Name ____ Date __________

Chapter 3 Maintaining Mathematical Proficiency

Simplify the expression.

1. $\sqrt{50}$

2. $-\sqrt{96}$

3. $\sqrt{\frac{3}{121}}$

4. $\sqrt{200}$

5. $\sqrt{\frac{75}{81}}$

6. $-\sqrt{\frac{14}{144}}$

7. $-\sqrt{54}$

8. $\sqrt{250}$

Factor the polynomial.

9. $x^2 - 100$

10. $4x^2 - 49$

11. $16x^2 - 9$

12. $x^2 - 30x + 225$

13. $x^2 + 16x + 64$

14. $25x^2 + 10x + 1$

15. Explain why the expression $81 - x^4$ *cannot* be factored into $(3 + x)^2(3 - x)^2$.

Name____ Date___________

3.1 Solving Quadratic Equations

For use with Exploration 3.1

Essential Question How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?

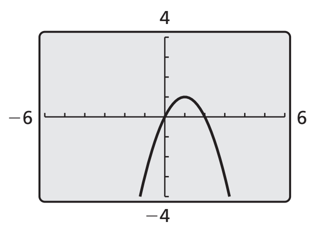
1 EXPLORATION: Matching a Quadratic Function with Its Graph

Work with a partner. Match each quadratic function with its graph. Explain your reasoning. Determine the number of x-intercepts of the graph.

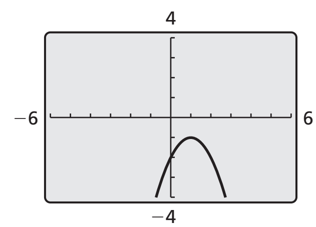
a. $f(x) = x^2 - 2x$ **b.** $f(x) = x^2 - 2x + 1$ **c.** $f(x) = x^2 - 2x + 2$

d. $f(x) = -x^2 + 2x$ **e.** $f(x) = -x^2 + 2x - 1$ **f.** $f(x) = -x^2 + 2x - 2$

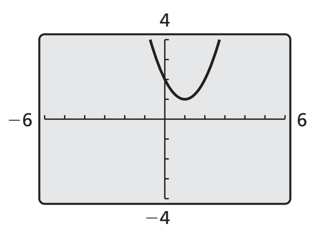
A.



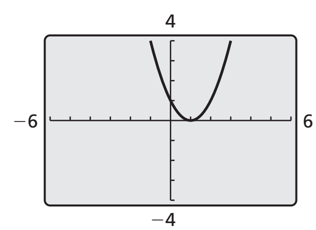
B.



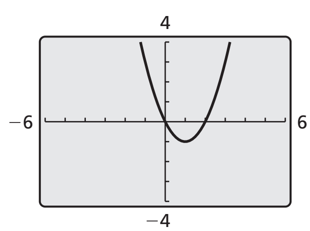
C.



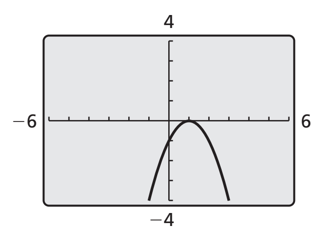
D.



E.



F.



2 EXPLORATION: Solving Quadratic Equations

Work with a partner. Use the results of Exploration 1 to find the real solutions (if any) of each quadratic equation.

a. $x^2 - 2x = 0$ **b.** $x^2 - 2x + 1 = 0$ **c.** $x^2 - 2x + 2 = 0$

d. $-x^2 + 2x = 0$ **e.** $-x^2 + 2x - 1 = 0$ **f.** $-x^2 + 2x - 2 = 0$

Communicate Your Answer

3. How can you use the graph of a quadratic equation to determine the number of real solutions of the equation?

4. How many real solutions does the quadratic equation $x^2 + 3x + 2 = 0$ have? How do you know? What are the solutions?

Name____ Date__________

3.1 Notetaking with Vocabulary
For use after Lesson 3.1

In your own words, write the meaning of each vocabulary term.

quadratic equation in one variable

root of an equation

zero of a function

Core Concepts

Solving Quadratic Equations

By graphing Find the x-intercepts of the related function $y = ax^2 + bx + c$.

Using square roots Write the equation in the form $u^2 = d$, where u is an algebraic expression, and solve by taking the square root of each side.

By factoring Write the polynomial equation $ax^2 + bx + c = 0$ in factored form and solve using the Zero-Product Property.

Notes:

Name ____ Date _________

3.1 Notetaking with Vocabulary (continued)

Zero-Product Property

Words If the product of two expressions is zero, then one or both of the expressions equal zero.

Algebra If A and B are expressions and $AB = 0$, then $A = 0$ or $B = 0$.

Notes:

Extra Practice

In Exercises 1–3, solve the equation by graphing.

1. $x^2 - 11x + 24 = 0$

2. $13 = -x^2 - 12$

3. $12x^2 = 5x + 2$

In Exercises 4–6, solve the equation using square roots.

4. $t^2 = 400$

5. $(2k + 3)^2 - 19 = 81$

6. $\frac{1}{7}p^2 = \frac{5}{7}p^2 - 20$

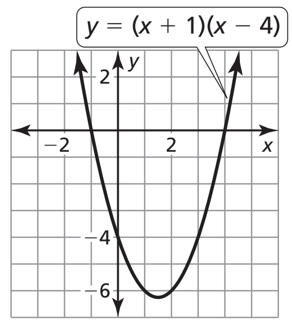
Name____ Date__________

3.1 Notetaking with Vocabulary (continued)

In Exercises 7–9, solve the equation by factoring.

7. $0 = x^2 - 12x + 36$ **8.** $x^2 = 14x - 40$ **9.** $5x^2 + 5x - 1 = -x^2 + 4x$

10. Which equations have roots that are equivalent to the x-intercepts of the graph shown?



A. $-2x^2 - 10x - 8 = 0$

B. $x^2 - 3x = 4$

C. $(x - 1)(x + 4) = 0$

D. $(x - 1)^2 + 4 = 0$

E. $6x^2 = 18x + 24$

11. A skydiver drops out of an airplane that is flying at an altitude of 4624 feet.

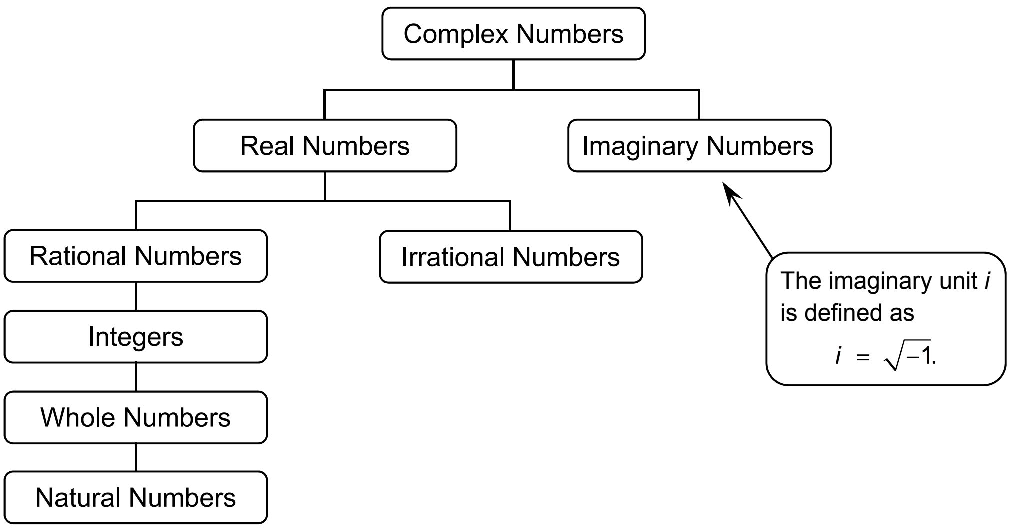
a. Use the formula $h = -16t^2 + h_0$ to write an equation that gives the skydiver's height h (in feet) during free fall t seconds after the skydiver drops out of the airplane.

b. It is possible for the skydiver to wait 18 seconds before pulling the parachute cord? Explain.

Name ____ Date __________

3.2 Complex Numbers
For use with Exploration 3.2

Essential Question What are the subsets of the set of complex numbers?



1 EXPLORATION: Classifying Numbers

Work with a partner. Determine which subsets of the set of complex numbers contain each number.

a. $\sqrt{9}$ **b.** $\sqrt{0}$ **c.** $-\sqrt{4}$

d. $\sqrt{\frac{4}{9}}$ **e.** $\sqrt{2}$ **f.** $\sqrt{-1}$

2 EXPLORATION: Complex Solutions of Quadratic Equations

Work with a partner. Use the definition of the imaginary unit i to match each quadratic equation with its complex solution. Justify your answers.

a. $x^2 - 4 = 0$ **b.** $x^2 + 1 = 0$ **c.** $x^2 - 1 = 0$

d. $x^2 + 4 = 0$ **e.** $x^2 - 9 = 0$ **f.** $x^2 + 9 = 0$

A. i **B.** $3i$ **C.** 3

D. $2i$ **E.** 1 **F.** 2

Communicate Your Answer

3. What are the subsets of the set of complex numbers? Give an example of a number in each subset.

4. Is it possible for a number to be both whole and natural? natural and rational? rational and irrational? real and imaginary? Explain your reasoning.

Name ____ Date __________

3.2 Notetaking with Vocabulary

For use after Lesson 3.2

In your own words, write the meaning of each vocabulary term.

imaginary unit i

complex number

imaginary number

pure imaginary number

Core Concepts

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.	$\sqrt{-3} = i\sqrt{3}$
2. By the first property, it follows that $\left(i\sqrt{r}\right)^2 = -r$.	$\left(i\sqrt{3}\right)^2 = i^2 \bullet 3 = -3$

Notes:

3.2 Notetaking with Vocabulary (continued)

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

Notes:

Extra Practice

In Exercises 1–6, find the square root of the number.

1. $\sqrt{-49}$

2. $\sqrt{-4}$

3. $\sqrt{-45}$

4. $-2\sqrt{-100}$

5. $6\sqrt{-121}$

6. $5\sqrt{-75}$

In Exercises 7 and 8, find the values of *x* and *y* that satisfy the equation.

7. $-10x + i = 30 - yi$

8. $44 - \frac{1}{2}yi = -\frac{1}{4}x - 7i$

In Exercises 9–14, simplify the expression. Then classify the result as a *real number* or *imaginary number*. If the result is an *imaginary number*, specify if it is a *pure imaginary number*.

9. $(-8 + 3i) + (-1 - 2i)$

10. $(36 - 3i) - (12 + 24i)$

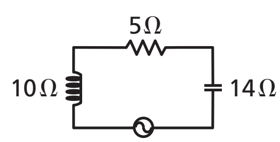
11. $(16 + i) + (-16 - 8i)$

12. $(-5 - 5i) - (-6 - 6i)$

13. $(-1 + 9i)(15 - i)$

14. $(13 + i)(13 - i)$

15. Find the impedance of the series circuit.



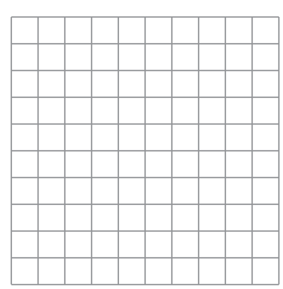
In Exercises 16–18, solve the equation. Check your solution(s).

16. $0 = 5x^2 + 25$

17. $x^2 - 10 = -18$

18. $-\frac{1}{3}x^2 = \frac{1}{5} + \frac{4}{3}x^2$

19. Sketch a graph of a function that has two real zeros at -2 and 2. Then sketch a graph on the same grid of a function that has two imaginary zeros of $-2i$ and $2i$. Explain the difference in the graphs of the two functions.



Name____ Date__________

3.3 Completing the Square
For use with Exploration 3.3

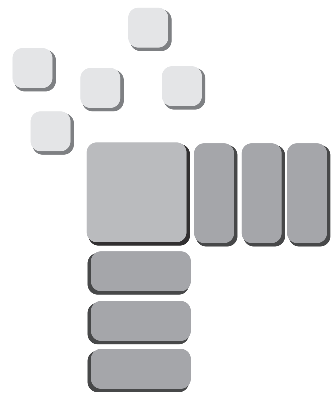
Essential Question How can you complete the square for a quadratic expression?

1 **EXPLORATION:** Using Algebra Tiles to Complete the Square

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use algebra tiles to complete the square for the expression $x^2 + 6x$.

a. You can model $x^2 + 6x$ using one x^2-tile and six x-tiles. Arrange the tiles in a square. Your arrangement will be incomplete in one of the corners.



b. How many 1-tiles do you need to complete the square?

c. Find the value of c so that the expression

$x^2 + 6x + c$

is a perfect square trinomial.

d. Write the expression in part (c) as the square of a binomial.

2 EXPLORATION: Drawing Conclusions

Work with a partner.

a. Use the method outlined in Exploration 1 to complete the table.

Expression	Value of *c* needed to complete the square	Expression written as a binomial squared
$x^2 + 2x + c$		
$x^2 + 4x + c$		
$x^2 + 8x + c$		
$x^2 + 10x + c$		

b. Look for patterns in the last column of the table. Consider the general statement $x^2 + bx + c = (x + d)^2$. How are d and b related in each case? How are c and d related in each case?

c. How can you obtain the values in the second column directly from the coefficients of x in the first column?

Communicate Your Answer

3. How can you complete the square for a quadratic expression?

4. Describe how you can solve the quadratic equation $x^2 + 6x = 1$ by completing the square.

Name____ Date__________

3.3 Notetaking with Vocabulary

For use after Lesson 3.3

In your own words, write the meaning of each vocabulary term.

completing the square

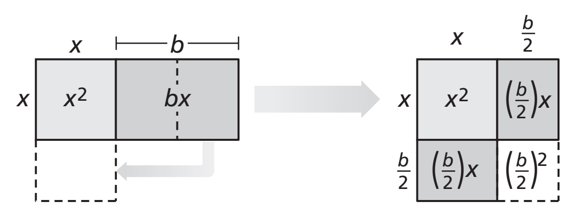
Core Concepts

Completing the Square

Words To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.

Diagrams In each diagram, the combined area of the shaded regions is $x^2 + bx$.

Adding $\left(\frac{b}{2}\right)^2$ completes the square in the second diagram.



Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right) = \left(x + \frac{b}{2}\right)^2$

Notes:

Extra Practice

In Exercises 1–3, solve the equation using square roots. Check your solution(s).

1. $x^2 + 4x + 4 = 2$

2. $t^2 - 40t + 400 = 300$

3. $9w^2 + 6w + 1 = -18$

In Exercises 4–6, find the value of *c* that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

4. $y^2 - 14y + c$

5. $s^2 + 17s + c$

6. $z^2 + 24z + c$

In Exercises 7–12, solve the equation by completing the square.

7. $r^2 - 6r - 2 = 0$

8. $x^2 + 10x + 28 = 0$

9. $y(y + 1) = \frac{3}{4}$

10. $2t^2 + 16t - 6 = 0$

11. $3x(2x + 10) = -24$

12. $4x^2 - 5x + 28 = 3x^2 + x$

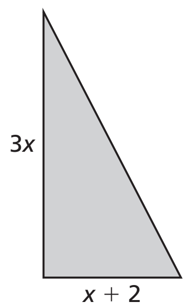
13. Explain how the expression $(4p + 1)^2 + 8(4p + 1) + 16$ is a perfect square trinomial. Then write the expression as a square of a binomial.

3.3 Notetaking with Vocabulary (continued)

In Exercises 14–17, determine whether you would use factoring, square roots, or completing the square to solve the equation. Explain your reasoning. Then solve the equation.

14. $x^2 + 7x = 0$ **15.** $(x - 1)^2 = 35$ **16.** $x^2 - 225 = 0$ **17.** $4x^2 + 8x + 12 = 0$

18. The area of the triangle is 30. Find the value of x.



19. Write the quadratic function $f(x) = x^2 + 6x + 22$ in vertex form. Then identify the vertex.

20. A golfer hits a golf ball on the fairway with an initial velocity of 80 feet per second. The height h (in feet) of the golf ball t seconds after it is hit can be modeled by the function $h(t) = -16t^2 + 80t + 0.1$.

a. Find the maximum height of the golf ball.

b. How long does the ball take to hit the ground?

Name ____ Date __________

3.4 Using the Quadratic Formula

For use with Exploration 3.4

Essential Question How can you derive a general formula for solving a quadratic equation?

1 EXPLORATION: Deriving the Quadratic Formula

Work with a partner. Analyze and describe what is done in each step in the development of the Quadratic Formula.

Step	Justification
$ax^2 + bx + c = 0$	______________________
$ax^2 + bx = -c$	______________________
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	______________________
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	______________________
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$	______________________
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	______________________
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	______________________
$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2\lvert a\rvert}$	______________________
The result is the Quadratic Formula. → $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	______________________

2 EXPLORATION: Using the Quadratic Formula

Work with a partner. Use the Quadratic Formula to solve each equation.

a. $x^2 - 4x + 3 = 0$

b. $x^2 - 2x + 2 = 0$

c. $x^2 + 2x - 3 = 0$

d. $x^2 + 4x + 4 = 0$

e. $x^2 - 6x + 10 = 0$

f. $x^2 + 4x + 6 = 0$

Communicate Your Answer

3. How can you derive a general formula for solving a quadratic equation?

4. Summarize the following methods you have learned for solving quadratic equations: graphing, using square roots, factoring, completing the square, and using the Quadratic Formula.

Name ____ Date __________

3.4 Notetaking with Vocabulary
For use after Lesson 3.4

In your own words, write the meaning of each vocabulary term.

Quadratic Formula

discriminant

Core Concepts

The Quadratic Formula

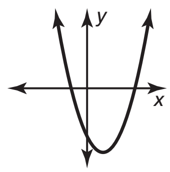
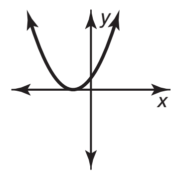
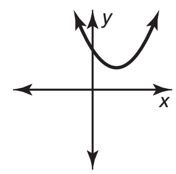
Let a, b, and c be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Notes:

Name____ Date__________

3.4 Notetaking with Vocabulary (continued)

Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercept	 No x-intercept

Notes:

Extra Practice

In Exercises 1–3, solve the equation using the Quadratic Formula. Use a graphing calculator to check your solution(s).

1. $x^2 - 7x - 18 = 0$

2. $w^2 = 4w - 1$

3. $-7z = -4z^2 - 3$

In Exercises 4–6, find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

4. $b^2 + 34b + 289 = 0$

5. $x^2 = 3 - 8x$

6. $4q^2 + 1 = 3q$

7. A baseball player hits a foul ball straight up in the air from a height of 4 feet off the ground with an initial velocity of 85 feet per second.

a. Write a quadratic function that represents the height h of the ball t seconds after it hits the bat.

b. When is the ball 110 feet off the ground? Explain your reasoning.

c. The catcher catches the ball 6 feet from the ground. How long is the ball in the air?

Name____________________ Date__________

3.5 Solving Nonlinear Systems

For use with Exploration 3.5

Essential Question How can you solve a nonlinear system of equations?

1 EXPLORATION: Solving Nonlinear Systems of Equations

Work with a partner. Match each system with its graph. Explain your reasoning. Then solve each system using the graph.

a. $y = x^2$
$y = x + 2$

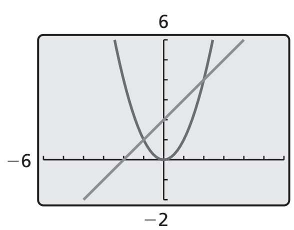
b. $y = x^2 + x - 2$
$y = x + 2$

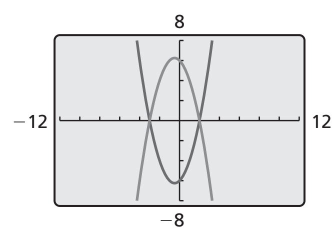
c. $y = x^2 - 2x - 5$
$y = -x + 1$

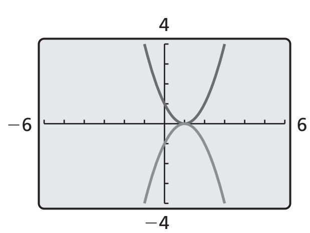
d. $y = x^2 + x - 6$
$y = -x^2 - x + 6$

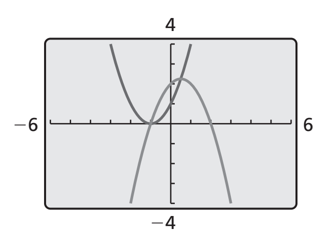
e. $y = x^2 - 2x + 1$
$y = -x^2 + 2x - 1$

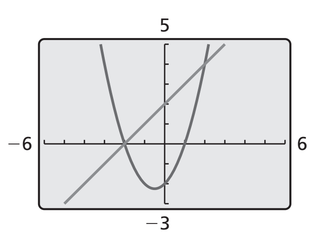
f. $y = x^2 + 2x + 1$
$y = -x^2 + x + 2$

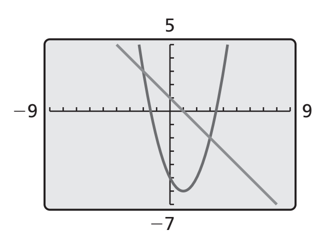
A.


B.


C.


D.


E.


F.


2 EXPLORATION: Solving Nonlinear Systems of Equations

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Look back at the nonlinear system in Exploration 1(f). Suppose you want a more accurate way to solve the system than using a graphical approach.

a. Show how you could use a *numerical approach* by creating a table. For instance, you might use a spreadsheet to solve the system.

b. Show how you could use an *analytical approach*. For instance, you might try solving the system by substitution or elimination.

Communicate Your Answer

3. How can you solve a nonlinear system of equations?

4. Would you prefer to use a graphical, numerical, or analytical approach to solve the given nonlinear system of equations? Explain your reasoning.

$$y = x^2 + 2x - 3$$
$$y = -x^2 - 2x + 4$$

Name____ Date__________

3.5 Notetaking with Vocabulary
For use after Lesson 3.5

In your own words, write the meaning of each vocabulary term.

system of nonlinear equations

Core Concepts

Solve Equations by Graphing

Step 1 To solve the equation $f(x) = g(x)$, write a system of two equations, $y = f(x)$ and $y = g(x)$.

Step 2 Graph the system of equations $y = f(x)$ and $y = g(x)$. The x-value of each solution of the system is a solution of the equation $f(x) = g(x)$.

Notes:

Name ______________________________ Date __________

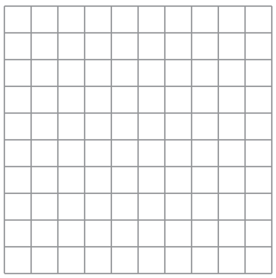
3.5 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–3, solve the system by graphing. Check your solution(s).

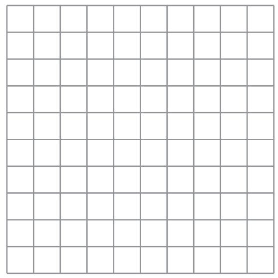
1. $y = \frac{1}{2}x^2 - 3$

$y = -4 - 2x^2$



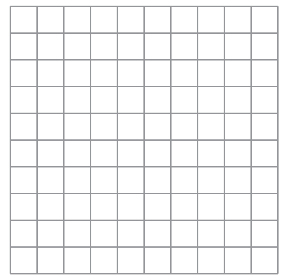
2. $y = (x - 2)^2$

$y = \frac{1}{4}x - \frac{1}{2}$



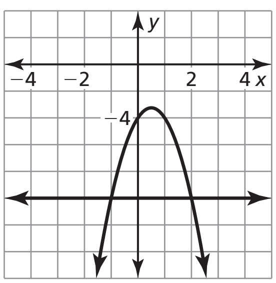
3. $y = -x^2 - 2$

$y = 4(x + 1) - 3$



In Exercises 4 and 5, solve the system of nonlinear equations by using the graph.

4.



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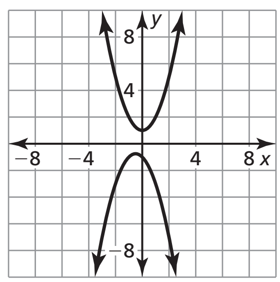


In Exercises 6–8, solve the system by substitution.

6. $y = x + 4$

$y = (x + 2)^2 + 1$

7. $x^2 + y^2 = 16$

$y = -x + 4$

8. $2x^2 + 10x + 48 = y - 10x$

$-4x^2 - 16x = y$

3.5 Notetaking with Vocabulary (continued)

In Exercises 9–11, solve the system by elimination.

9. $x^2 - 7x + 11 = y - 1$
$-x + y = -4$

10. $y = 9x^2 + 6x + 2$
$y = x^2 - 8x - 19$

11. $-5x + 29 = y - x^2$
$x^2 + y = 2x^2 - 1$

12. Consider the following system.

$x^2 = 9 - y^2$
$x + 2y = 2x^2 + 7 + x$

a. Which method would you use to solve the system? Explain your reasoning.

b. Would you have used a different method if the system had been as follows? Explain.

$x = 9 - y$
$x + 2y = 2x^2 + 7 + x$

13. The sum of two numbers is -5, and the sum of the squares of the two numbers is 17. What are the two numbers? Explain your reasoning.

Name ______________________________ Date __________

3.6 Quadratic Inequalities

For use with Exploration 3.6

Essential Question

How can you solve a quadratic inequality?

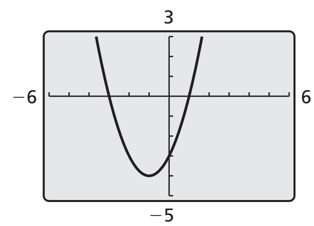
1 EXPLORATION: Solving a Quadratic Inequality

Work with a partner. The graphing calculator screen shows the graph of

$f(x) = x^2 + 2x - 3.$

Explain how you can use the graph to solve the inequality

$x^2 + 2x - 3 \le 0.$



Then solve the inequality.

2 EXPLORATION: Solving Quadratic Inequalities

Work with a partner. Match each inequality with the graph of its related quadratic function on the next page. Then use the graph to solve the inequality.

a. $x^2 - 3x + 2 > 0$

b. $x^2 - 4x + 3 \le 0$

c. $x^2 - 2x - 3 < 0$

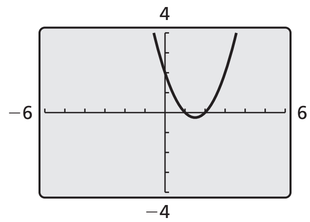
d. $x^2 + x - 2 \ge 0$

e. $x^2 - x - 2 < 0$

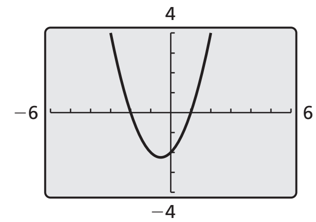
f. $x^2 - 4 > 0$

3.6 Quadratic Inequalities (continued)

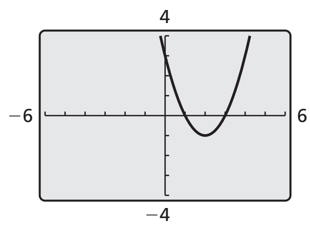
A.



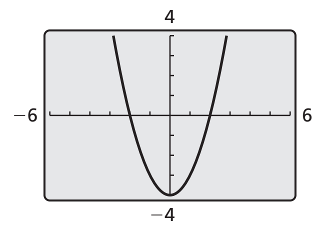
B.



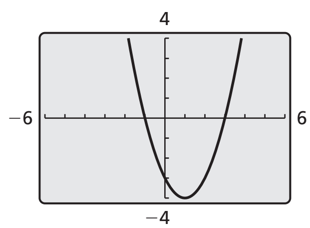
C.



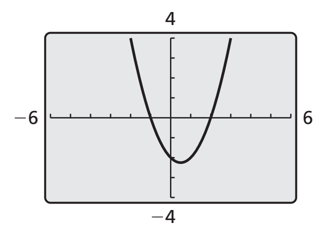
D.



E.



F.



Communicate Your Answer

3. How can you solve a quadratic inequality?

4. Explain how you can use the graph in Exploration 1 to solve each inequality. Then solve each inequality.

a. $x^2 + 2x - 3 > 0$ **b.** $x^2 + 2x - 3 < 0$ **c.** $x^2 + 2x - 3 \geq 0$

Name ____ Date __________

3.6 Notetaking with Vocabulary
For use after Lesson 3.6

In your own words, write the meaning of each vocabulary term.

quadratic inequality in two variables

quadratic inequality in one variable

Core Concepts

Graphing a Quadratic Inequality in Two Variables

To graph a quadratic inequality in one of the following forms,

$y < ax^2 + bx + c$ $\quad$ $y > ax^2 + bx + c$

$y \leq ax^2 + bx + c$ $\quad$ $y \geq ax^2 + bx + c$,

follow these steps.

Step 1 Graph the parabola with the equation $y = ax^2 + bx + c$. Make the parabola *dashed* for inequalities with < or > and *solid* for inequalities with ≤ or ≥.

Step 2 Test a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.

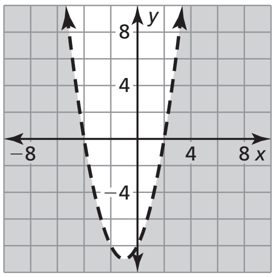
Step 3 Shade the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.

Notes:

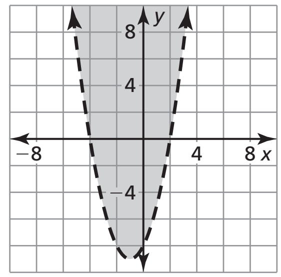
Extra Practice

In Exercises 1–4, match the graph with its inequality. Explain your reasoning.

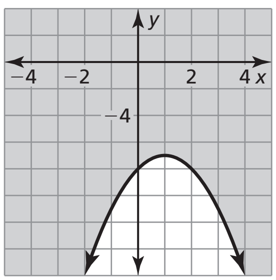
1.



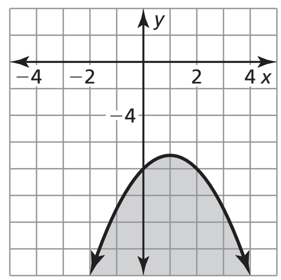
2.



3.



4.



A. $y < x^2 + 2x - 8$

B. $y \leq -x^2 + 2x - 8$

C. $y > x^2 + 2x - 8$

D. $y \geq -x^2 + 2x - 8$

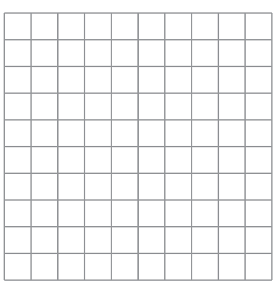
In Exercises 5–8, graph the inequality.

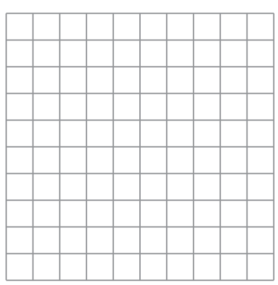
5. $y < x^2 + 2$

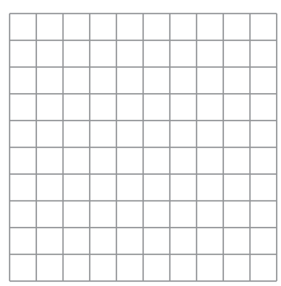
6. $y \leq -5x^2$

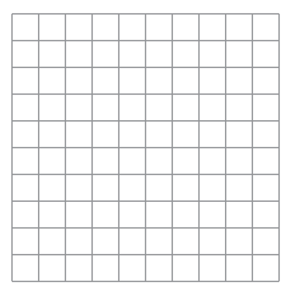
7. $y \geq -(x + 4)^2 - 1$

8. $y < 4x^2 + 4x + 1$









9. Accident investigators use the formula $d = 0.01875v^2$, where d is the braking distance of a car (in feet) and v is the speed of the car (in miles per hour) to determine how fast a car is going at the time of an accident. For what speeds v would a car leave a tire mark on the road of over 1 foot?

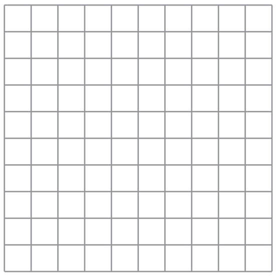
Name ____ Date __________

3.6 Notetaking with Vocabulary (continued)

In Exercises 10–12, graph the system of quadratic inequalities.

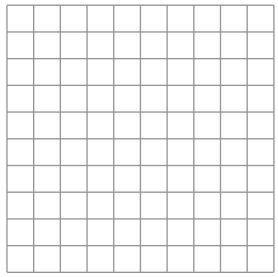
10. $y \le -x^2$

$y > -3x^2 + 3$



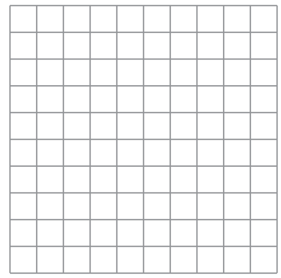
11. $y \ge x^2 + 5x$

$y \ge (x + 2)^2 - 1$



12. $y > x^2 - 7x - 8$

$y < -x^2 + 6x + 5$



In Exercises 13–15, solve the inequality algebraically.

13. $16x^2 > 100$

14. $x^2 \le 15x - 34$

15. $-\frac{1}{5}x^2 + 10x \ge -25$

16. The profit for a hot dog company is given by the equation $y = -0.02x^2 + 140x - 2500$, where x is the number of hot dogs produced and y is the profit (in dollars). How many hot dogs must be produced so that the company will generate a profit of at least \$150,000?