

# 6.4

## Transformations of Exponential and Logarithmic Functions

For use with Exploration 6.4

**Essential Question** How can you transform the graphs of exponential and logarithmic functions?

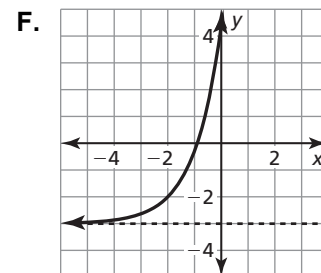
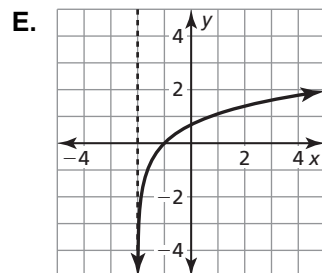
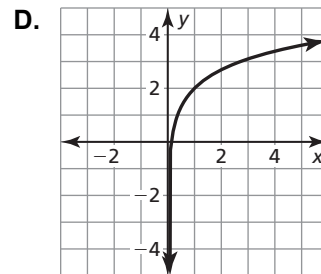
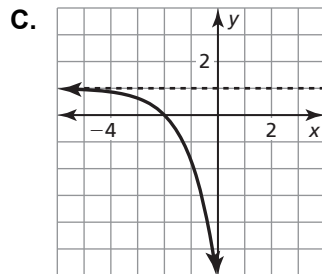
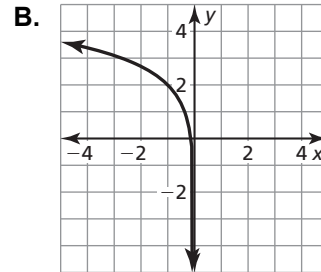
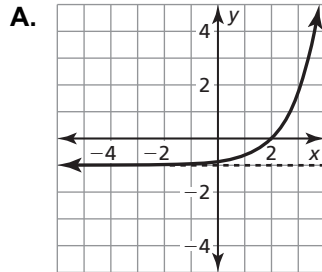
**1 EXPLORATION: Identifying Transformations**

**Work with a partner.** Each graph shown is a transformation of the parent function

$$f(x) = e^x \quad \text{or} \quad f(x) = \ln x.$$

Match each function with its graph. Explain your reasoning. Then describe the transformation of  $f$  represented by  $g$ .

- |                         |                          |                         |
|-------------------------|--------------------------|-------------------------|
| a. $g(x) = e^{x+2} - 3$ | b. $g(x) = -e^{x+2} + 1$ | c. $g(x) = e^{x-2} - 1$ |
| d. $g(x) = \ln(x + 2)$  | e. $g(x) = 2 + \ln x$    | f. $g(x) = 2 + \ln(-x)$ |



**6.4 Transformations of Exponential and Logarithmic Functions (continued)****2 EXPLORATION: Characteristics of Graphs**

**Work with a partner.** Determine the domain, range, and asymptote of each function in Exploration 1. Justify your answers.

**Communicate Your Answer**

3. How can you transform the graphs of exponential and logarithmic functions?
  
  
  
  
  
  
  
  
  
  
4. Find the inverse of each function in Exploration 1. Then check your answer by using a graphing calculator to graph each function and its inverse in the same viewing window.

**6.4**

**Notetaking with Vocabulary**  
For use after Lesson 6.4

In your own words, write the meaning of each vocabulary term.

exponential function

logarithmic function

transformations

**Core Concepts**

Transformation	$f(x)$ Notation	Examples
<b>Horizontal Translation</b> Graph shifts left or right.	$f(x - h)$	$g(x) = 4^{x-3}$ 3 units right $g(x) = 4^{x+2}$ 2 units left
<b>Vertical Translation</b> Graph shifts up or down.	$f(x) + k$	$g(x) = 4^x + 5$ 5 units up $g(x) = 4^x - 1$ 1 unit down
<b>Reflection</b> Graph flips over $x$ - or $y$ -axis.	$f(-x)$ $-f(x)$	$g(x) = 4^{-x}$ over $y$ -axis $g(x) = -4^x$ over $x$ -axis
<b>Horizontal Stretch or Shrink</b> Graph stretches away from or shrinks toward $y$ -axis	$f(ax)$	$g(x) = 4^{2x}$ shrink by $\frac{1}{2}$ $g(x) = 4^{x/2}$ stretch by 2
<b>Vertical Stretch or Shrink</b> Graph stretches away from or shrinks toward $x$ -axis	$a \cdot f(x)$	$g(x) = 3(4^x)$ stretch by 3 $g(x) = \frac{1}{4}(4^x)$ shrink by $\frac{1}{4}$

**Notes:**

**6.4** Notetaking with Vocabulary (continued)

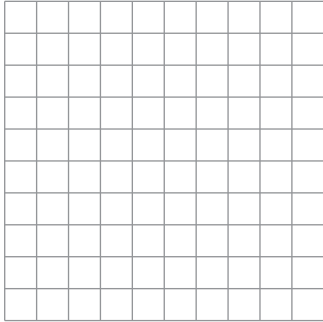
Transformation	$f(x)$ Notation	Examples
<b>Horizontal Translation</b> Graph shifts left or right.	$f(x - h)$	$g(x) = \log(x - 4)$ 4 units right $g(x) = \log(x + 7)$ 7 units left
<b>Vertical Translation</b> Graph shifts up or down.	$f(x) + k$	$g(x) = \log x + 3$ 3 units up $g(x) = \log x - 1$ 1 unit down
<b>Reflection</b> Graph flips over $x$ - or $y$ -axis.	$f(-x)$ $-f(x)$	$g(x) = \log(-x)$ over $y$ -axis $g(x) = -\log x$ over $x$ -axis
<b>Horizontal Stretch or Shrink</b> Graph stretches away from or shrinks toward $y$ -axis	$f(ax)$	$g(x) = \log(4x)$ shrink by $\frac{1}{4}$ $g(x) = \log\left(\frac{1}{3}x\right)$ stretch by 3
<b>Vertical Stretch or Shrink</b> Graph stretches away from or shrinks toward $x$ -axis	$a \cdot f(x)$	$g(x) = 5 \log x$ stretch by 5 $g(x) = \frac{2}{3} \log x$ shrink by $\frac{2}{3}$

**Notes:**

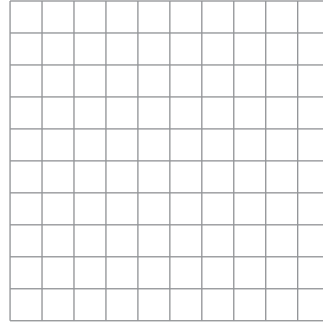
**6.4** Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–6, describe the transformation of  $f$  represented by  $g$ . Then graph each function.

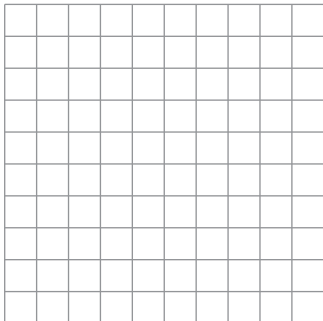
1.  $f(x) = 6^x, g(x) = 6^x + 6$



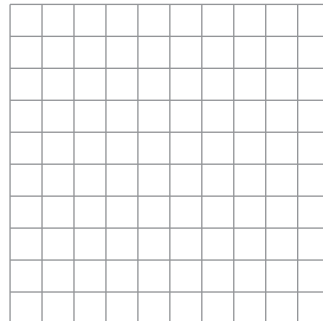
2.  $f(x) = e^x, g(x) = e^{x-4}$



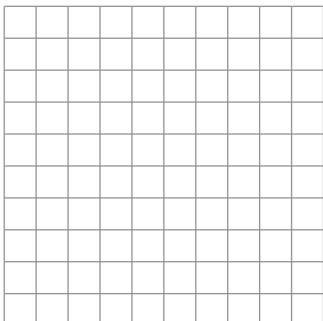
3.  $f(x) = \log_5 x, g(x) = \frac{1}{2} \log_5(x + 7)$



4.  $f(x) = \log_{1/3} x, g(x) = \log_{1/3} x - \frac{4}{3}$



5.  $f(x) = \left(\frac{1}{5}\right)^x, g(x) = \left(\frac{1}{5}\right)^{-3x} + 4$



6.  $f(x) = \log x, g(x) = -3 \log(x - 2)$

