Factoring Polynomials

For use with Exploration 4.4

Essential Question How can you factor a polynomial?

EXPLORATION: Factoring Polynomials

Work with a partner. Match each polynomial equation with the graph of its related polynomial function. Use the x-intercepts of the graph to write each polynomial in factored form. Explain your reasoning.

a.
$$x^2 + 5x + 4 = 0$$

b.
$$x^3 - 2x^2 - x + 2 = 0$$

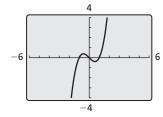
c.
$$x^3 + x^2 - 2x = 0$$

d.
$$x^3 - x = 0$$

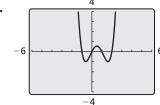
e.
$$x^4 - 5x^2 + 4 = 0$$

f.
$$x^4 - 2x^3 - x^2 + 2x = 0$$

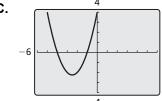
A.



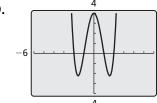
В.



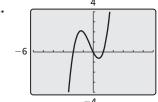
C.



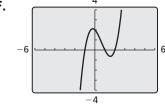
D.



E.



F.



4.4 Factoring Polynomials (continued)

EXPLORATION: Factoring Polynomials

Work with a partner. Use the *x*-intercepts of the graph of the polynomial function to write each polynomial in factored form. Explain your reasoning. Check your answers by multiplying.

a.
$$f(x) = x^2 - x - 2$$

b.
$$f(x) = x^3 - x^2 - 2x$$

c.
$$f(x) = x^3 - 2x^2 - 3x$$

d.
$$f(x) = x^3 - 3x^2 - x + 3$$

e.
$$f(x) = x^4 + 2x^3 - x^2 - 2x$$

f.
$$f(x) = x^4 - 10x^2 + 9$$

Communicate Your Answer

3. How can you factor a polynomial?

4. What information can you obtain about the graph of a polynomial function written in factored form?

Notetaking with Vocabulary For use after Lesson 4.4

In your own words, write the meaning of each vocabulary term.

factored completely

factor by grouping

quadratic form

Core Concepts

Special Factoring Patterns

Sum of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Difference of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Notes:

Example

$$64x^{3} + 1 = (4x)^{3} + 1^{3}$$
$$= (4x + 1)(16x^{2} - 4x + 1)$$

Example

$$27x^{3} - 8 = (3x)^{3} - 2^{3}$$
$$= (3x - 2)(9x^{2} + 6x + 4)$$

4.4 Notetaking with Vocabulary (continued)

The Factor Theorem

A polynomial f(x) has a factor x - k if and only if f(k) = 0.

Notes:

Extra Practice

In Exercises 1–14, factor the polynomial completely.

1.
$$20x^3 - 220x^2 + 600x$$

2.
$$m^5 - 81m$$

3.
$$27a^3 + 8b^3$$

4.
$$5t^6 + 2t^5 - 5t^4 - 2t^3$$

5.
$$y^4 - 13y^2 - 48$$

6.
$$5p^3 + 5p - 5p^2 - 5$$

7.
$$810k^4 - 160$$

8.
$$a^5 + a^3 - a^2 - 1$$

4.4 Notetaking with Vocabulary (continued)

9.
$$2x^6 - 8x^5 - 42x^4$$

10.
$$5z^3 + 5z^2 - 6z - 6$$

11.
$$12x^2 - 22x - 20$$

12.
$$3m^2 - 48m^6$$

13.
$$4x^3 - 4x^2 + x$$

14.
$$5m^4 - 70m^3 + 245m^2$$

In Exercises 15–17, show that the binomial is a factor of the polynomial. Then factor the function completely.

15.
$$f(x) = x^3 - 13x - 12; x + 1$$

16.
$$f(x) = 6x^3 + 8x^2 - 34x - 12; x - 2$$

17.
$$f(x) = 2x^4 - 12x^3 + 6x^2 + 20x$$
; $x - 5$