

**Chapter
2****Maintaining Mathematical Proficiency**

Find the x -intercept of the graph of the linear equation.

1. $y = 4x + 36$

2. $y = -3x + 5$

3. $y = -10x - 75$

4. $y = 2(x - 9)$

5. $y = -7(x + 10)$

6. $5x + 15y = 60$

Find the distance between the two points.

7. $(1, 3), (-2, 8)$

8. $(-5, 0), (-9, 2)$

9. $(3, 7), (10, 4)$

10. $(6, -2), (-3, 0)$

11. $(9, -1), (9, 8)$

12. $(0, 5), (-4, -6)$

13. A student uses the Distance Formula to find the distance between two points (a, b) and (c, d) . What does the step $\sqrt{(c - a)^2 + (0)^2}$ tell the student about the relationship between the two points? How could the student have found the distance between the two points using another method?

2.1

Transformations of Quadratic Functions

For use with Exploration 2.1

Essential Question How do the constants a , h , and k affect the graph of the quadratic function $g(x) = a(x - h)^2 + k$?

1 EXPLORATION: Identifying Graphs of Quadratic Functions

Work with a partner. Match each quadratic function with its graph. Explain your reasoning. Then use a graphing calculator to verify that your answer is correct.

a. $g(x) = -(x - 2)^2$

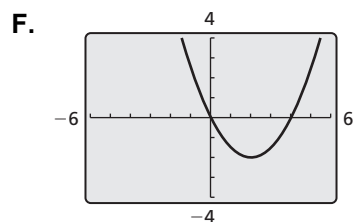
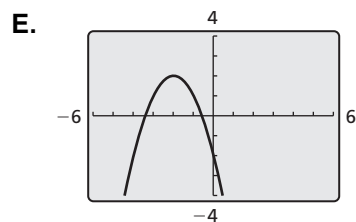
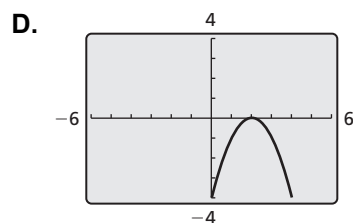
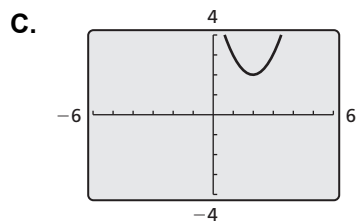
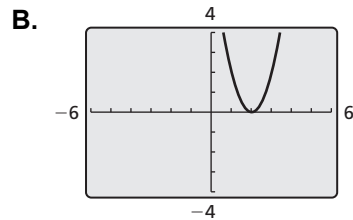
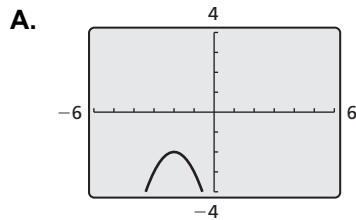
b. $g(x) = (x - 2)^2 + 2$

c. $g(x) = -(x + 2)^2 - 2$

d. $g(x) = 0.5(x - 2)^2 - 2$

e. $g(x) = 2(x - 2)^2$

f. $g(x) = -(x + 2)^2 + 2$

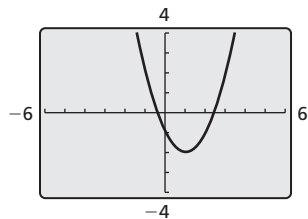


2.1 Transformations of Quadratic Functions (continued)**Communicate Your Answer**

2. How do the constants a , h , and k affect the graph of the quadratic function

$$g(x) = a(x - h)^2 + k?$$

3. Write the equation of the quadratic function whose graph is shown. Explain your reasoning. Then use a graphing calculator to verify that your equation is correct.



2.1**Notetaking with Vocabulary**

For use after Lesson 2.1

In your own words, write the meaning of each vocabulary term.

quadratic function

parabola

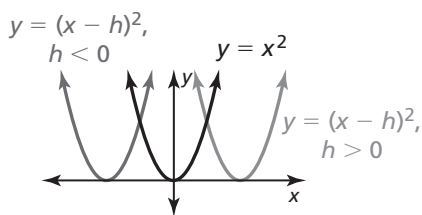
vertex of a parabola

vertex form

Core Concepts**Horizontal Translations**

$$f(x) = x^2$$

$$f(x - h) = (x - h)^2$$

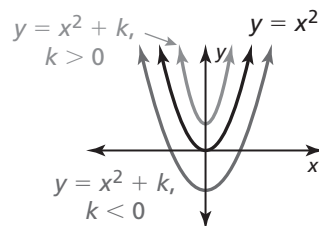


- shifts left when $h < 0$
- shifts right when $h > 0$

Vertical Translations

$$f(x) = x^2$$

$$f(x) + k = x^2 + k$$



- shifts down when $k < 0$
- shifts up when $k > 0$

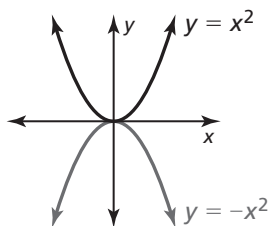
Notes:

2.1 Notetaking with Vocabulary (continued)

Reflections in the x-Axis

$$f(x) = x^2$$

$$-f(x) = -(x^2) = -x^2$$

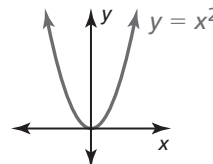


flips over the x -axis

Reflections in the y-Axis

$$f(x) = x^2$$

$$f(-x) = (-x)^2 = x^2$$

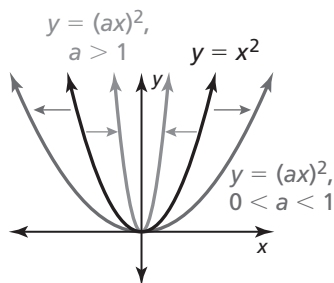


$y = x^2$ is its own reflection in the y -axis.

Horizontal Stretches and Shrinks

$$f(x) = x^2$$

$$f(ax) = (ax)^2$$

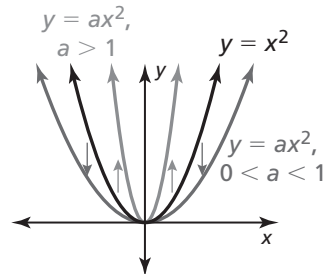


- horizontal stretch (away from y -axis) when $0 < a < 1$
- horizontal shrink (toward y -axis) when $a > 1$

Vertical Stretches and Shrinks

$$f(x) = x^2$$

$$a \cdot f(x) = ax^2$$



- vertical stretch (away from x -axis) when $a > 1$
- vertical shrink (toward x -axis) when $0 < a < 1$

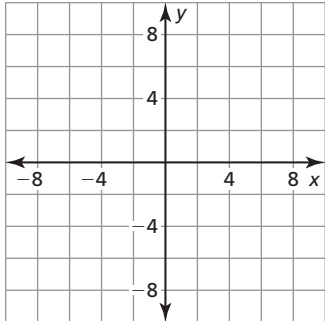
Notes:

2.1 Notetaking with Vocabulary (continued)**Extra Practice**

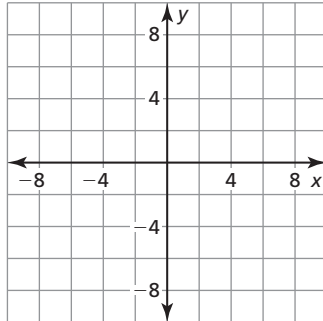
In Exercises 1–6, describe the transformation of $f(x) = x^2$ represented by g .

Then graph the function.

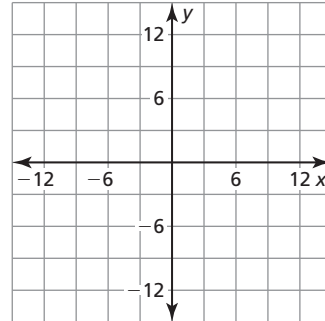
1. $g(x) = x^2 + 4$



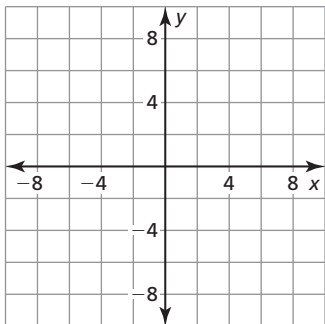
2. $g(x) = (x - 1)^2 - 3$



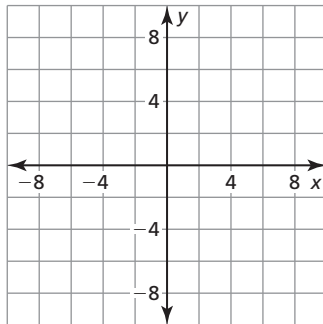
3. $g(x) = -(x + 9)^2$



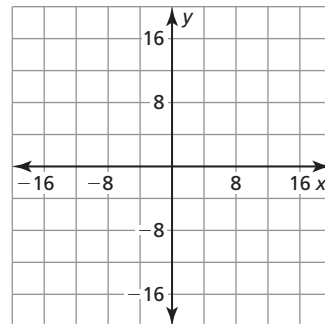
4. $g(x) = x^2 - 7$



5. $g(x) = \frac{1}{3}x^2 - 6$



6. $g(x) = (-4x)^2$



7. Consider the function $f(x) = -10(x - 5)^2 + 7$. Describe the transformation of the graph of the parent quadratic function. Then identify the vertex.

2.2

Characteristics of Quadratic Functions

For use with Exploration 2.2

Essential Question What type of symmetry does the graph of $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?

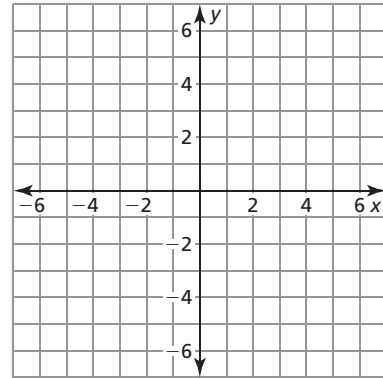
1 EXPLORATION: Parabolas and Symmetry

Work with a partner.

- a. Complete the table. Then use the values in the table to sketch the graph of the function $f(x) = \frac{1}{2}x^2 - 2x - 2$ on graph paper.

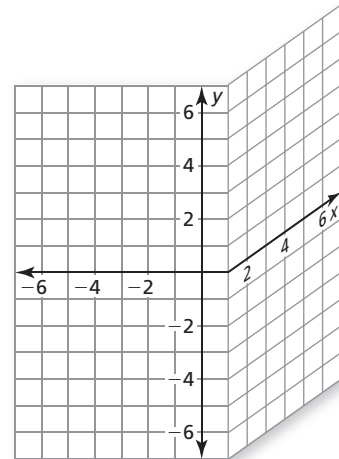
x	-2	-1	0	1	2
f(x)					

x	3	4	5	6
f(x)				



- b. Use the results in part (a) to identify the vertex of the parabola.

- c. Find a vertical line on your graph paper so that when you fold the paper, the left portion of the graph coincides with the right portion of the graph. What is the equation of this line? How does it relate to the vertex?



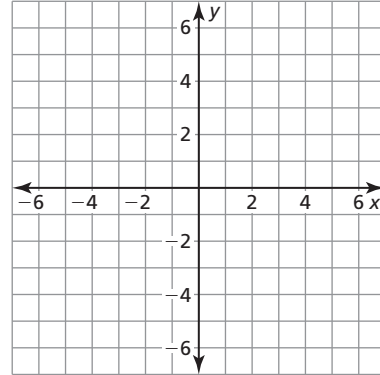
- d. Show that the vertex form $f(x) = \frac{1}{2}(x - 2)^2 - 4$ is equivalent to the function given in part (a).

2.2 Characteristics of Quadratic Functions (continued)**2** **EXPLORATION:** Parabolas and Symmetry

Work with a partner. Repeat Exploration 1 for the function given by $f(x) = -\frac{1}{3}x^2 + 2x + 3 = -\frac{1}{3}(x - 3)^2 + 6$.

x	-2	-1	0	1	2
f(x)					

x	3	4	5	6
f(x)				

**Communicate Your Answer**

3. What type of symmetry does the graph of the parabola $f(x) = a(x - h)^2 + k$ have and how can you describe this symmetry?
4. Describe the symmetry of each graph. Then use a graphing calculator to verify your answer.
- a. $f(x) = -(x - 1)^2 + 4$ b. $f(x) = (x + 1)^2 - 2$ c. $f(x) = 2(x - 3)^2 + 1$
- d. $f(x) = \frac{1}{2}(x + 2)^2$ e. $f(x) = -2x^2 + 3$ f. $f(x) = 3(x - 5)^2 + 2$

2.2**Notetaking with Vocabulary**

For use after Lesson 2.2

In your own words, write the meaning of each vocabulary term.

axis of symmetry

standard form

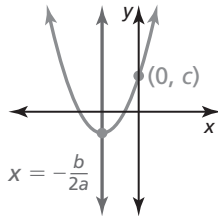
minimum value

maximum value

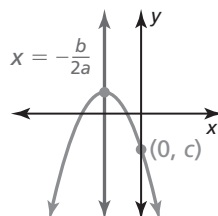
intercept form

Core Concepts**Properties of the graph of $f(x) = ax^2 + bx + c$**

$$y = ax^2 + bx + c, a > 0$$



$$y = ax^2 + bx + c, a < 0$$



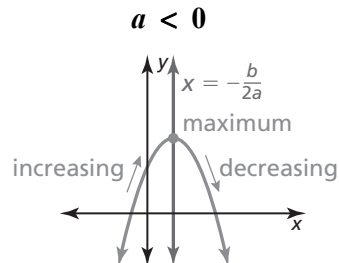
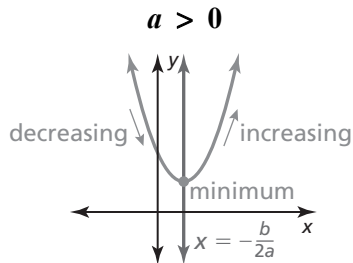
- The parabola opens up when $a > 0$ and open down when $a < 0$.
- The graph is narrower than the graph of $f(x) = x^2$ when $|a| > 1$ and wider when $|a| < 1$.
- The axis of symmetry is $x = -\frac{b}{2a}$ and the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- The y-intercept is c . So, the point $(0, c)$ is on the parabola.

Notes:

2.2 Notetaking with Vocabulary (continued)

Minimum and Maximum Values

For the quadratic function $f(x) = ax^2 + bx + c$, the y -coordinate of the vertex is the **minimum value** of the function when $a > 0$ and the **maximum value** when $a < 0$.



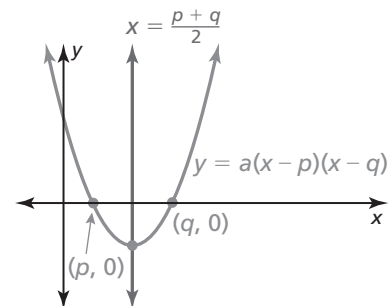
- Minimum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \geq f\left(-\frac{b}{2a}\right)$
- Decreasing to the left of $x = -\frac{b}{2a}$
- Increasing to the right of $x = -\frac{b}{2a}$

- Maximum value: $f\left(-\frac{b}{2a}\right)$
- Domain: All real numbers
- Range: $y \leq f\left(-\frac{b}{2a}\right)$
- Increasing to the left of $x = -\frac{b}{2a}$
- Decreasing to the right of $x = -\frac{b}{2a}$

Notes:

Properties of the graph of $f(x) = a(x - p)(x - q)$

- Because $f(p) = 0$ and $f(q) = 0$, p and q are the x -intercepts of the graph of the function.
- The axis of symmetry is halfway between $(p, 0)$ and $(q, 0)$.
So, the axis of symmetry is $x = \frac{p + q}{2}$.
- The parabola opens up when $a > 0$ and opens down when $a < 0$.



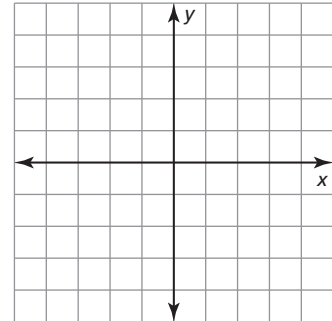
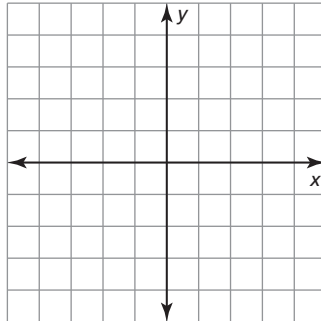
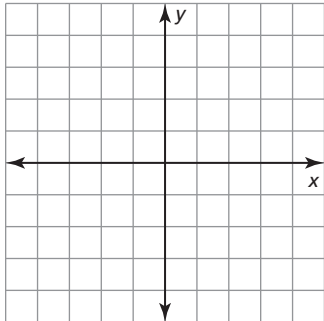
Notes:

2.2 Notetaking with Vocabulary (continued)

Extra Practice

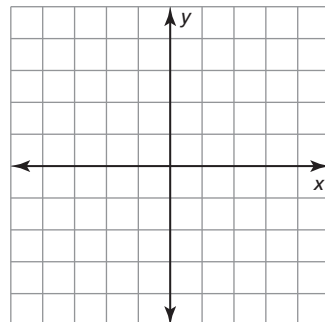
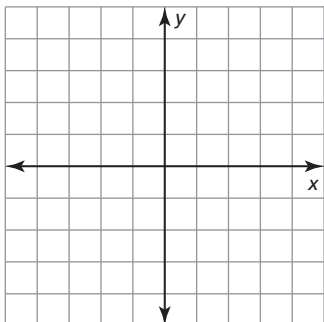
In Exercises 1–3, graph the function. Label the vertex and axis of symmetry. Find the minimum or maximum value of the function. Describe the domain and range of the function, and where the function is increasing and decreasing.

1. $f(x) = (x + 1)^2$ 2. $y = -2(x - 4)^2 - 5$ 3. $t(x) = \frac{3}{2}x^2 - 3x - 1$



In Exercises 4 and 5, graph the function. Label the x-intercept(s), vertex, and axis of symmetry.

4. $f(x) = 4(x + 4)(x - 3)$ 5. $f(x) = -7x(x - 6)$



6. A softball player hits a ball whose path is modeled by $f(x) = -0.0005x^2 + 0.2127x + 3$, where x is the distance from home plate (in feet) and y is the height of the ball above the ground (in feet). What is the highest point this ball will reach? If the ball was hit to center field which has an 8 foot fence located 410 feet from home plate, was this hit a home run? Explain.

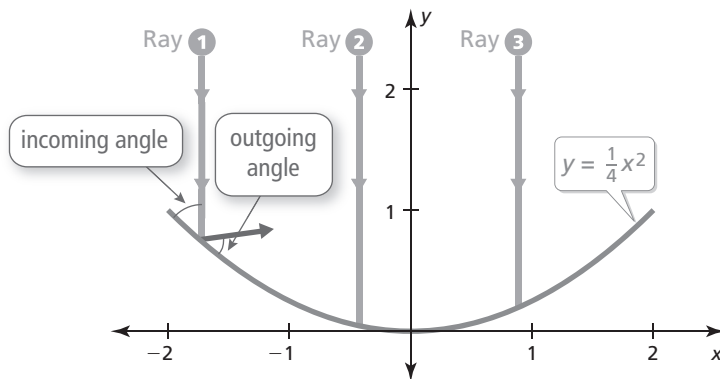
2.3**Focus of a Parabola**

For use with Exploration 2.3

Essential Question What is the focus of a parabola?**1 EXPLORATION:** Analyzing Satellite DishesGo to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Vertical rays enter a satellite dish whose cross section is a parabola. When the rays hit the parabola, they reflect at the same angle at which they entered. (See Ray 1 in the figure.)

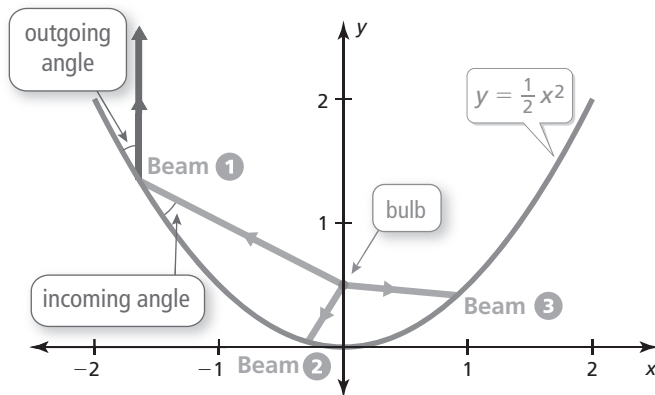
- Draw the reflected rays so that they intersect the y -axis.
- What do the reflected rays have in common?
- The optimal location for the receiver of the satellite dish is at a point called the *focus* of the parabola. Determine the location of the focus. Explain why this makes sense in this situation.



2.3 Focus of a Parabola (continued)**2** **EXPLORATION:** Analyzing Spotlights

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Beams of light are coming from the bulb in a spotlight, located at the focus of the parabola. When the beams hit the parabola, they reflect at the same angle at which they hit. (See Beam 1 in the figure.) Draw the reflected beams. What do they have in common? Would you consider this to be the optimal result? Explain.

**Communicate Your Answer**

3. What is the focus of a parabola?
4. Describe some of the properties of the focus of a parabola.

2.3

Notetaking with Vocabulary

For use after Lesson 2.3

In your own words, write the meaning of each vocabulary term.

focus

directrix

Core Concepts

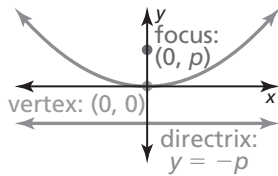
Standard Equations of a Parabola with Vertex at the Origin

Vertical axis of symmetry ($x = 0$)

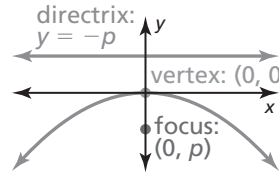
Equation: $y = \frac{1}{4p}x^2$

Focus: $(0, p)$

Directrix: $y = -p$



$p > 0$



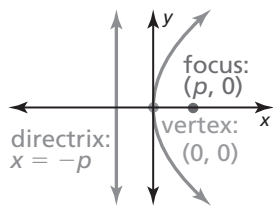
$p < 0$

Horizontal axis of symmetry ($y = 0$)

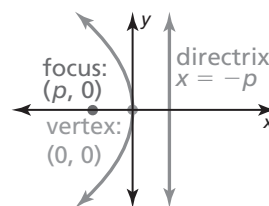
Equation: $x = \frac{1}{4p}y^2$

Focus: $(p, 0)$

Directrix: $x = -p$



$p > 0$



$p < 0$

Notes:

2.3 Notetaking with Vocabulary (continued)

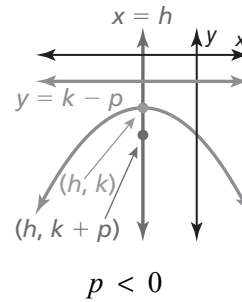
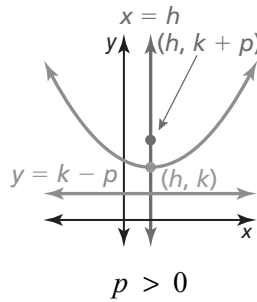
Standard Equations of a Parabola with Vertex at (h, k)

Vertical axis of symmetry ($x = h$)

Equation: $y = \frac{1}{4p}(x - h)^2 + k$

Focus: $(h, k + p)$

Directrix: $y = k - p$

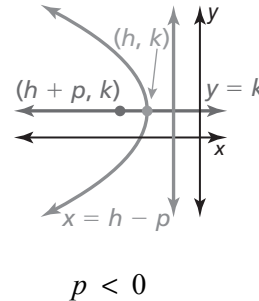
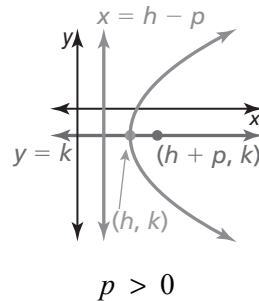


Horizontal axis of symmetry ($y = k$)

Equation: $x = \frac{1}{4p}(y - k)^2 + h$

Focus: $(h + p, k)$

Directrix: $x = h - p$



Notes:

Extra Practice

In Exercises 1 and 2, use the Distance Formula to write an equation of the parabola.

1. focus: $(0, -8)$ directrix: $y = 8$

2. vertex: $(0, 0)$ focus: $(0, 1)$

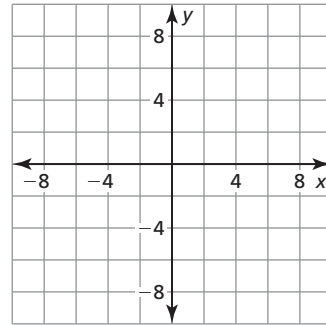
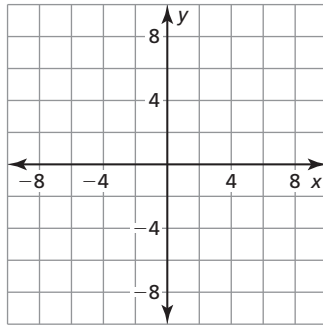
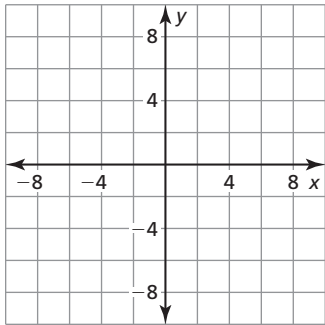
2.3 Notetaking with Vocabulary (continued)

In Exercises 3–5, identify the focus, directrix, and axis of symmetry of the parabola. Graph the equation.

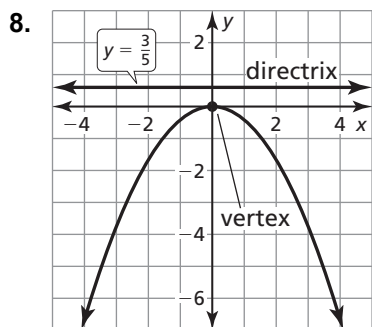
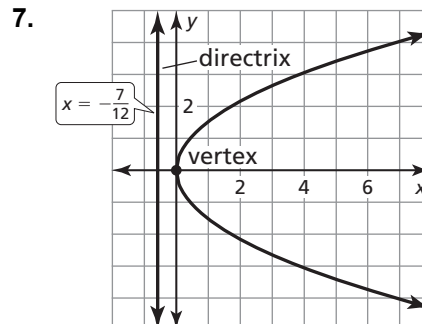
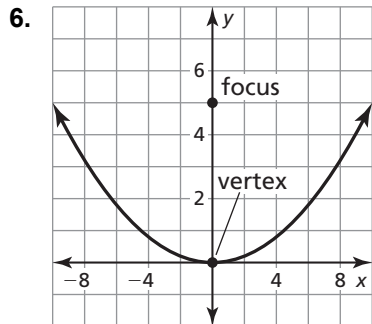
3. $x^2 = -2y$

4. $-5x + \frac{1}{3}y^2 = 0$

5. $y = -2(x + 1)^2 - 3$



In Exercises 6–8, write an equation of the parabola shown.



9. The cross section of a parabolic sound reflector at the Olympics has a diameter of 20 inches and is 25 inches deep. Write an equation that represents the cross section of the reflector with its vertex at $(0, 0)$ and its focus to the left of the vertex.

2.4

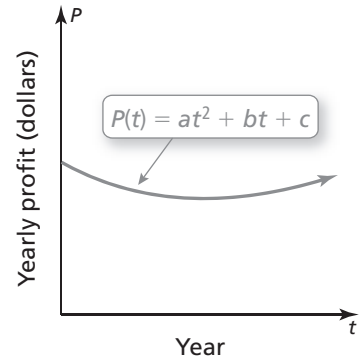
Modeling with Quadratic Functions

For use with Exploration 2.4

Essential Question How can you use a quadratic function to model a real-life situation?

1 EXPLORATION: Modeling with a Quadratic Function

Work with a partner. The graph shows a quadratic function of the form $P(t) = at^2 + bt + c$ which approximates the yearly profits for a company, where $P(t)$ is the profit in year t .



- a. Is the value of a positive, negative, or zero? Explain.
- b. Write an expression in terms of a and b that represents the year t when the company made the least profit.
- c. The company made the same yearly profits in 2004 and 2012. Estimate the year in which the company made the least profit.
- d. Assume that the model is still valid today. Are the yearly profits currently increasing, decreasing, or constant? Explain.

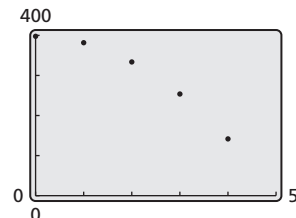
2 EXPLORATION: Modeling with a Graphing Calculator

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. The table shows the heights h (in feet) of a wrench t seconds after it has been dropped from a building under construction.

Time, t	0	1	2	3	4
Height, h	400	384	336	256	144

- a. Use a graphing calculator to create a scatter plot of the data, as shown at the right. Explain why the data appear to fit a quadratic model.



2.4 Modeling with Quadratic Functions (continued)**2** **EXPLORATION:** Modeling with a Graphing Calculator (continued)

- b. Use the *quadratic regression* feature to find a quadratic model for the data.

- c. Graph the quadratic function on the same screen as the scatter plot to verify that it fits the data.

- d. When does the wrench hit the ground? Explain.

Communicate Your Answer

3. How can you use a quadratic function to model a real-life situation?

4. Use the Internet or some other reference to find examples of real-life situations that can be modeled by quadratic functions.

2.4**Notetaking with Vocabulary**

For use after Lesson 2.4

In your own words, write the meaning of each vocabulary term.

average rate of change

system of three linear equations

Core Concepts**Writing Quadratic Equations****Given a point and the vertex (h, k)** Use vertex form: $y = a(x - h)^2 + k$ **Given a point and x-intercepts p and q** Use intercept form: $y = a(x - p)(x - q)$ **Given three points**

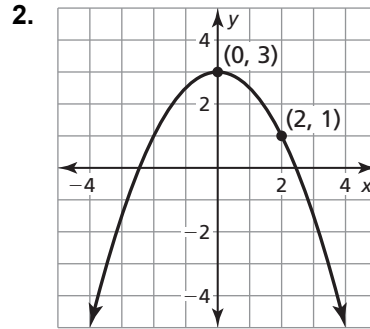
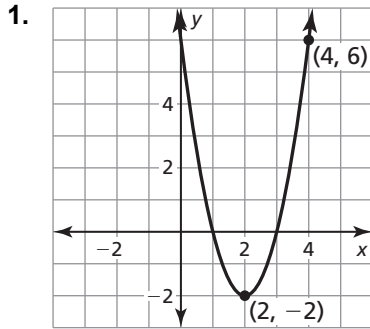
Write and solve a system of three equations in three variables.

Notes:

2.4 Notetaking with Vocabulary (continued)

Extra Practice

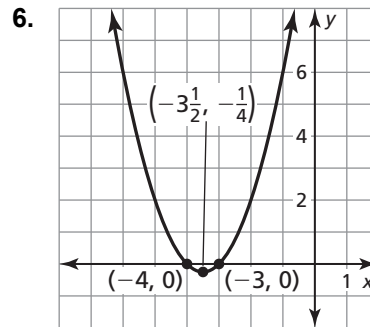
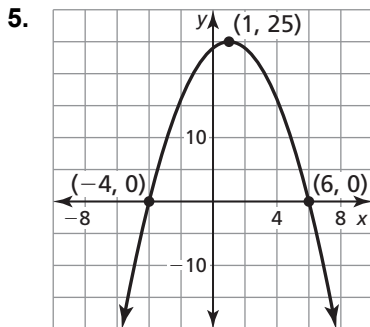
In Exercises 1–4, write an equation of the parabola in vertex form.



3. passes through $(-3, 0)$ and has vertex $(-1, -8)$

4. passes through $(-4, 7)$ and has vertex $(-2, 5)$

In Exercises 5–8, write an equation of the parabola in intercept form.



7. x-intercepts of -5 and 8 ; passes through $(1, 84)$

8. x-intercepts of 7 and 10 ; passes through $(-2, 27)$

2.4 Notetaking with Vocabulary (continued)

In Exercises 9–11, analyze the differences in the outputs to determine whether the data are *linear*, *quadratic* or *neither*. If linear or quadratic, write an equation that fits the data.

9.

Time (seconds), x	1	2	3	4	5	6
Distance (feet), y	424	416	376	304	200	64

10.

Time (days), x	0	3	6	9	12	15
Height (inches), y	36	30	24	18	12	6

11.

Time (years), x	1	2	3	4	5	6
Profit (dollars), y	5	15	45	135	405	1215

12. The table shows a university's budget (in millions of dollars) over a 10-year period, where $x = 0$ represents the first year in the 10-year period.

Years, x	0	1	2	3	4	5	6	7	8	9
Budget, y	65	32	22	40	65	92	114	128	140	150

- Use a graphing calculator to create a scatter plot. Which better represents the data, a line or a parabola? Explain.
- Use the *regression* feature of your calculator to find the model that best fits the data.
- Use the model in part (b) to predict when the budget of the university is \$500,000,000.00.