Name _____ Date _____

6.3 Logarithms and Logarithmic Functions

For use with Exploration 6.3

Essential Question What are some of the characteristics of the graph of a logarithmic function?

Every exponential function of the form $f(x) = b^x$, where b is a positive real number other than 1, has an inverse function that you can denote by $g(x) = \log_b x$. This inverse function is called a *logarithmic function with base b*.

1 EXPLORATION: Rewriting Exponential Equations

Work with a partner. Find the value of x in each exponential equation. Explain your reasoning. Then use the value of x to rewrite the exponential equation in its equivalent logarithmic form, $x = \log_b y$.

a. $2^x = 8$

b. $3^x = 9$

c. $4^x = 2$

d. $5^x = 1$

e. $5^x = \frac{1}{5}$

f. $8^x = 4$

2 EXPLORATION: Graphing Exponential and Logarithmic Functions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Complete each table for the given exponential function. Use the results to complete the table for the given logarithmic function. Explain your reasoning. Then sketch the graphs of f and g in the same coordinate plane.

a.

x	-2	-1	0	1	2
$f(x) = 2^x$					

x					
$g(x) = \log_2 x$	-2	-1	0	1	2

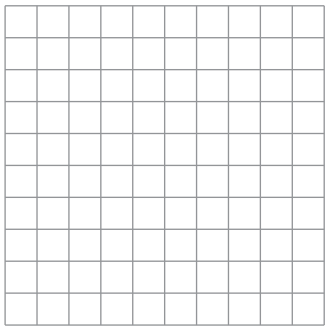
6.3 Logarithms and Logarithmic Functions (continued)

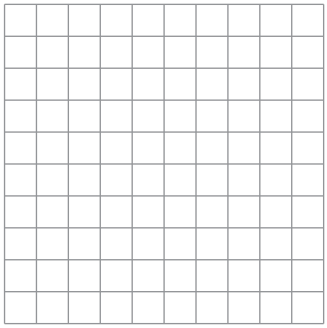
2 EXPLORATION: Graphing Exponential and Logarithmic Functions (continued)

b.

x	-2	-1	0	1	2
$f(x) = 10^x$					

x					
$g(x) = \log_{10} x$	-2	-1	0	1	2





3 EXPLORATION: Characteristics of Graphs of Logarithmic Functions

Work with a partner. Use the graphs you sketched in Exploration 2 to determine the domain, range, x-intercept, and asymptote of the graph of $g(x) = \log_b x$, where b is a positive real number other than 1. Explain your reasoning.

Communicate Your Answer

4. What are some of the characteristics of the graph of a logarithmic function?

5. How can you use the graph of an exponential function to obtain the graph of a logarithmic function?

Name _____ Date _____

6.3 Notetaking with Vocabulary

For use after Lesson 6.3

In your own words, write the meaning of each vocabulary term.

logarithm of y with base b function

common logarithm

natural logarithm

Core Concepts

Definition of Logarithm with Base *b*

Let b and y be positive real numbers with $b \neq 1$. The **logarithm of *y* with base *b*** is denoted by $\log_b y$ and is defined as

$$\log_b y = x \quad \text{if and only if} \quad b^x = y.$$

The expression $\log_b y$ is read as "log base b of y."

Notes:

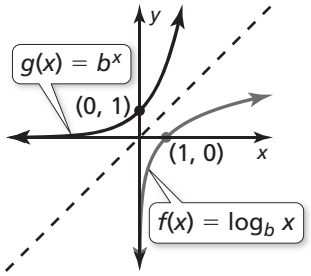
Name_____ Date _____

6.3 Notetaking with Vocabulary (continued)

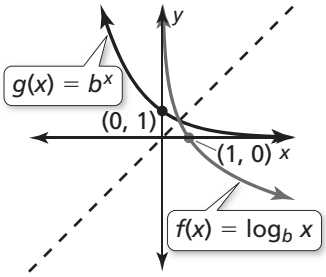
Parent Graphs for Logarithmic Functions

The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$.
Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of $f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$



Graph of $f(x) = \log_b x$ for $0 < b < 1$



Note that the y-axis is a vertical asymptote of the graph of $f(x) = \log_b x$. The domain of $f(x) = \log_b x$ is $x > 0$, and the range is all real numbers.

Notes:

Extra Practice

In Exercises 1–4, rewrite the equation in exponential form.

1. $\log_{10} 1000 = 3$ **2.** $\log_5 \frac{1}{25} = -2$ **3.** $\log_{10} 1 = 0$ **4.** $\log_{1/4} 64 = -3$

Name _____ Date _____

6.3 Notetaking with Vocabulary (continued)

In Exercises 5–8, rewrite the equation in logarithmic form.

5. $12^2 = 144$ **6.** $20^{-1} = \frac{1}{20}$ **7.** $216^{1/3} = 6$ **8.** $4^0 = 1$

In Exercises 9–12, evaluate the logarithm.

9. $\log_4 64$ **10.** $\log_{1/8} 1$ **11.** $\log_2 \frac{1}{32}$ **12.** $\log_{1/25} \frac{1}{5}$

In Exercises 13 and 14, simplify the expression.

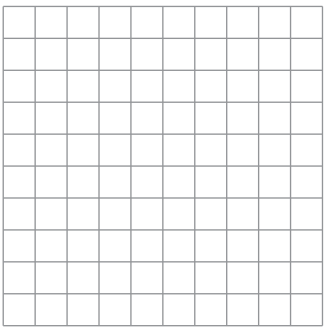
13. $13^{\log_{13} 6}$ **14.** $\ln e^{x^3}$

In Exercises 15 and 16, find the inverse of the function.

15. $y = 15^x + 10$ **16.** $y = \ln(2x) - 8$

In Exercises 17 and 18, graph the function. Determine the asymptote of the function.

17. $y = \log_2(x + 1)$



18. $y = \log_{1/2} x - 4$