Name____ Date__________

4.6 The Fundamental Theorem of Algebra

For use with Exploration 4.6

Essential Question How can you determine whether a polynomial equation has imaginary solutions?

1 EXPLORATION: Cubic Equations and Imaginary Solutions

Work with a partner. Match each cubic polynomial equation with the graph of its related polynomial function. Then find *all* solutions. Make a conjecture about how you can use a graph or table of values to determine the number and types of solutions of a cubic polynomial equation.

a. $x^3 - 3x^2 + x + 5 = 0$

b. $x^3 - 2x^2 - x + 2 = 0$

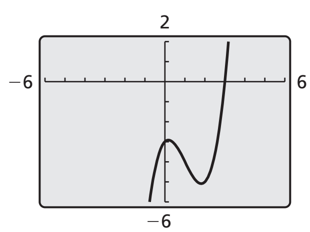
c. $x^3 - x^2 - 4x + 4 = 0$

d. $x^3 + 5x^2 + 8x + 6 = 0$

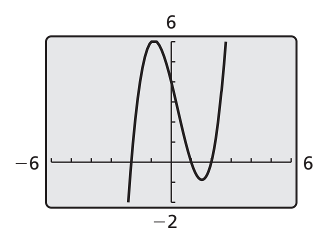
e. $x^3 - 3x^2 + x - 3 = 0$

f. $x^3 - 3x^2 + 2x = 0$

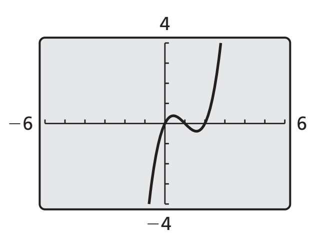
A.



B.



C.



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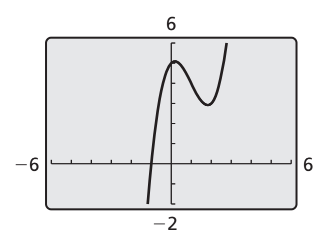
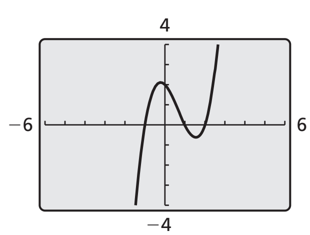
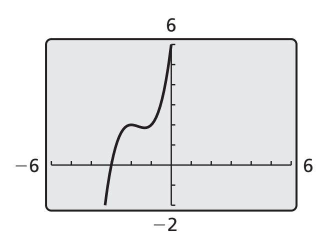


E.



F.



2 EXPLORATION: Quartic Equations and Imaginary Solutions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use the graph of the related quartic function, or a table of values, to determine whether each quartic equation has imaginary solutions. Explain your reasoning. Then find *all* solutions.

a. $x^4 - 2x^3 - x^2 + 2x = 0$

b. $x^4 - 1 = 0$

c. $x^4 + x^3 - x - 1 = 0$

d. $x^4 - 3x^3 + x^2 + 3x - 2 = 0$

Communicate Your Answer

3. How can you determine whether a polynomial equation has imaginary solutions?

4. Is it possible for a cubic equation to have three imaginary solutions? Explain your reasoning.

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4.6 Notetaking with Vocabulary

For use after Lesson 4.6

In your own words, write the meaning of each vocabulary term.

complex conjugates

Core Concepts

The Fundamental Theorem of Algebra

Theorem If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

Notes:

The Complex Conjugates Theorem

If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.

Notes:

Descartes's Rule of Signs

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial function with real coefficients.

- The number of *positive real zeros* of f is equal to the number of changes in sign of the coefficients of $f(x)$ or is less than this by an even number.

- The number of *negative real zeros* of f is equal to the number of changes in sign of the coefficients of $f(-x)$ or is less than this by an even number.

Notes:

Extra Practice

In Exercises 1–4, find all zeros of the polynomial function.

1. $h(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$

2. $f(x) = x^3 - 3x^2 - 15x + 125$

3. $g(x) = x^4 - 48x^2 - 49$

4. $h(x) = -5x^3 + 9x^2 - 18x - 4$

In Exercises 5–8, write a polynomial function *f* of least degree that has rational coefficients, a leading coefficient of 1, and the given zeros.

5. $-4, 1, 7$

6. $10, -\sqrt{5}$

7. $8, 3 - i$

8. $0, 2 - \sqrt{2}, 2 + 3i$