Name_____ Date _____

Chapter 6 Maintaining Mathematical Proficiency
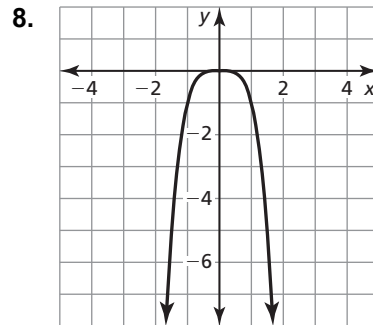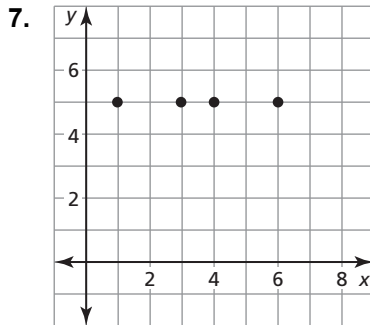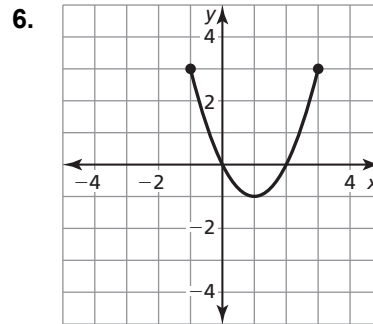
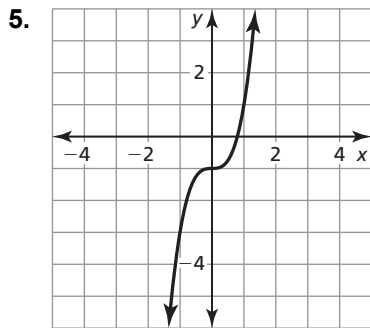**Evaluate the expression.**

1. $-4 \cdot 5^3$

2. $(-3)^4$

3. $-\left(\frac{7}{8}\right)^2$

4. $\left(\frac{3}{10}\right)^3$

Find the domain and range of the function represented by the graph.

5.

6.

7.

8.

9. Is the expression "the sum of the square of x and the square of the opposite of x" equivalent to 0 or $2x^2$? Explain your reasoning.

Name _____ Date _____

Exponential Growth and Decay Functions
For use with Exploration 6.1

Essential Question What are some of the characteristics of the graph of an exponential function?

1 EXPLORATION: Identifying Graphs of Exponential Functions

Work with a partner. Match each exponential function with its graph. Use a table of values to sketch the graph of the function, if necessary.

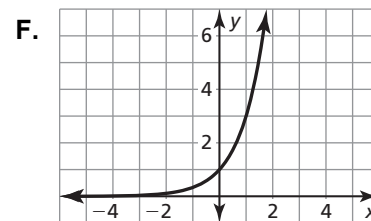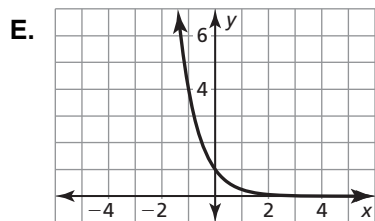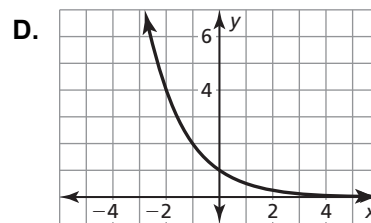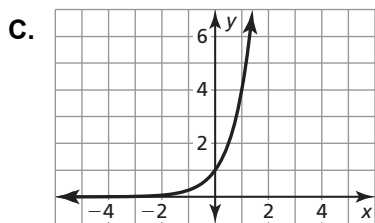**a.** $f(x) = 2^x$ **b.** $f(x) = 3^x$ **c.** $f(x) = 4^x$

d. $f(x) = \left(\dfrac{1}{2}\right)^x$ **e.** $f(x) = \left(\dfrac{1}{3}\right)^x$ **f.** $f(x) = \left(\dfrac{1}{4}\right)^x$

A.

B.

C.

D.

E.

F.
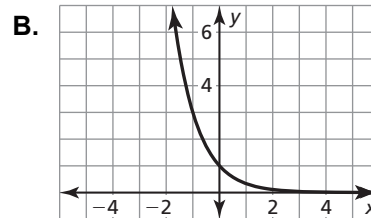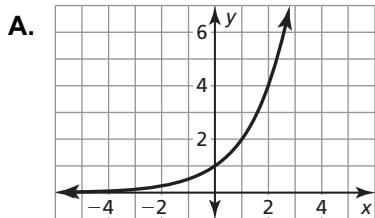
6.1 **Exponential Growth and Decay Functions** (continued)

2 **EXPLORATION:** Characteristics of Graphs of Exponential Functions

Work with a partner. Use the graphs in Exploration 1 to determine the domain, range, and y-intercept of the graph of $f(x) = b^x$, where b is a positive real number other than 1. Explain your reasoning.

Communicate Your Answer

3. What are some of the characteristics of the graph of an exponential function?

4. In Exploration 2, is it possible for the graph of $f(x) = b^x$ to have an x-intercept? Explain your reasoning.

Name _____ Date _____

6.1 Notetaking with Vocabulary
For use after Lesson 6.1

In your own words, write the meaning of each vocabulary term.

exponential function

exponential growth function

growth factor

asymptote

exponential decay function

decay factor

Core Concepts

Parent Function for Exponential Growth Functions

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base b. The graph shows the general shape of an exponential growth function.

The *x*-axis is an *asymptote* of the graph. An **asymptote** is a line that a graph approaches more and more closely.

$f(x) = b^x$ ($b > 1$)

(0, 1)

(1, *b*)

The graph rises from left to right, passing through the points (0, 1) and (1, *b*).
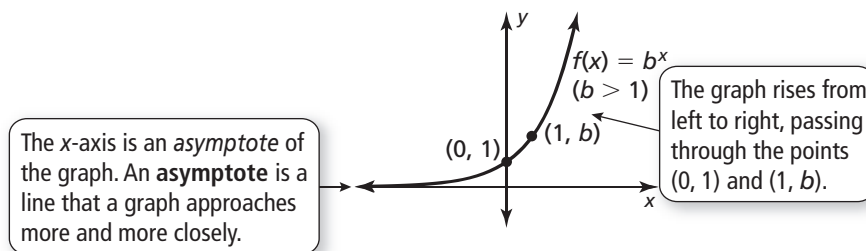
The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Notes:

6.1 **Notetaking with Vocabulary** (continued)

Parent Function for Exponential Decay Functions

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base b. The graph shows the general shape of an exponential decay function.
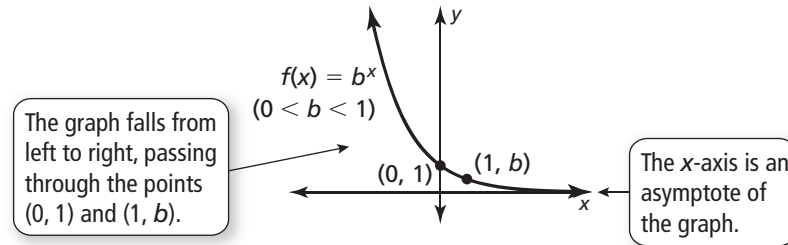

The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

Notes:

Compound Interest

Consider an initial principal P deposited in an account that pays interest at an annual rate r (expressed as a decimal), compounded n times per year. The amount A in the account after t years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

Notes:

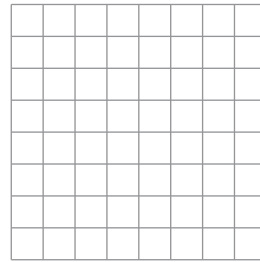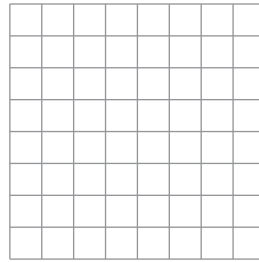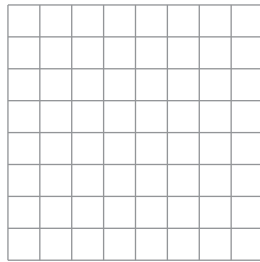**6.1** **Notetaking with Vocabulary** (continued)

Extra Practice

In Exercises 1–4, tell whether the function represents *exponential growth* or *exponential decay.* Then graph the function.

1. $y = \left(\dfrac{1}{12}\right)^x$ **2.** $y = (1.5)^x$ **3.** $y = \left(\dfrac{7}{2}\right)^x$ **4.** $y = (0.8)^x$

5. The number of bacteria y (in thousands) in a culture can be approximated by the model $y = 100(1.99)^t$, where t is the number of hours the culture is incubated.

   a. Tell whether the model represents exponential growth or exponential decay.

   b. Identify the hourly percent increase or decrease in the number of bacteria.

   c. Estimate when the number of bacteria will be 1,000,000.

In Exercises 6 and 7, use the given information to find the amount A in the account earning compound interest after 5 years when the principal is $1250.

   6. $r = 2.25\%$, compounded quarterly

   7. $r = 1.25\%$, compounded daily

Name_____ Date _____

6.2 The Natural Base e
For use with Exploration 6.2

Essential Question What is the natural base *e*?

So far in your study of mathematics, you have worked with special numbers such as π and i. Another special number is called the *natural base* and is denoted by *e*. The natural base *e* is irrational, so you cannot find its exact value.

1 EXPLORATION: Approximating the Natural Base *e*

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. One way to approximate the natural base *e* is to approximate the sum

$$1 + \frac{1}{1} + \frac{1}{1 \bullet 2} + \frac{1}{1 \bullet 2 \bullet 3} + \frac{1}{1 \bullet 2 \bullet 3 \bullet 4} + \cdots.$$

Use a spreadsheet or a graphing calculator to approximate this sum. Explain the steps you used. How many decimal places did you use in your approximation?

2 EXPLORATION: Approximating the Natural Base *e*

Work with a partner. Another way to approximate the natural base *e* is to consider the expression

$$\left(1 + \frac{1}{x}\right)^x.$$

As *x* increases, the value of this expression approaches the value of *e*. Complete the table. Then use the results in the table to approximate *e*. Compare this approximation to the one you obtained in Exploration 1.

x	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1 + \dfrac{1}{x}\right)^x$						

**6.2** **The Natural Base *e* (continued)**

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3 **EXPLORATION:** Graphing a Natural Base Function

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use your approximate value of *e* in Exploration 1 or 2 to complete the table. Then sketch the graph of the *natural base exponential function* $y = e^x$. You can use a graphing calculator and the $\boxed{e^x}$ key to check your graph. What are the domain and range of $y = e^x$? Justify your answers.
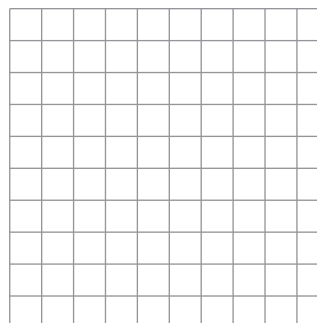
x	-2	-1	0	1	2
$y = e^x$					
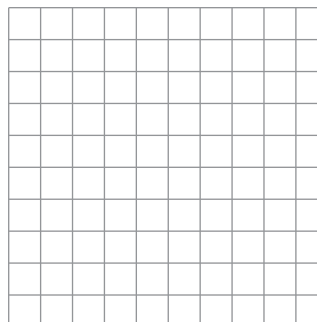
Communicate Your Answer

4. What is the natural base *e*?

5. Repeat Exploration 3 for the natural base exponential function $y = e^{-x}$. Then compare the graph of $y = e^x$ to the graph of $y = e^{-x}$.

x	-2	-1	0	1	2
$y = e^{-x}$					

6. The natural base *e* is used in a wide variety of real-life applications. Use the Internet or some other reference to research some of the real-life applications of *e*.

Name_____ Date_____

6.2 Notetaking with Vocabulary
For use after Lesson 6.2

In your own words, write the meaning of each vocabulary term.

natural base *e*

Core Concepts

The Natural Base e

The natural base *e* is irrational. It is defined as follows:

As *x* approaches $+\infty$, $\left(1 + \frac{1}{x}\right)^x$ approaches $e \approx 2.71828182846$.

Notes:

Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function*.

- When $a > 0$ and $r > 0$, the function is an exponential growth function.

- When $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown.

Notes:

Name _____ Date _____
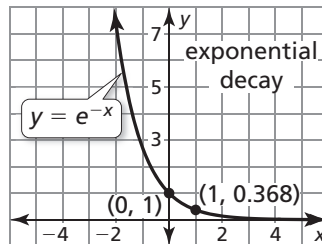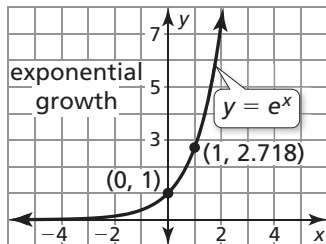
Continuously Compounded Interest

When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

where P is the principal and r is the annual interest rate expressed as a decimal.

Notes:

Extra Practice

In Exercises 1–4, simplify the expression.

1. $e^{-9} \bullet e^{12}$

2. $\dfrac{25e^2}{35e^7}$

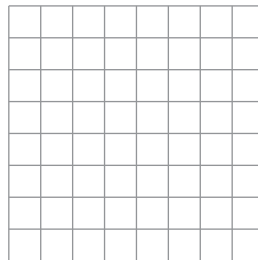**3.** $\left(2e^{-3x}\right)^5 \bullet 2e^{x+1}$
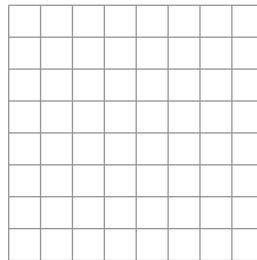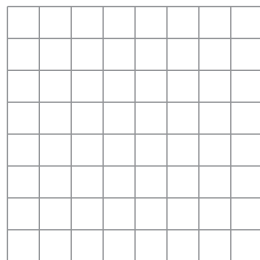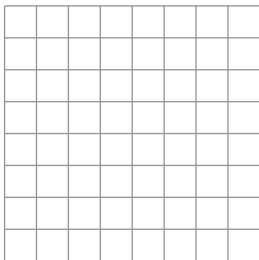
4. $\sqrt[4]{16e^{24x}}$

In Exercises 5–8, tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

5. $y = 2e^{-x}$
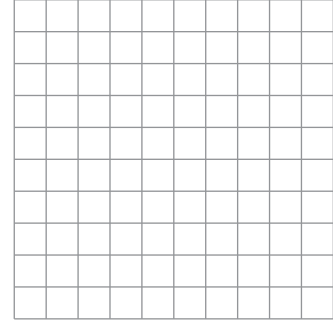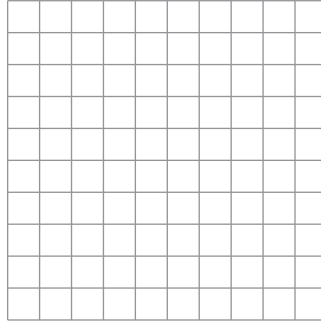
6. $y = 0.75e^{4x}$

7. $y = 5e^{0.25x}$

8. $y = 0.8e^{-3x}$

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In Exercises 9–11, use a table of values or a graphing calculator to graph the function. Then identify the domain and range.

9. $y = e^x - 4$

10. $y = 2e^{x+3}$

11. $y = -e^x + 5$
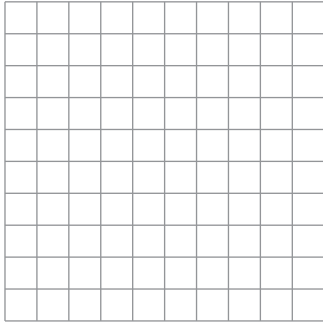
12. The population of Evans City is currently 48,500 and is declining at a rate of 2.5% each year. You can model the population of Evans City by the equation $P_t = P_c e^{rt}$, where P_c is the current population, P_t is the population after t years, and r is the decimal rate of decline per year. Predict the population of Evans City after 25 years.

13. Your parents will need $25,000 in 10 years to pay for your brother's college tuition. They can invest in an account with an interest rate of 9.8% that compounds continuously. How much should your parents invest today in order to have your brother's full tuition available in 10 years?

6.3 **Logarithms and Logarithmic Functions**
For use with Exploration 6.3

Essential Question What are some of the characteristics of the graph of a logarithmic function?

Every exponential function of the form $f(x) = b^x$, where b is a positive real number other than 1, has an inverse function that you can denote by $g(x) = \log_b x$. This inverse function is called a *logarithmic function with base b*.

---

1 **EXPLORATION:** Rewriting Exponential Equations

Work with a partner. Find the value of x in each exponential equation. Explain your reasoning. Then use the value of x to rewrite the exponential equation in its equivalent logarithmic form, $x = \log_b y$.

a. $2^x = 8$ **b.** $3^x = 9$ **c.** $4^x = 2$

d. $5^x = 1$ **e.** $5^x = \dfrac{1}{5}$ **f.** $8^x = 4$

---

2 **EXPLORATION:** Graphing Exponential and Logarithmic Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Complete each table for the given exponential function. Use the results to complete the table for the given logarithmic function. Explain your reasoning. Then sketch the graphs of f and g in the same coordinate plane.

a.

x	-2	-1	0	1	2
$f(x) = 2^x$					

x					
$g(x) = \log_2 x$	-2	-1	0	1	2

Name_____ Date_____

2 **EXPLORATION:** Graphing Exponential and Logarithmic Functions (continued)

b.

x	−2	−1	0	1	2
$f(x) = 10^x$					

x					
$g(x) = \log_{10} x$	−2	−1	0	1	2
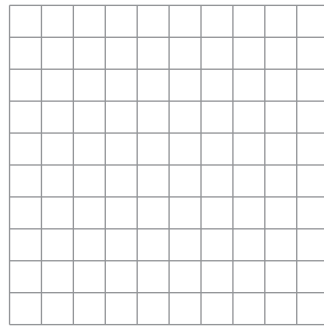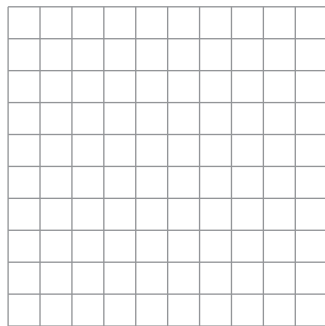
3 **EXPLORATION:** Characteristics of Graphs of Logarithmic Functions

Work with a partner. Use the graphs you sketched in Exploration 2 to determine the domain, range, *x*-intercept, and asymptote of the graph of $g(x) = \log_b x$, where *b* is a positive real number other than 1. Explain your reasoning.

Communicate Your Answer

4. What are some of the characteristics of the graph of a logarithmic function?

5. How can you use the graph of an exponential function to obtain the graph of a logarithmic function?

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Notetaking with Vocabulary
For use after Lesson 6.3

In your own words, write the meaning of each vocabulary term.

logarithm of y with base b function

common logarithm

natural logarithm

Core Concepts

Definition of Logarithm with Base b

Let b and y be positive real numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as

$$\log_b y = x \qquad \text{if and only if} \qquad b^x = y.$$

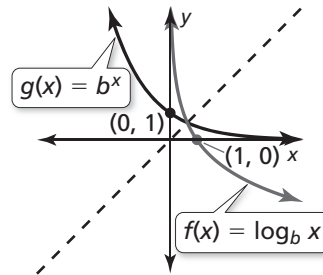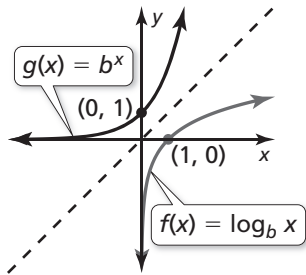The expression $\log_b y$ is read as "log base b of y."

Notes:

6.3 **Notetaking with Vocabulary (continued)**

Parent Graphs for Logarithmic Functions

The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$.
Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of
$f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$ **Graph of $f(x) = \log_b x$ for $0 < b < 1$**

Note that the y-axis is a vertical asymptote of the graph of $f(x) = \log_b x$. The domain of
$f(x) = \log_b x$ is $x > 0$, and the range is all real numbers.

Notes:

Extra Practice

In Exercises 1–4, rewrite the equation in exponential form.

1. $\log_{10} 1000 = 3$ **2.** $\log_5 \dfrac{1}{25} = -2$ **3.** $\log_{10} 1 = 0$ **4.** $\log_{1/4} 64 = -3$

Name _____ Date _____

In Exercises 5–8, rewrite the equation in logarithmic form.

5. $12^2 = 144$ **6.** $20^{-1} = \dfrac{1}{20}$ **7.** $216^{1/3} = 6$ **8.** $4^0 = 1$

In Exercises 9–12, evaluate the logarithm.

9. $\log_4 64$ **10.** $\log_{1/8} 1$ **11.** $\log_2 \dfrac{1}{32}$ **12.** $\log_{1/25} \dfrac{1}{5}$

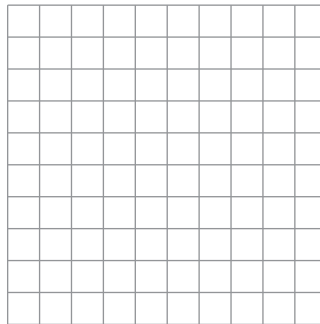**In Exercises 13 and 14, simplify the expression.**
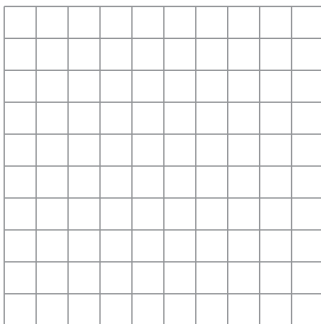
13. $13^{\log_{13} 6}$ **14.** $\ln e^{x^3}$

In Exercises 15 and 16, find the inverse of the function.

15. $y = 15^x + 10$ **16.** $y = \ln(2x) - 8$

In Exercises 17 and 18, graph the function. Determine the asymptote of the function.

17. $y = \log_2(x + 1)$ **18.** $y = \log_{1/2} x - 4$

Name_____ Date _____

6.4 Transformations of Exponential and Logarithmic Functions
For use with Exploration 6.4

Essential Question How can you transform the graphs of exponential and logarithmic functions?
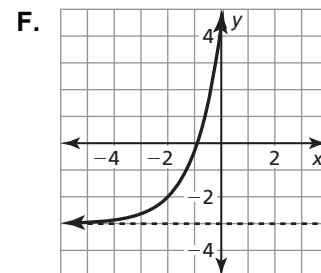
1 EXPLORATION: Identifying Transformations

Work with a partner. Each graph shown is a transformation of the parent function

$$f(x) = e^x \qquad \text{or} \qquad f(x) = \ln x.$$

Match each function with its graph. Explain your reasoning. Then describe the transformation of f represented by g.

a. $g(x) = e^{x+2} - 3$　　　　**b.** $g(x) = -e^{x+2} + 1$　　　　**c.** $g(x) = e^{x-2} - 1$

d. $g(x) = \ln(x + 2)$　　　　**e.** $g(x) = 2 + \ln x$　　　　**f.** $g(x) = 2 + \ln(-x)$
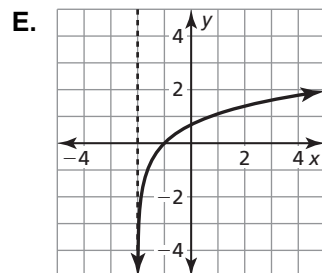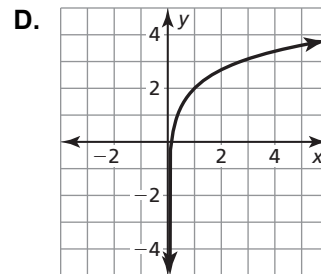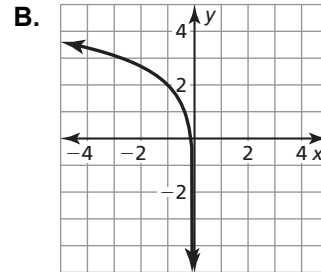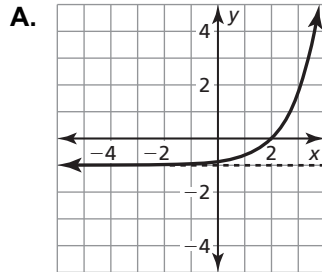
A.

B.

C.

D.

E.

F.

6.4 **Transformations of Exponential and Logarithmic Functions** (continued)

2 **EXPLORATION:** Characteristics of Graphs

Work with a partner. Determine the domain, range, and asymptote of each function in Exploration 1. Justify your answers.

Communicate Your Answer

3. How can you transform the graphs of exponential and logarithmic functions?

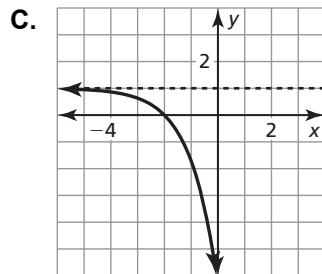**4.** Find the inverse of each function in Exploration 1. Then check your answer by using a graphing calculator to graph each function and its inverse in the same viewing window.

Name_____ Date_____

Notetaking with Vocabulary
For use after Lesson 6.4

In your own words, write the meaning of each vocabulary term.

exponential function

logarithmic function

transformations

Core Concepts

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x - h)$	$g(x) = 4^{x-3}$ $g(x) = 4^{x+2}$	3 units right 2 units left
Vertical Translation Graph shifts up or down.	$f(x) + k$	$g(x) = 4^x + 5$ $g(x) = 4^x - 1$	5 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = 4^{-x}$ $g(x) = -4^x$	over y-axis over x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis	$f(ax)$	$g(x) = 4^{2x}$ $g(x) = 4^{x/2}$	shrink by $\frac{1}{2}$ stretch by 2
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis	$a \bullet f(x)$	$g(x) = 3(4^x)$ $g(x) = \frac{1}{4}(4^x)$	stretch by 3 shrink by $\frac{1}{4}$

Notes:

6.4   Notetaking with Vocabulary (continued)

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x) = \log(x-4)$ $g(x) = \log(x+7)$	4 units right 7 units left
Vertical Translation Graph shifts up or down.	$f(x)+k$	$g(x) = \log x + 3$ $g(x) = \log x - 1$	3 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = \log(-x)$ $g(x) = -\log x$	over y-axis over x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis	$f(ax)$	$g(x) = \log(4x)$ $g(x) = \log\left(\frac{1}{3}x\right)$	shrink by $\frac{1}{4}$ stretch by 3
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis	$a \bullet f(x)$	$g(x) = 5\log x$ $g(x) = \frac{2}{3}\log x$	stretch by 5 shrink by $\frac{2}{3}$

Notes:

Name_____ Date_____

Extra Practice

In Exercises 1–6, describe the transformation of *f* represented by *g*. Then graph each function.
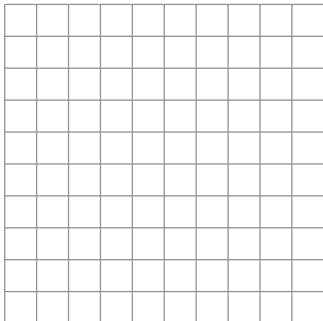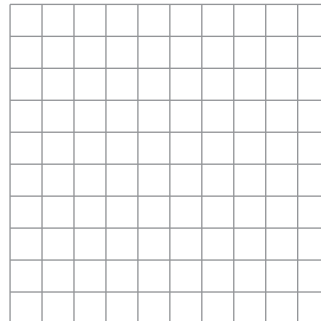
1. $f(x) = 6^x, g(x) = 6^x + 6$
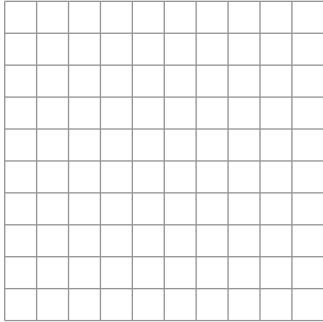
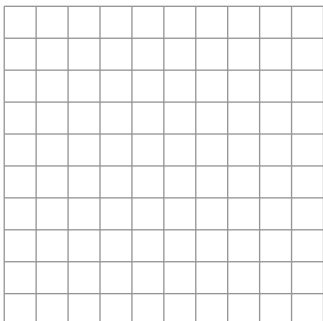**2.** $f(x) = e^x, g(x) = e^{x-4}$
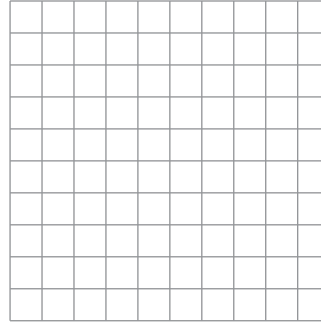
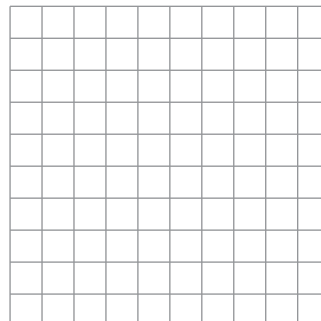**3.** $f(x) = \log_5 x, g(x) = \frac{1}{2}\log_5(x + 7)$

4. $f(x) = \log_{1/3} x, g(x) = \log_{1/3} x - \frac{4}{3}$

5. $f(x) = \left(\frac{1}{5}\right)^x, g(x) = \left(\frac{1}{5}\right)^{-3x} + 4$

6. $f(x) = \log x, g(x) = -3\log(x - 2)$

6.5 Properties of Logarithms
For use with Exploration 6.5

Essential Question   How can you use properties of exponents to derive properties of logarithms?

Let $x = \log_b m$ and $y = \log_b n.$

The corresponding exponential forms of these two equations are

$b^x = m$ and $b^y = n$.

---

1 EXPLORATION: Product Property of Logarithms

Work with a partner. To derive the Product Property, multiply m and n to obtain

$mn = b^x b^y = b^{x+y}.$

The corresponding logarithmic form of $mn = b^{x+y}$ is $\log_b mn = x + y$. So,

$\log_b mn =$ _____.   Product Property of Logarithms

---

2 EXPLORATION: Quotient Property of Logarithms

Work with a partner. To derive the Quotient Property, divide m by n to obtain

$\dfrac{m}{n} = \dfrac{b^x}{b^y} = b^{x-y}.$

The corresponding logarithmic form of $\dfrac{m}{n} = b^{x-y}$ is $\log_b \dfrac{m}{n} = x - y$. So,

$\log_b \dfrac{m}{n} =$ _____.   Quotient Property of Logarithms

---

3 EXPLORATION: Power Property of Logarithms

Work with a partner. To derive the Power Property, substitute b^x for m in the expression $\log_b m^n$, as follows.

$$\log_b m^n = \log_b \left(b^x\right)^n \qquad \text{Substitute } b^x \text{ for } m.$$
$$= \log_b b^{nx} \qquad \text{Power of a Power Property of Exponents}$$
$$= nx \qquad \text{Inverse Property of Logarithms}$$

6.5   **Properties of Logarithms** (continued)

3 **EXPLORATION:** Power Property of Logarithms (continued)

So, substituting $\log_b m$ for x, you have

$$\log_b m^n = \underline{\hspace{1.5cm}}. \qquad \text{Power Property of Logarithms}$$

Communicate Your Answer

4. How can you use properties of exponents to derive properties of logarithms?

5. Use the properties of logarithms that you derived in Explorations 1–3 to evaluate each logarithmic expression.

a. $\log_4 16^3$

b. $\log_3 81^{-3}$

c. $\ln e^2 + \ln e^5$

d. $2 \ln e^6 - \ln e^5$

e. $\log_5 75 - \log_5 3$

f. $\log_4 2 + \log_4 32$

6.5 Notetaking with Vocabulary
For use after Lesson 6.5

In your own words, write the meaning of each vocabulary term.

base

properties of exponents

Core Concepts

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

Product Property $\quad \log_b mn = \log_b m + \log_b n$

Quotient Property $\quad \log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\quad \log_b m^n = n \log_b m$

Notes:

Change-of-Base Formula

If a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$, then

$$\log_c a = \frac{\log_b a}{\log_b c}.$$

In particular, $\log_c a = \dfrac{\log a}{\log c}$ and $\log_c a = \dfrac{\ln a}{\ln c}$.

Notes:

Name_____ Date_____

Extra Practice

In Exercises 1–4, use $\log_2 5 \approx 2.322$ and $\log_2 12 \approx 3.585$ to evaluate the logarithm.

1. $\log_2 60$

2. $\log_2 \dfrac{1}{144}$

3. $\log_2 \dfrac{12}{25}$

4. $\log_2 720$

In Exercises 5–8, expand the logarithmic expression.

5. $\log 10x$

6. $\ln 2x^6$

7. $\log_3 \dfrac{x^4}{3y^3}$

8. $\ln \sqrt[4]{3y^2}$

In Exercises 9–13, condense the logarithmic expression.

9. $\log_2 3 + \log_2 8$

10. $\log_5 4 - 2\log_5 5$

11. $3\ln 6x + \ln 4y$

12. $\log_2 625 - \log_2 125 + \dfrac{1}{3}\log_2 27$

13. $-\log_6 6 - \log_6 2y + 2\log_6 3x$

6.5 **Notetaking with Vocabulary** (continued)

In Exercises 14–17, use the change-of-base formula to evaluate the logarithm.

14. $\log_3 17$ **15.** $\log_9 294$ **16.** $\log_7 \frac{4}{9}$ **17.** $\log_6 \frac{1}{10}$

18. For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function $L(I) = 10 \log \dfrac{I}{I_0}$, where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). The intensity of the sound of a certain children's television show is half the intensity of the adult show that is on before it. By how many decibels does the loudness decrease?

19. Hick's Law states that given n equally probable choices, such as choices on a menu, the average human's reaction time T (in seconds) required to choose from those choices is approximately $T = a + b \bullet \log_2(n + 1)$ where a and b are constants. If $a = 4$ and $b = 1$, how much longer would it take a customer to choose what to eat from a menu of 40 items than from a menu of 10 items?

Name_____ Date_____

6.6 Solving Exponential and Logarithmic Equations
For use with Exploration 6.6

Essential Question How can you solve exponential and logarithmic equations?

1 EXPLORATION: Solving Exponential and Logarithmic Equations

Work with a partner. Match each equation with the graph of its related system of equations. Explain your reasoning. Then use the graph to solve the equation.
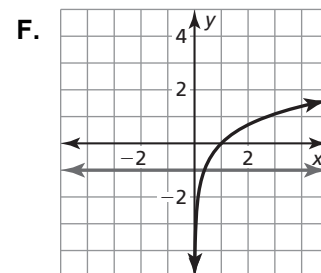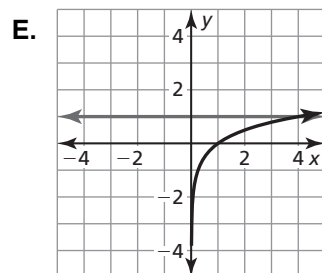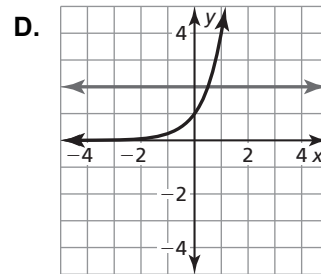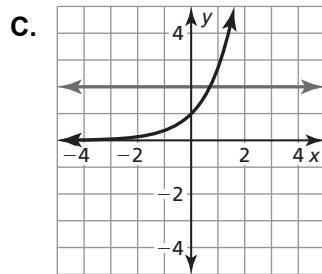
a. $e^x = 2$

b. $\ln x = -1$

c. $2^x = 3^{-x}$

d. $\log_4 x = 1$

e. $\log_5 x = \dfrac{1}{2}$

f. $4^x = 2$

A.

B.

C.

D.

E.

F.
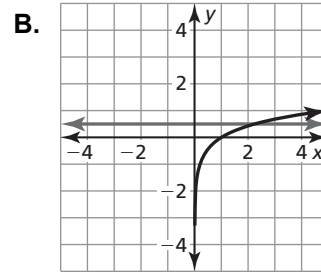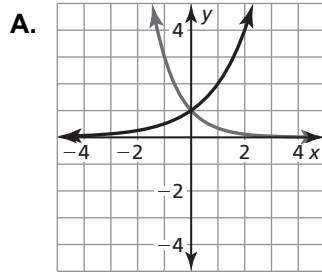
6.6   **Solving Exponential and Logarithmic Equations** (continued)

2 **EXPLORATION:** Solving Exponential and Logarithmic Equations

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Look back at the equations in Explorations 1(a) and 1(b). Suppose you want a more accurate way to solve the equations than using a graphical approach.

   a. Show how you could use a *numerical approach* by creating a table. For instance, you might use a spreadsheet to solve the equations.

   b. Show how you could use an *analytical approach*. For instance, you might try solving the equations by using the inverse properties of exponents and logarithms.

Communicate Your Answer

   3. How can you solve exponential and logarithmic equations?

   4. Solve each equation using any method. Explain your choice of method.

   a. $16^x = 2$

   b. $2^x = 4^{2x+1}$

   c. $2^x = 3^{x+1}$

   d. $\log x = \dfrac{1}{2}$

   e. $\ln x = 2$

   f. $\log_3 x = \dfrac{3}{2}$

6.6 Notetaking with Vocabulary
For use after Lesson 6.6

In your own words, write the meaning of each vocabulary term.

exponential equations

logarithmic equations

Core Concepts

Property of Equality for Exponential Equations

Algebra   If b is a positive real number other than 1, then $b^x = b^y$ if and only if $x = y$.

Example   If $3^x = 3^5$, then $x = 5$. If $x = 5$, then $3^x = 3^5$.

Notes:

Property of Equality for Logarithmic Equations

Algebra   If b, x, and y are positive real numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Example   If $\log_2 x = \log_2 7$, then $x = 7$. If $x = 7$, then $\log_2 x = \log_2 7$.

Notes:

6.6 **Notetaking with Vocabulary** (continued)

Extra Practice

In Exercises 1–6, solve the equation.

1. $5^{2x+4} = 5^{5x-8}$

2. $4^{2x-1} = 8^{x+2}$

3. $3^{x+3} = 5$

4. $\left(\dfrac{1}{5}\right)^{3x-2} = \sqrt{25^x}$

5. $12e^{1-x} = 500$

6. $-14 + 3e^x = 11$

In Exercises 7–11, solve the equation. Check for extraneous solutions.

7. $2 = \log_3(4x)$

8. $\ln(x^2 + 3) = \ln(4)$

9. $\log_8(x^2 - 5) = \dfrac{2}{3}$

10. $\ln x + \ln(x + 2) = \ln(x + 6)$

11. $\log_2(x + 5) - \log_2(x - 2) = 3$

6.6 **Notetaking with Vocabulary** (continued)

12. Solve the inequality $\log x \le \dfrac{1}{2}$.

13. Your parents buy juice for your graduation party and leave it in their hot car. When they take the cans out of the car and move them to the basement, the temperature of the juice is 80°F. The room temperature of the basement is 60°F, and the cooling rate of the juice is $r = 0.0147$. Using Newton's Law of Cooling, how long will it take to cool the juice to 63°F?

14. Earthquake intensity is measured by the formula $R = \log\left(\dfrac{I}{I_0}\right)$ where R is the Richter scale

rating of an earthquake, I is the intensity of the earthquake, and I_0 is the intensity of the smallest detectable wave. In 1906, an earthquake in San Francisco had an estimated measure of 7.8 on the Richter scale. In the same year, another earthquake had an intensity level four times stronger than the San Francisco earthquake giving it a Richter scale rating of $R_2 = \log\left(\dfrac{4I}{I_0}\right)$. What was

the Richter scale rating on a scale of 1–10 of the other earthquake?
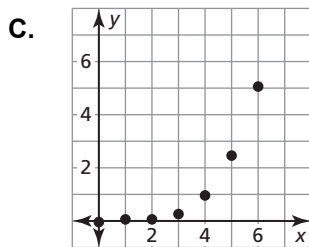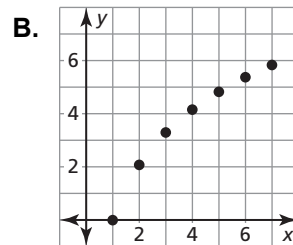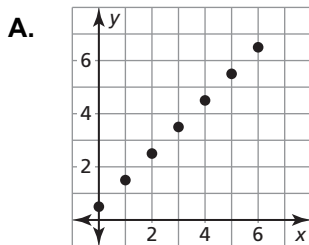
Name _____ Date _____

Essential Question How can you recognize polynomial, exponential, and logarithmic models?

1 EXPLORATION: Recognizing Different Types of Models

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Match each type of model with the appropriate scatter plot. Use a regression program to find a model that fits the scatter plot.

    a. linear (positive slope)   **b.** linear (negative slope)   **c.** quadratic

    d. cubic   **e.** exponential   **f.** logarithmic

A.

B.

C.

D.

E.

F.
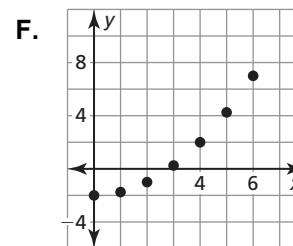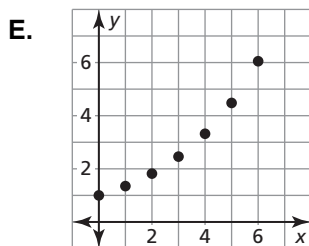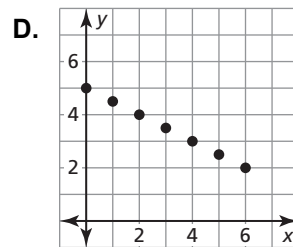
6.7 **Modeling with Exponential and Logarithmic Functions** (continued)

2 **EXPLORATION:** Exploring Gaussian and Logistic Models

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Two common types of functions that are related to exponential functions are given. Use a graphing calculator to graph each function. Then determine the domain, range, intercept, and asymptote(s) of the function.

a. Gaussian Function: $f(x) = e^{-x^2}$

b. Logistic Function: $f(x) = \dfrac{1}{1 + e^{-x}}$

Communicate Your Answer

3. How can you recognize polynomial, exponential, and logarithmic models?

4. Use the Internet or some other reference to find real-life data that can be modeled using one of the types given in Exploration 1. Create a table and a scatter plot of the data. Then use a regression program to find a model that fits the data.
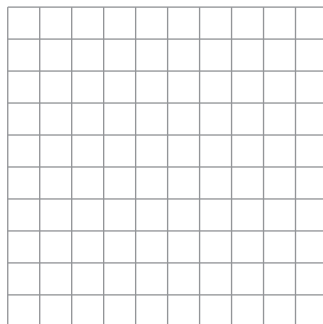
6.7 Notetaking with Vocabulary
For use after Lesson 6.7

In your own words, write the meaning of each vocabulary term.

finite differences

common ratio

point-slope form

Notes:

6.7 **Notetaking with Vocabulary** (continued)

Extra Practice

In Exercises 1 and 2, determine the type of function represented by the table. Explain your reasoning.

1.

x	6	7	8	9	10	11
y	34	47	62	79	98	119

2.

x	−5	−3	−1	1	3	5
y	$\dfrac{1}{5}$	$\dfrac{3}{5}$	$\dfrac{9}{5}$	$\dfrac{27}{5}$	$\dfrac{81}{5}$	$\dfrac{243}{5}$

In Exercises 3–6, write an exponential function $y = ab^x$ whose graph passes through the given points.

3. $(1, 12), (3, 108)$

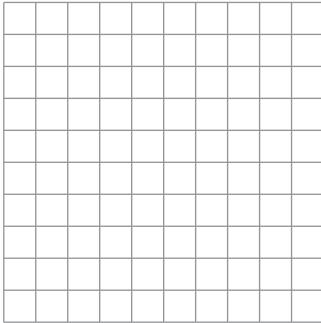**4.** $(-1, 2), (3, 32)$

5. $(2, 9), (4, 324)$

6. $(-2, 2), (1, 0.25)$

6.7 Notetaking with Vocabulary (continued)

7. An Olympic swimmer starts selling a new type of goggles. The table shows the number y of goggles sold during a 6-month period.

Months, x	1	2	3	4	5	6
Goggles sold, y	28	47	64	79	97	107

a. Create a scatterplot of the data.

b. Create a scatterplot of the data pairs $(x, \ln y)$ to show that an exponential model should be a good fit for the original data pairs (x, y). Write a function that models the data.
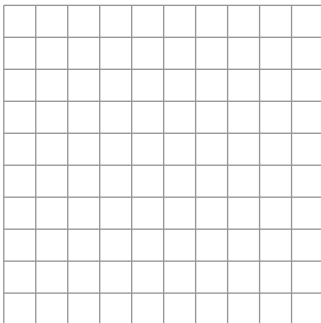
c. Use a graphing calculator to write an exponential model for the data.

d. Use each model to predict the number of goggles sold after 1 year.