3.2 Complex Numbers
For use with Exploration 3.2

Essential Question  What are the subsets of the set of complex numbers?

Work with a partner. Determine which subsets of the set of complex numbers contain each number.

a. \( \sqrt{9} \)  
b. \( \sqrt{0} \)  
c. \( -\sqrt{4} \)

d. \( \sqrt{\frac{4}{9}} \)  
e. \( \sqrt{2} \)  
f. \( \sqrt{-1} \)
EXPLORATION: Complex Solutions of Quadratic Equations

Work with a partner. Use the definition of the imaginary unit $i$ to match each quadratic equation with its complex solution. Justify your answers.

a. $x^2 - 4 = 0$

b. $x^2 + 1 = 0$

c. $x^2 - 1 = 0$

d. $x^2 + 4 = 0$

e. $x^2 - 9 = 0$

f. $x^2 + 9 = 0$

A. $i$

B. $3i$

C. 3

D. $2i$

E. 1

F. 2

Communicate Your Answer

3. What are the subsets of the set of complex numbers? Give an example of a number in each subset.

4. Is it possible for a number to be both whole and natural? natural and rational? rational and irrational? real and imaginary? Explain your reasoning.
3.2 Notetaking with Vocabulary
For use after Lesson 3.2

In your own words, write the meaning of each vocabulary term.

imaginary unit \(i\)

complex number

imaginary number

pure imaginary number

Core Concepts
The Square Root of a Negative Number

Property \hspace{1cm} Example
1. If \(r\) is a positive real number, then \(\sqrt{-r} = i\sqrt{r}\). \(\sqrt{-3} = i\sqrt{3}\)

2. By the first property, it follows that \(\left(i\sqrt{r}\right)^2 = -r\). \(\left(i\sqrt{3}\right)^2 = i^2 \cdot 3 = -3\)

Notes:
3.2 Notetaking with Vocabulary (continued)

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: \((a + bi) + (c + di) = (a + c) + (b + d)i\)

Difference of complex numbers: \((a + bi) - (c + di) = (a - c) + (b - d)i\)

Notes:

Extra Practice

In Exercises 1–6, find the square root of the number.

1. \(\sqrt{-49}\)  
2. \(\sqrt{-4}\)  
3. \(\sqrt{-45}\)  
4. \(-2\sqrt{-100}\)  
5. \(6\sqrt{-121}\)  
6. \(5\sqrt{-75}\)

In Exercises 7 and 8, find the values of \(x\) and \(y\) that satisfy the equation.

7. \(-10x + i = 30 - yi\)  
8. \(44 - \frac{1}{2}yi = -\frac{1}{4}x - 7i\)
In Exercises 9–14, simplify the expression. Then classify the result as a real number or imaginary number. If the result is an imaginary number, specify if it is a pure imaginary number.

9. \((-8 + 3i) + (-1 - 2i)\)
10. \((36 - 3i) - (12 + 24i)\)

11. \((16 + i) + (-16 - 8i)\)
12. \((-5 - 5i) - (-6 - 6i)\)

13. \((-1 + 9i)(15 - i)\)
14. \((13 + i)(13 - i)\)

15. Find the impedance of the series circuit.

16. \(0 = 5x^2 + 25\)
17. \(x^2 - 10 = -18\)
18. \(-\frac{1}{3}x^2 = \frac{1}{5} + \frac{4}{3}x^2\)

19. Sketch a graph of a function that has two real zeros at \(-2\) and \(2\). Then sketch a graph on the same grid of a function that has two imaginary zeros of \(-2i\) and \(2i\). Explain the difference in the graphs of the two functions.