

5.5**Performing Function Operations**

For use with Exploration 5.5

Essential Question How can you use the graphs of two functions to sketch the graph of an arithmetic combination of the two functions?

Just as two real numbers can be combined by the operations of addition, subtraction, multiplication, and division to form other real numbers, two functions can be combined to form other functions. For example, the functions $f(x) = 2x - 3$ and $g(x) = x^2 - 1$ can be combined to form the sum, difference, product, or quotient of f and g .

$$f(x) + g(x) = (2x - 3) + (x^2 - 1) = x^2 + 2x - 4 \quad \text{sum}$$

$$f(x) - g(x) = (2x - 3) - (x^2 - 1) = -x^2 + 2x - 2 \quad \text{difference}$$

$$f(x) \cdot g(x) = (2x - 3)(x^2 - 1) = 2x^3 - 3x^2 - 2x + 3 \quad \text{product}$$

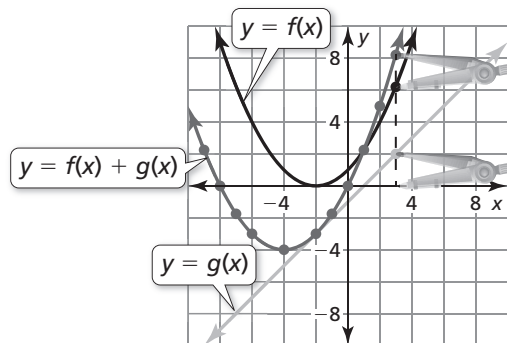
$$\frac{f(x)}{g(x)} = \frac{2x - 3}{x^2 - 1}, x \neq \pm 1 \quad \text{quotient}$$

1 EXPLORATION: Graphing the Sum of Two Functions

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

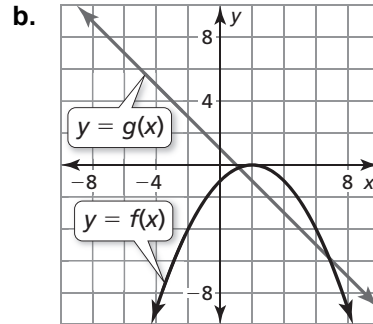
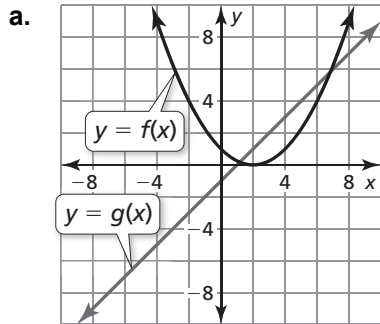
Work with a partner. Use the graphs of f and g to sketch the graph of $f + g$. Explain your steps.

Sample Use a compass or a ruler to measure the distance from a point on the graph of g to the x -axis. Then add this distance to the point with the same x -coordinate on the graph of f . Plot the new point. Repeat this process for several points. Finally, draw a smooth curve through the new points to obtain the graph of $f + g$.



5.5 Performing Function Operations (continued)

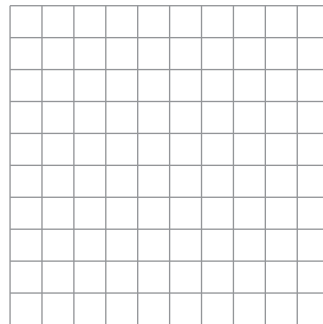
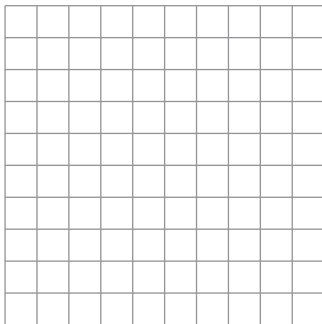
1 **EXPLORATION:** Graphing the Sum of Two Functions (continued)



Communicate Your Answer

2. How can you use the graphs of two functions to sketch the graph of an arithmetic combination of the two functions?

3. Check your answers in Exploration 1 by writing equations for f and g , adding the functions, and graphing the sum.



5.5**Notetaking with Vocabulary**

For use after Lesson 5.5

In your own words, write the meaning of each vocabulary term.

domain

scientific notation

Core Concepts**Operations on Functions**

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g .

Operation	Definition	Example: $f(x) = 5x$, $g(x) = x + 2$
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$	$(fg)(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{5x}{x + 2}$

The domains of the sum, difference, product, and quotient functions consist of the x -values that are in the domains of both f and g . Additionally, the domain of the quotient does not include x -values for which $g(x) = 0$.

Notes:

5.5 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–4, find $(f + g)(x)$ and $(f - g)(x)$ and state the domain of each. Then evaluate $f + g$ and $f - g$ for the given value of x .

1. $f(x) = -\frac{1}{2}\sqrt[3]{x}$, $g(x) = \frac{9}{2}\sqrt[3]{x}$; $x = -1000$

2. $f(x) = -x^2 - 3x + 8$, $g(x) = 6x - 3x^2$; $x = -1$

3. $f(x) = 4x^3 + 12$, $g(x) = 2x^2 - 3x^3 + 9$; $x = 2$

4. $f(x) = 5\sqrt[4]{x} + 1$, $g(x) = -3\sqrt[4]{x} - 2$; $x = 1$

In Exercises 5–8, find $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$ and state the domain of each. Then evaluate fg and $\frac{f}{g}$ for the given value of x .

5. $f(x) = -x^3$, $g(x) = 2\sqrt[3]{x}$; $x = -64$

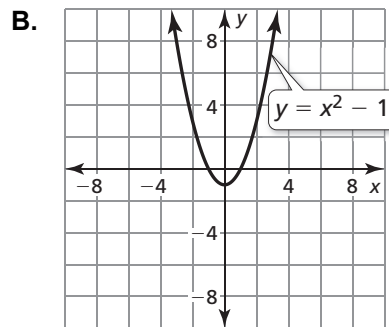
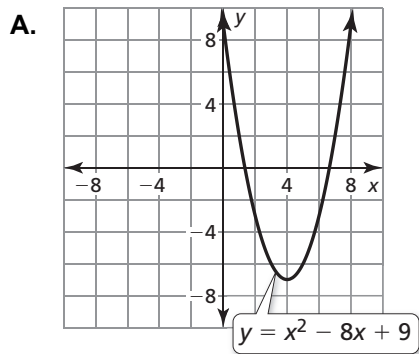
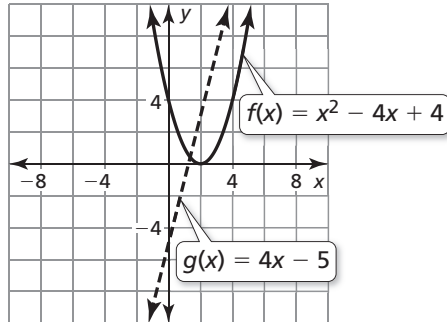
6. $f(x) = 12x$, $g(x) = 11x^{1/2}$; $x = 4$

7. $f(x) = 0.25x^{1/3}$, $g(x) = -4x^{3/2}$; $x = 1$

8. $f(x) = 36x^{7/4}$, $g(x) = 4x^{1/2}$; $x = 16$

5.5 Notetaking with Vocabulary (continued)

9. The graphs of the functions $f(x) = x^2 - 4x + 4$ and $g(x) = 4x - 5$ are shown. Which graph represents the function $f + g$? the function $f - g$? Explain your reasoning.



10. The variable x represents the number of pages of a textbook to be printed. The cost C to print the textbook can be modeled by the equation $C(x) = 0.2x^2 + 10$. The selling price P of the textbook can be modeled by the equation $P(x) = 0.05x^2 + 20$.

- a. Find $(P - C)(x)$.

- b. Explain what $(P - C)(x)$ represents.