6.2 The Natural Base e
For use with Exploration 6.2

Essential Question What is the natural base e?
So far in your study of mathematics, you have worked with special numbers such as \( \pi \) and \( i \). Another special number is called the natural base and is denoted by \( e \). The natural base \( e \) is irrational, so you cannot find its exact value.

1 EXPLORATION: Approximating the Natural Base e

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. One way to approximate the natural base \( e \) is to approximate the sum

\[
1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \ldots.
\]

Use a spreadsheet or a graphing calculator to approximate this sum. Explain the steps you used. How many decimal places did you use in your approximation?

2 EXPLORATION: Approximating the Natural Base e

Work with a partner. Another way to approximate the natural base \( e \) is to consider the expression

\[
\left( 1 + \frac{1}{x} \right)^x.
\]

As \( x \) increases, the value of this expression approaches the value of \( e \). Complete the table. Then use the results in the table to approximate \( e \). Compare this approximation to the one you obtained in Exploration 1.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 10^1 )</th>
<th>( 10^2 )</th>
<th>( 10^3 )</th>
<th>( 10^4 )</th>
<th>( 10^5 )</th>
<th>( 10^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \left( 1 + \frac{1}{x} \right)^x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3 EXPLORATION: Graphing a Natural Base Function

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. Use your approximate value of \( e \) in Exploration 1 or 2 to complete the table. Then sketch the graph of the natural base exponential function \( y = e^x \). You can use a graphing calculator and the \( e^x \) key to check your graph.

What are the domain and range of \( y = e^x \)? Justify your answers.

\[
\begin{array}{c|ccccc}
 x & -2 & -1 & 0 & 1 & 2 \\
 \hline
 y = e^x & & & & & \\
\end{array}
\]

Communicate Your Answer

4. What is the natural base \( e \)?

5. Repeat Exploration 3 for the natural base exponential function \( y = e^{-x} \). Then compare the graph of \( y = e^x \) to the graph of \( y = e^{-x} \).

\[
\begin{array}{c|ccccc}
 x & -2 & -1 & 0 & 1 & 2 \\
 \hline
 y = e^{-x} & & & & & \\
\end{array}
\]

6. The natural base \( e \) is used in a wide variety of real-life applications. Use the Internet or some other reference to research some of the real-life applications of \( e \).
Core Concepts

The Natural Base e

The natural base e is irrational. It is defined as follows:

As \( x \) approaches \(+\infty\), \( \left(1 + \frac{1}{x}\right)^x \) approaches \( e \approx 2.71828182846 \).

Notes:

Natural Base Functions

A function of the form \( y = ae^{rx} \) is called a natural base exponential function.

- When \( a > 0 \) and \( r > 0 \), the function is an exponential growth function.
- When \( a > 0 \) and \( r < 0 \), the function is an exponential decay function.

The graphs of the basic functions \( y = e^x \) and \( y = e^{-x} \) are shown.

Notes:
6.2 Notetaking with Vocabulary (continued)

Continuously Compounded Interest

When interest is compounded *continuously*, the amount \( A \) in an account after \( t \) years is given by the formula

\[
A = Pe^{rt}
\]

where \( P \) is the principal and \( r \) is the annual interest rate expressed as a decimal.

Notes:

Extra Practice

In Exercises 1–4, simplify the expression.

1. \( e^{-9} \cdot e^{12} \)
2. \( \frac{25e^2}{35e^7} \)
3. \( (2e^{-3x})^5 \cdot 2e^{x+1} \)
4. \( \sqrt[4]{16e^{24x}} \)

In Exercises 5–8, tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function.

5. \( y = 2e^{-x} \)
6. \( y = 0.75e^{4x} \)
7. \( y = 5e^{0.25x} \)
8. \( y = 0.8e^{-3x} \)
In Exercises 9–11, use a table of values or a graphing calculator to graph the function. Then identify the domain and range.

9. \( y = e^x - 4 \)  
10. \( y = 2e^{x+3} \)  
11. \( y = -e^x + 5 \)

12. The population of Evans City is currently 48,500 and is declining at a rate of 2.5% each year. You can model the population of Evans City by the equation \( P_t = P_c e^{rt} \), where \( P_c \) is the current population, \( P_t \) is the population after \( t \) years, and \( r \) is the decimal rate of decline per year. Predict the population of Evans City after 25 years.

13. Your parents will need $25,000 in 10 years to pay for your brother’s college tuition. They can invest in an account with an interest rate of 9.8% that compounds continuously. How much should your parents invest today in order to have your brother’s full tuition available in 10 years?