

5.4

Solving Radical Equations and Inequalities

For use with Exploration 5.4

Essential Question How can you solve a radical equation?

1 EXPLORATION: Solving Radical Equations

Work with a partner. Match each radical equation with the graph of its related radical function. Explain your reasoning. Then use the graph to solve the equation, if possible. Check your solutions.

a. $\sqrt{x-1} - 1 = 0$

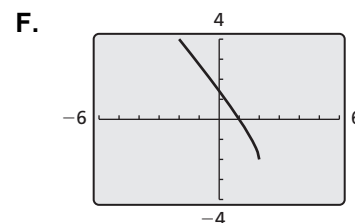
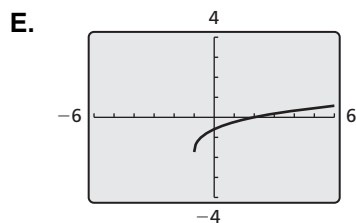
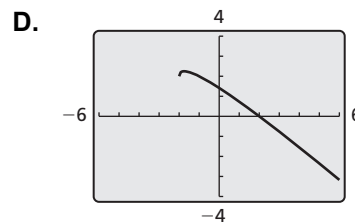
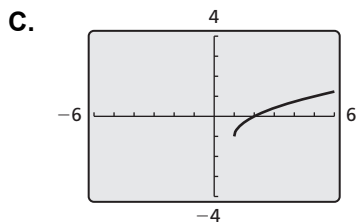
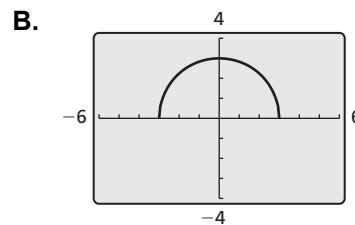
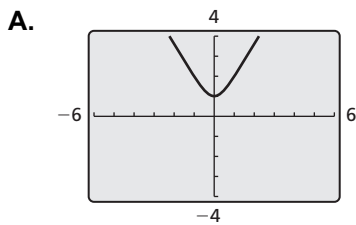
b. $\sqrt{2x+2} - \sqrt{x+4} = 0$

c. $\sqrt{9-x^2} = 0$

d. $\sqrt{x+2} - x = 0$

e. $\sqrt{-x+2} - x = 0$

f. $\sqrt{3x^2+1} = 0$



5.4 Solving Radical Equations and Inequalities (continued)**2 EXPLORATION: Solving Radical Equations**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Look back at the radical equations in Exploration 1. Suppose that you did not know how to solve the equations using a graphical approach.

- a. Show how you could use a *numerical approach* to solve one of the equations. For instance, you might use a spreadsheet to create a table of values.

- b. Show how you could use an *analytical approach* to solve one of the equations. For instance, look at the similarities between the equations in Exploration 1. What first step may be necessary so you could square each side to eliminate the radical(s)? How would you proceed to find the solution?

Communicate Your Answer

3. How can you solve a radical equation?

4. Would you prefer to use a graphical, numerical, or analytical approach to solve the given equation? Explain your reasoning. Then solve the equation.

$$\sqrt{x + 3} - \sqrt{x - 2} = 1$$

5.4

Notetaking with Vocabulary

For use after Lesson 5.4

In your own words, write the meaning of each vocabulary term.

radical equation

extraneous solutions

Core Concepts

Solving Radical Equations

To solve a radical equation, follow these steps:

Step 1 Isolate the radical on one side of the equation, if necessary.

Step 2 Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3 Solve the resulting equation using techniques you learned in previous chapters.
Check your solution.

Notes:

5.4 Notetaking with Vocabulary (continued)**Extra Practice**

In Exercises 1–10, solve the equation. Check your solution(s).

1. $\sqrt{1-x} = 7$

2. $\sqrt[3]{5x+1} = -4$

3. $\frac{1}{4}\sqrt[4]{2x} + 6 = 10$

4. $2\sqrt[3]{13x-5} = 10$

5. $x - 7 = \sqrt{x-5}$

6. $\sqrt[3]{486 - 27x^3} = 3x$

7. $4\sqrt{x+1} = x+1$

8. $\sqrt{2x+2} - 3\sqrt{x+1} = 0$

9. $2 - \sqrt[4]{2x-6} = 14$

10. $\sqrt{x+7} + 2 = \sqrt{3-x}$

5.4 Notetaking with Vocabulary (continued)

In Exercises 11 and 12, solve the equation. Check your solution(s).

11. $\frac{1}{2}x^{5/2} = 16$

12. $(6x + 10)^{7/3} + 28 = 156$

In Exercises 13–15, solve the inequality.

13. $-4\sqrt{x-1} + 3 \geq -1$

14. $\sqrt[3]{\frac{2}{3}x + 1} < 6$

15. $2\sqrt{\frac{3}{4}x} - 39 \leq -25$

16. In basketball, the term “hang time” is the amount of time that a player is suspended in the air when making a basket. To win a slam-dunk contest, players try to maximize their hang time. A player’s hang time is given by the equation $t = 0.5\sqrt{h}$, where t is the time (in seconds) and h is the height (in feet) of the jump. The second-place finisher of a slam-dunk contest had a hang time of 1 second, and the winner had a hang time of 1.2 seconds. How many feet higher did the winner jump than the second-place finisher?