Name ____ Date __________

4.5 Solving Polynomial Equations
For use with Exploration 4.5

Essential Question How can you determine whether a polynomial equation has a repeated solution?

1 EXPLORATION: Cubic Equations and Repeated Solutions

Work with a partner. Some cubic equations have three distinct solutions. Others have repeated solutions. Match each cubic polynomial equation with the graph of its related polynomial function. Then solve each equation. For those equations that have repeated solutions, describe the behavior of the related function near the repeated zero using the graph or a table of values.

a. $x^3 - 6x^2 + 12x - 8 = 0$

b. $x^3 + 3x^2 + 3x + 1 = 0$

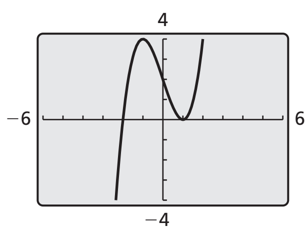
c. $x^3 - 3x + 2 = 0$

d. $x^3 + x^2 - 2x = 0$

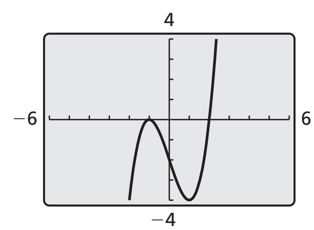
e. $x^3 - 3x - 2 = 0$

f. $x^3 - 3x^2 + 2x = 0$

A.



B.



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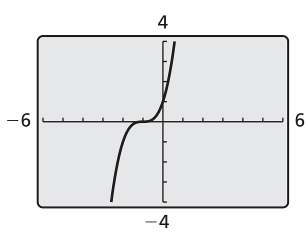

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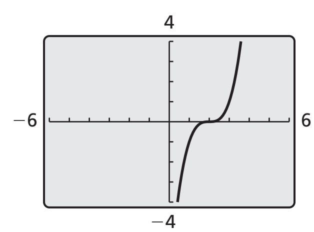
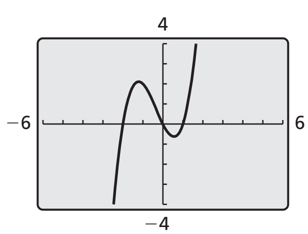
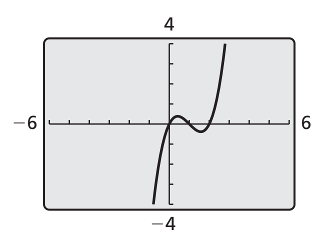


E.



F.



2 EXPLORATION: Quartic Equations and Repeated Solutions

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Determine whether each quartic equation has repeated solutions using the graph of the related quartic function or a table of values. Explain your reasoning. Then solve each equation.

a. $x^4 - 4x^3 + 5x^2 - 2x = 0$

b. $x^4 - 2x^3 - x^2 + 2x = 0$

c. $x^4 - 4x^3 + 4x^2 = 0$

d. $x^4 + 3x^3 = 0$

Communicate Your Answer

3. How can you determine whether a polynomial equation has a repeated solution?

4. Write a cubic or a quartic polynomial equation that is different from the equations in Explorations 1 and 2 and has a repeated solution.

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4.5 Notetaking with Vocabulary

For use after Lesson 4.5

In your own words, write the meaning of each vocabulary term.

repeated solution

Core Concepts

The Rational Root Theorem

If $f(x) = a_n x^n + \cdots + a_1 x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Notes:

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.

Notes:

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Extra Practice

In Exercises 1–6, solve the equation.

1. $36r^3 - r = 0$

2. $20x^3 + 80x^2 = -60x$

3. $3m^2 = 75m^4$

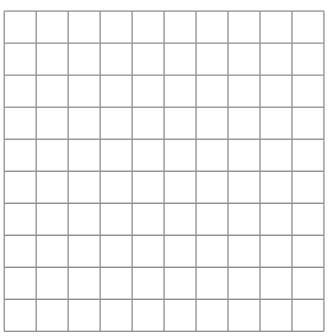
4. $-13y^2 + 36 = -y^4$

5. $2x^3 - x^2 - 2x = -1$

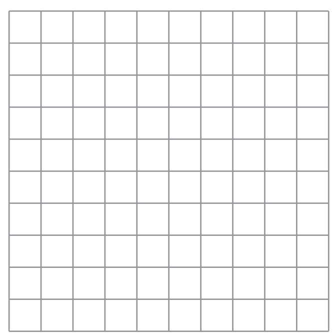
6. $-20c^2 + 50c = 8c^3 - 125$

In Exercises 7–10, find the zeros of the function. Then sketch a graph of the function.

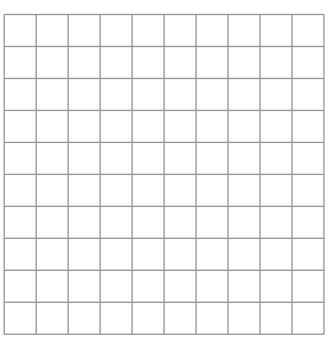
7. $f(x) = x^4 - x^3 - 12x^2$



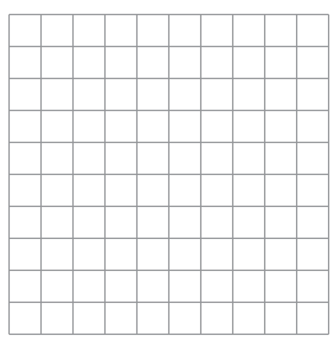
8. $f(x) = -4x^3 + 12x^2 - 9x$



9. $f(x) = x^3 + 4x^2 - 6x - 24$



10. $f(x) = x^4 - 18x^2 + 81$



4.5 Notetaking with Vocabulary (continued)

11. According to the Rational Root Theorem, which is *not* a possible solution of the equation $2x^4 + 3x^3 - 6x + 7 = 0$?

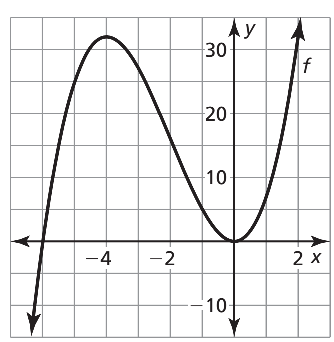
A. 3.5 **B.** 0.5 **C.** 7 **D.** 2

12. Find all the real zeros of the function $f(x) = 3x^4 + 11x^3 - 40x^2 - 132x + 48$.

13. Write a polynomial function g of least degree that has rational coefficients, a leading coefficient of 1, and the zeros -5 and $4 + \sqrt{2}$.

14. Use the information in the graph to answer the questions.

a. What are the real zeros of the function f?



b. Write an equation of the cubic function in factored form.