the number n of farm workers. A farmer can harvest his crop in 20 days with 7 farm workers. How long will it take to harvest the crop if he hires 10 farm workers?

Name____ Date__________

7.2 Graphing Rational Functions

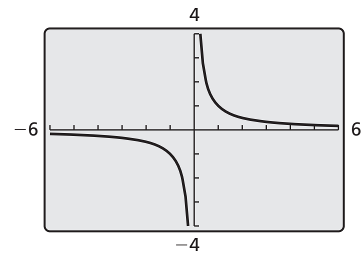
For use with Exploration 7.2

Essential Question What are some of the characteristics of the graph of a rational function?

The parent function for rational functions with a linear numerator and a linear denominator is

$f(x) = \frac{1}{x}$. Parent function

The graph of this function, shown at the right, is a *hyperbola*.



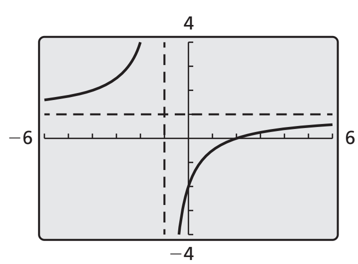
1 EXPLORATION: Identifying Graphs of Rational Functions

Work with a partner. Each function is a transformation of the graph of the parent function $f(x) = \frac{1}{x}$. Match the function with its graph. Explain your reasoning. Then describe the transformation.

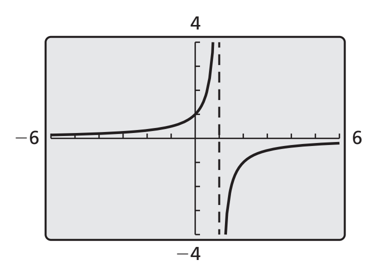
a. $g(x) = \frac{1}{x - 1}$ **b.** $g(x) = \frac{-1}{x - 1}$ **c.** $g(x) = \frac{x + 1}{x - 1}$

d. $g(x) = \frac{x - 2}{x + 1}$ **e.** $g(x) = \frac{x}{x + 2}$ **f.** $g(x) = \frac{-x}{x + 2}$

A.



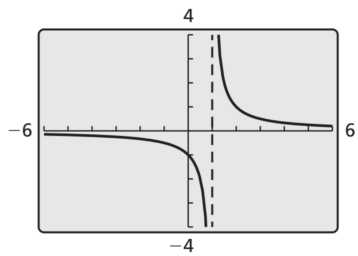
B.



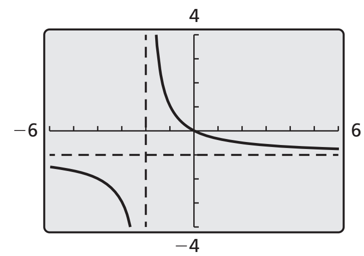
7.2 Graphing Rational Functions (continued)

1 EXPLORATION: Identifying Graphs of Rational Functions (continued)

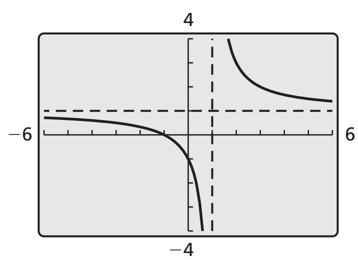
C.



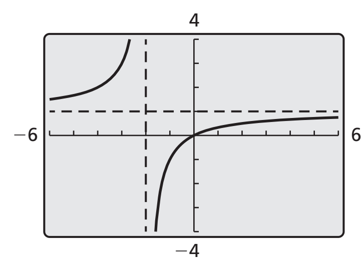
D.



E.



F.



Communicate Your Answer

2. What are some of the characteristics of the graph of a rational function?

3. Determine the intercepts, asymptotes, domain, and range of the rational function $g(x) = \dfrac{x - a}{x - b}$.

Name____ Date__________

7.2 Notetaking with Vocabulary
For use after Lesson 7.2

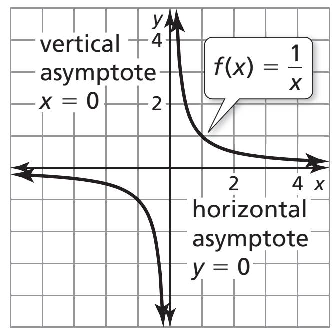
In your own words, write the meaning of each vocabulary term.

rational function

Core Concepts

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \frac{1}{x}$ is a *hyperbola*, which consists of two symmetrical parts called branches. The domain and range are all nonzero real numbers.



Any function of the form $g(x) = \frac{a}{x} (a \neq 0)$ has the same asymptotes, domain, and range as the function $f(x) = \frac{1}{x}$.

Notes:

Name ____ Date __________

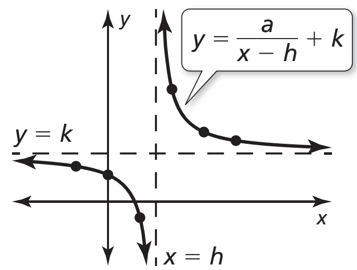
Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \frac{a}{x - h} + k$, follow these steps:

Step 1 Draw the asymptotes $x = h$ and $y = k$.

Step 2 Plot points to the left and to the right of the vertical asymptote.

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.

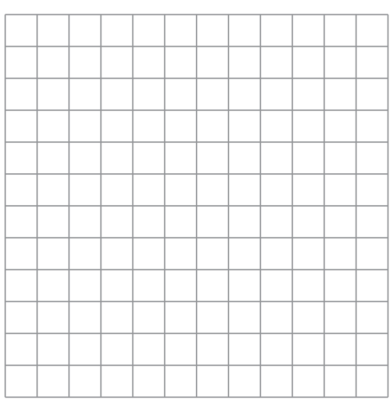


Notes:

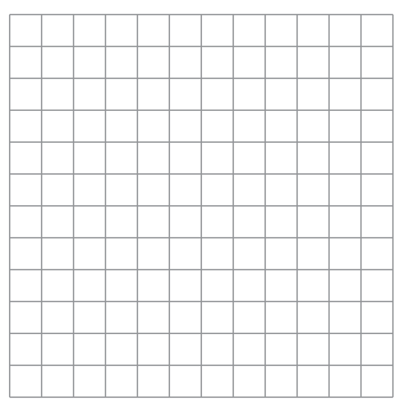
Extra Practice

In Exercises 1 and 2, graph the function. Compare the graph with the graph of $f(x) = \frac{1}{x}$.

1. $g(x) = \frac{0.25}{x}$



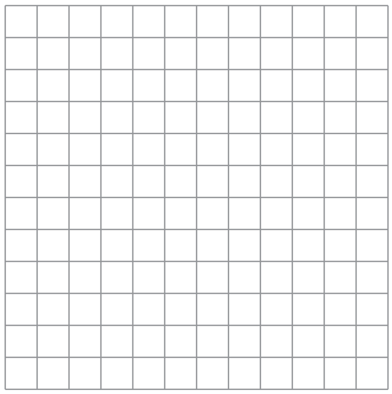
2. $h(x) = \frac{-2}{x}$



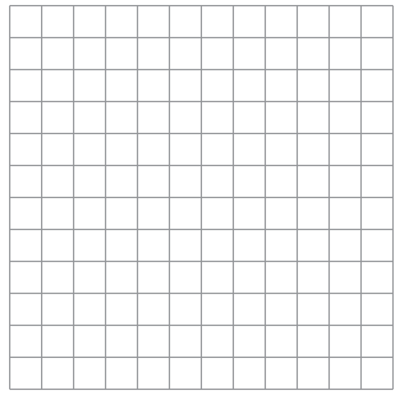
7.2 Notetaking with Vocabulary (continued)

In Exercises 3 and 4, graph the function. State the domain and range.

3. $k(x) = \dfrac{1}{x - 3} + 5$

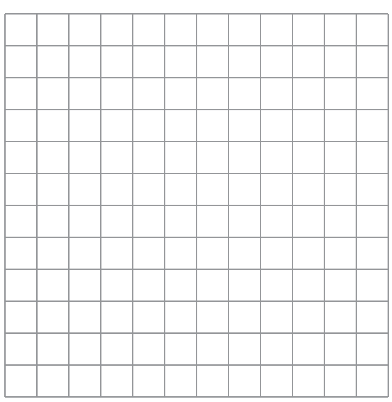


4. $m(x) = \dfrac{-3}{x} - 4$

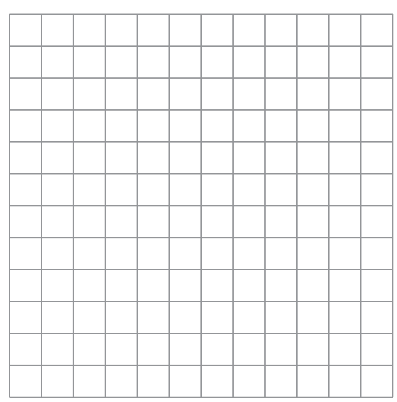


In Exercises 5 and 6, rewrite the function in the form $g(x) = \dfrac{a}{x - h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \dfrac{a}{x}$.

5. $g(x) = \dfrac{x + 2}{x - 5}$



6. $g(x) = \dfrac{2x + 8}{3x - 12}$



Name ____ Date __________

7.3 Multiplying and Dividing Rational Expressions

For use with Exploration 7.3

Essential Question How can you determine the excluded values in a product or quotient of two rational expressions?

1 EXPLORATION: Multiplying and Dividing Rational Expressions

Work with a partner. Find the product or quotient of the two rational expressions. Then match the product or quotient with its excluded values. Explain your reasoning.

Product or Quotient	Excluded Values
a. $\frac{1}{x-1} \bullet \frac{x-2}{x+1} =$	**A.** $-1, 0,$ and 2
b. $\frac{1}{x-1} \bullet \frac{-1}{x-1} =$	**B.** -2 and 1
c. $\frac{1}{x-2} \bullet \frac{x-2}{x+1} =$	**C.** $-2, 0,$ and 1
d. $\frac{x+2}{x-1} \bullet \frac{-x}{x+2} =$	**D.** -1 and 2
e. $\frac{x}{x+2} \div \frac{x+1}{x+2} =$	**E.** $-1, 0,$ and 1
f. $\frac{x}{x-2} \div \frac{x+1}{x} =$	**F.** -1 and 1
g. $\frac{x}{x+2} \div \frac{x}{x-1} =$	**G.** -2 and -1
h. $\frac{x+2}{x} \div \frac{x+1}{x-1} =$	**H.** 1

2 EXPLORATION: Writing a Product or Quotient

Work with a partner. Write a product or quotient of rational expressions that has the given excluded values. Justify your answer.

a. -1

b. -1 and 3

c. $-1, 0,$ and 3

Communicate Your Answer

3. How can you determine the excluded values in a product or quotient of two rational expressions?

4. Is it possible for the product or quotient of two rational expressions to have *no* excluded values? Explain your reasoning. If it is possible, give an example.

Name ____ Date __________

7.3 Notetaking with Vocabulary

For use after Lesson 7.3

In your own words, write the meaning of each vocabulary term.

rational expression

simplified form of a rational expression

Core Concepts

Simplifying Rational Expressions

Let a, b, and c be expressions with $b \neq 0$ and $c \neq 0$.

Property $\dfrac{a\cancel{c}}{b\cancel{c}} = \dfrac{a}{b}$ Divide out common factor c.

Examples $\dfrac{15}{65} = \dfrac{3 \bullet \cancel{5}}{13 \bullet \cancel{5}} = \dfrac{3}{13}$ Divide out common factor 5.

$\dfrac{4\cancel{(x + 3)}}{(x + 3)\cancel{(x + 3)}} = \dfrac{4}{x + 3}$ Divide out common factor $x + 3$.

Notes:

Multiplying Rational Expressions

Let a, b, c, and d be expressions with $b \neq 0$ and $d \neq 0$.

Property $\quad \dfrac{a}{b} \bullet \dfrac{c}{d} = \dfrac{ac}{bd}$ $\qquad$ Simplify $\dfrac{ac}{bd}$ if possible.

____

Example

$$\frac{5x^2}{2xy^2} \bullet \frac{6xy^3}{10y} = \frac{30x^3y^3}{20xy^3} = \frac{\cancel{10} \bullet 3 \bullet \cancel{x} \bullet x^2 \bullet \cancel{y^3}}{\cancel{10} \bullet 2 \bullet \cancel{x} \bullet \cancel{y^3}} = \frac{3x^2}{2}, x \neq 0, y \neq 0$$

Notes:

Dividing Rational Expressions

Let a, b, c, and d be expressions with $b \neq 0, c \neq 0,$ and $d \neq 0$.

Property $\quad \dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \bullet \dfrac{d}{c} = \dfrac{ad}{bc}$ $\qquad$ Simplify $\dfrac{ad}{bc}$ if possible.

____

Example $\quad \dfrac{7}{x+1} \div \dfrac{x+2}{2x-3} = \dfrac{7}{x+1} \bullet \dfrac{2x-3}{x+2} = \dfrac{7(2x-3)}{(x+1)(x+2)}, \ x \neq \dfrac{3}{2}$

Notes:

Extra Practice

In Exercises 1–4, simplify the expression, if possible.

1. $\dfrac{2x^3 - 8x^2}{6x^2}$

2. $\dfrac{5xy^3 - 2x^2y^2}{x^2y^2}$

3. $\dfrac{x^2 - 5x + 4}{x^2 - 2x + 1}$

4. $\dfrac{x^3 + 3x^2}{x^2 - 5x - 24}$

In Exercises 5–10, find the product or the quotient.

5. $\dfrac{3xy}{xy^2} \bullet \dfrac{y}{2x}$

6. $\dfrac{x + y}{7xy} \div \dfrac{4x}{y}$

7. $\dfrac{x(x + 1)}{x - 2} \div \dfrac{(x + 1)(x - 6)}{(x - 6)(x - 9)}$

8. $\dfrac{x^2 - 2x - 3}{x^2 - 1} \bullet \dfrac{x^2 - 2x - 63}{x^2 + 4x - 21}$

9. $\dfrac{x^2 - 2x}{x + 7} \bullet \dfrac{x^3 + 8}{x^3 - 4x}$

10. $\dfrac{x^2 + 2x - 15}{x^2 - 3x - 40} \div \dfrac{x^2 + 8x - 9}{x^2 + x - 72}$

Name____ Date__________

7.4 Adding and Subtracting Rational Expressions

For use with Exploration 7.4

Essential Question How can you determine the domain of the sum or difference of two rational expressions?

1 EXPLORATION: Adding and Subtracting Rational Expressions

Work with a partner. Find the sum or difference of the two rational expressions. Then match the sum or difference with its domain. Explain your reasoning.

Sum or Difference	Domain
a. $\frac{1}{x-1} + \frac{3}{x-1} =$	**A.** all real numbers except -2
b. $\frac{1}{x-1} + \frac{1}{x} =$	**B.** all real numbers except -1 and 1
c. $\frac{1}{x-2} + \frac{1}{2-x} =$	**C.** all real numbers except 1
d. $\frac{1}{x-1} + \frac{-1}{x+1} =$	**D.** all real numbers except 0
e. $\frac{x}{x+2} - \frac{x+1}{2+x} =$	**E.** all real numbers except -2 and 1
f. $\frac{x}{x-2} - \frac{x+1}{x} =$	**F.** all real numbers except 0 and 1
g. $\frac{x}{x+2} - \frac{x}{x-1} =$	**G.** all real numbers except 2
h. $\frac{x+2}{x} - \frac{x+1}{x} =$	**H.** all real numbers except 0 and 2

2 EXPLORATION: Writing a Sum or Difference

Work with a partner. Write a sum or difference of rational expressions that has the given domain. Justify your answer.

a. all real numbers except -1

b. all real numbers except -1 and 3

c. all real numbers except -1, 0, and 3

Communicate Your Answer

3. How can you determine the domain of the sum or difference of two rational expressions?

4. Your friend found a sum as follows. Describe and correct the error(s).

$$\frac{x}{x+4} + \frac{3}{x-4} = \frac{x+3}{2x}$$

Name____ Date__________

7.4 Notetaking with Vocabulary

For use after Lesson 7.4

In your own words, write the meaning of each vocabulary term.

complex fraction

Core Concepts

Adding or Subtracting with Like Denominators

Let a, b, and c be expressions with $c \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$$

Subtraction

$$\frac{a}{c} - \frac{b}{c} = \frac{a - b}{c}$$

Notes:

Adding or Subtracting with Unlike Denominators

Let a, b, c, and d be expressions with $c \neq 0$ and $d \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad + bc}{cd}$$

Subtraction

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad - bc}{cd}$$

Notes:

Name ____ Date __________

7.4 Notetaking with Vocabulary (continued)

Simplifying Complex Fractions

Method 1 If necessary, simplify the numerator and denominator by writing each as a single fraction. Then divide by multiplying the numerator by the reciprocal of the denominator.

Method 2 Multiply the numerator and the denominator by the LCD of *every* fraction in the numerator and denominator. Then simplify.

Notes:

Extra Practice

In Exercises 1–4, find the sum or difference.

1. $\frac{1}{x-1} - \frac{5}{x-1}$

2. $\frac{4x}{3x-5} + \frac{x}{3x-5}$

3. $\frac{6x}{x+4} + \frac{24}{x+4}$

4. $\frac{2x^2}{x-7} - \frac{14x}{x-7}$

7.4 Notetaking with Vocabulary (continued)

In Exercises 5–7, find the least common multiple of the expressions.

5. $9x^3, 3x^2 - 21x$

6. $x + 5, 2x^2 + 11x + 5$

7. $x^2 + 5x + 6, x^2 - 3x - 18$

In Exercises 8–11, find the sum or the difference.

8. $\frac{3}{2x} + \frac{11}{5x}$

9. $\frac{15}{x - 2} + \frac{3}{x + 8}$

10. $\frac{3x}{2x + 1} + \frac{10}{2x^2 - 5x - 3}$

11. $\frac{x}{x - 7} - \frac{2}{x + 1} - \frac{8x}{x^2 - 6x - 7}$

In Exercises 12 and 13, simplify the complex fraction.

12. $\dfrac{\frac{x}{10} - 3}{5 + \frac{1}{x}}$

13. $\dfrac{\frac{12}{x^2 - 7x - 44}}{\frac{2}{x - 11} + \frac{1}{x + 4}}$

Name ____ Date __________

7.5 Solving Rational Equations
For use with Exploration 7.5

Essential Question How can you solve a rational equation?

1 EXPLORATION: Solving Rational Equations

Work with a partner. Match each equation with the graph of its related system of equations. Explain your reasoning. Then use the graph to solve the equation.

a. $\frac{2}{x-1} = 1$

b. $\frac{2}{x-2} = 2$

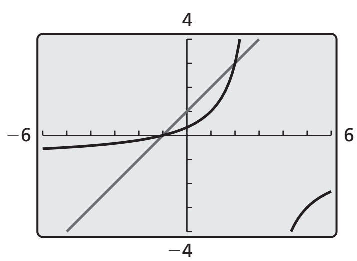
c. $\frac{-x-1}{x-3} = x + 1$

d. $\frac{2}{x-1} = x$

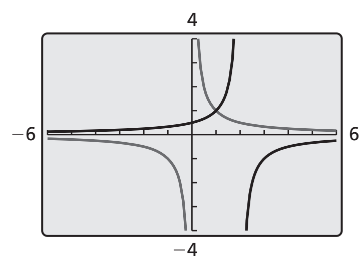
e. $\frac{1}{x} = \frac{-1}{x-2}$

f. $\frac{1}{x} = x^2$

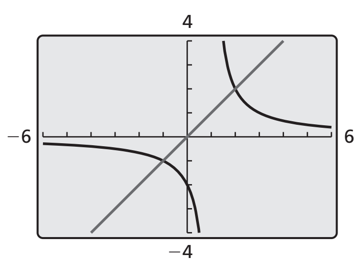
A.



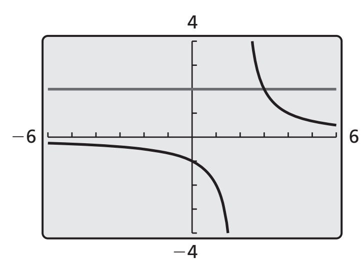
B.



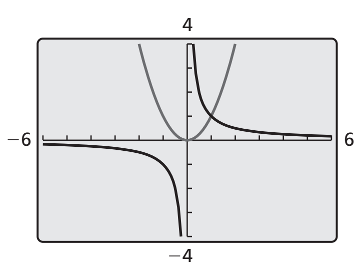
C.



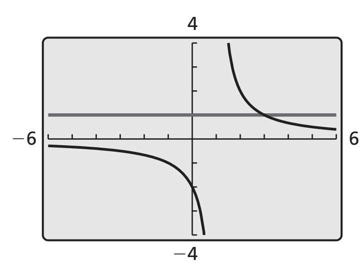
D.



E.



F.



2 EXPLORATION: Solving Rational Equations

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Look back at the equations in Explorations 1(d) and 1(e). Suppose you want a more accurate way to solve the equations than using a graphical approach.

a. Show how you could use a *numerical approach* by creating a table. For instance, you might use a spreadsheet to solve the equations.

b. Show how you could use an *analytical approach*. For instance, you might use the method you used to solve proportions.

Communicate Your Answer

3. How can you solve a rational equation?

4. Use the method in either Exploration 1 or 2 to solve each equation.

a. $\dfrac{x+1}{x-1} = \dfrac{x-1}{x+1}$ **b.** $\dfrac{1}{x+1} = \dfrac{1}{x^2+1}$ **c.** $\dfrac{1}{x^2-1} = \dfrac{1}{x-1}$

Name ____ Date __________

7.5 Notetaking with Vocabulary

For use after Lesson 7.5

In your own words, write the meaning of each vocabulary term.

cross multiplying

Notes:

Extra Practice

In Exercises 1–4, solve the equation by cross multiplying. Check your solution(s).

1. $\dfrac{2}{x + 8} = \dfrac{5}{2x - 7}$

2. $\dfrac{x}{x + 1} = \dfrac{-4}{x}$

3. $\dfrac{x + 1}{x - 3} = \dfrac{x + 2}{x - 6}$

4. $\dfrac{-2}{x - 3} = \dfrac{x + 9}{x + 21}$

In Exercises 5–12, solve the equation by using the LCD. Check your solution(s).

5. $\dfrac{4}{7} - \dfrac{1}{x} = 6$

6. $\dfrac{3}{x + 1} + \dfrac{4}{x + 2} = \dfrac{15}{x + 2}$

7. $\dfrac{12}{x + 4} - \dfrac{7}{x} = \dfrac{22}{x^2 + 4x}$

8. $3 - \dfrac{18}{x - 1} = -\dfrac{12}{x}$

7.5 Notetaking with Vocabulary (continued)

9. $\dfrac{2}{x-5} + \dfrac{3}{x} = \dfrac{10}{x^2 - 5x}$

10. $\dfrac{x+6}{x-4} - \dfrac{30}{x^2 - 5x + 4} = \dfrac{3}{x-1}$

11. $\dfrac{x}{x-5} + \dfrac{2}{x+2} = \dfrac{11}{x^2 - 3x - 10}$

12. $\dfrac{x-2}{x-4} - \dfrac{2}{x-1} = \dfrac{12}{x^2 - 5x + 4}$

In Exercises 13 and 14, determine whether the inverse of *f* is a function. Then find the inverse.

13. $f(x) = \dfrac{8}{x-3}$

14. $f(x) = \dfrac{12}{x} + 9$

15. You can complete the yard work at your friend's home in 5 hours. Working together, you and your friend can complete the yard work in 3 hours. How long would it take your friend to complete the yard work when working alone?

Let t be the time (in hours) your friend would take to complete the yard work when working alone.

	Work Rate	Time	Work Done
You	$\dfrac{1 \text{ yard}}{5 \text{ hours}}$	3 hours	
Friend		3 hours	